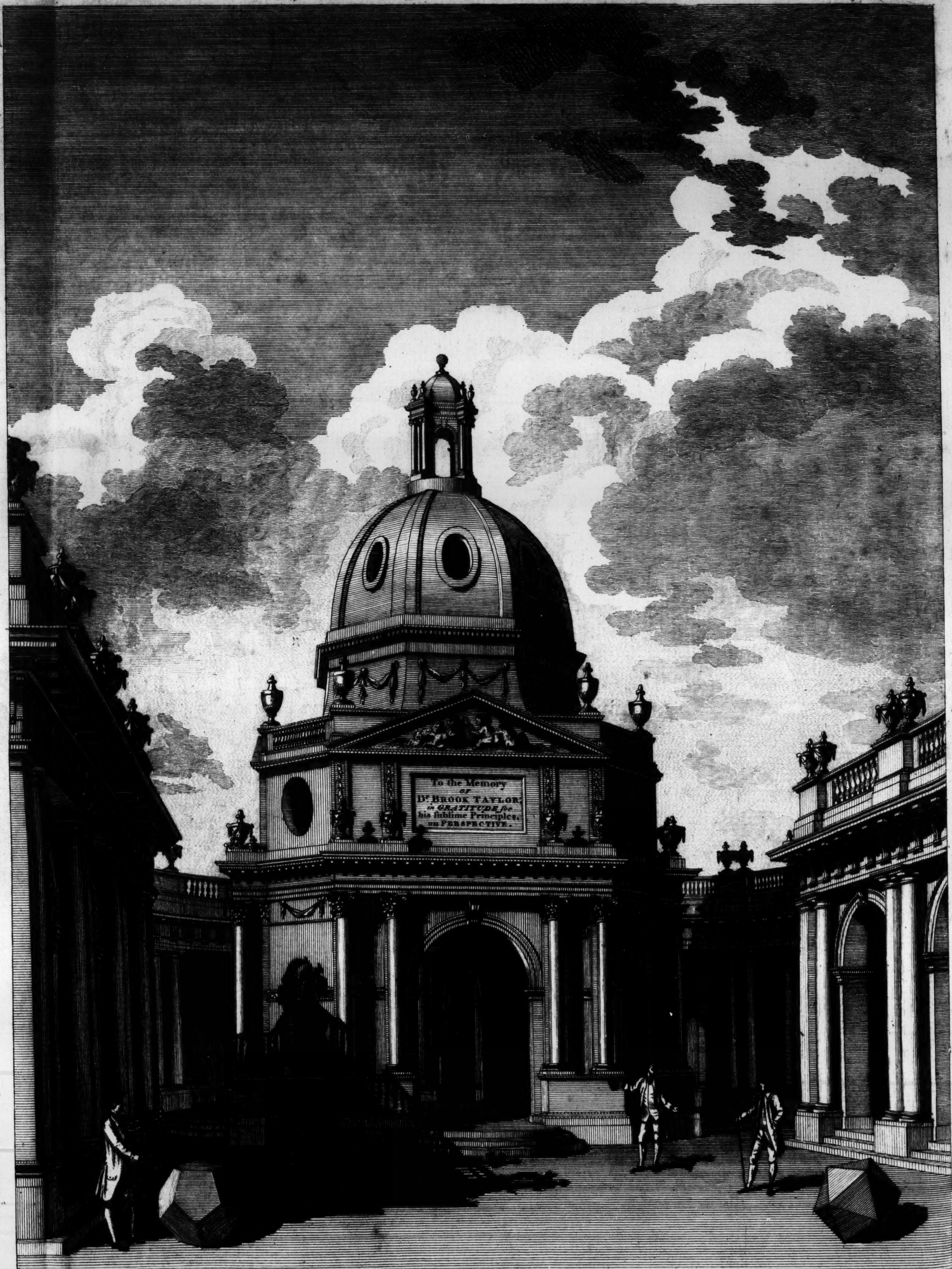

P. Mazell sculp.

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A

COMPLEAT TREATISE
ON
PERSPECTIVE,
IN
THEORY AND PRACTICE;
ON THE PRINCIPLES OF
DR. BROOK TAYLOR.
MADE CLEAR, BY
VARIOUS MOVEABLE SCHEMES, AND DIAGRAMS,
IN THE
MOST INTELLIGENT MANNER.

IN FOUR BOOKS.

EMBELLISHED WITH
AN ELEGANT FRONTISPIECE AND FORTY-EIGHT PLATES.

CONTAINING
DIAGRAMS, VIEWS, AND ORIGINAL DESIGNS, IN ARCHITECTURE, &c. BY THE AUTHOR;
ELEGANTLY ENGRAVED.

BOOK I.

Treats on Optics and Viſion, a neceſſary Introduction to the Theory of Perſpective; and contains ſome Objections to the received Opinions of Light and Colour; Reflection, &c.

BOOK II.

Contains the whole uſeful Theory of Perſpective, both rectilinear and curvilinear; with Remarks, and familiar Examples, to illuſtrate and evince the univerſality of its Principles; with a full refutation of the abſurd opinions which ſeveral Perſons entertain of Perſpective.

BOOK III.

Is a copious Treatiſe on practical Perſpective. In which, is firſt diſplayed the true Elements of the whole, as deduced from the foregoing Theory; their extenſive application is pointed out, and exemplified throughout the whole Book; and, by the moſt ſimple means poſſible, is ſhewn how to project, perſpectively, all kinds of regular objects, from the ſimpleſt to the moſt complex; alſo, how far it is applicable to irregular Objects. Comprized in twelve Sections, on various Subjects, and adapted to various Profeſſions.

BOOK IV.

Treats on ſhadows in general, in Theory and Practice, projected by the Sun, alſo by a Torch or Candle; of reflected Light on Objects, and the reflected Images of Objects, on the Surface of Water, and poliſhed, plane Surfaces, of Aireal Perſpective, or the effect of Diſtance, &c. In ſix Sections, containing nine Plates, which illuſtrate the whole.

By THOMAS MALTON.

LONDON:
Printed for the Author; and ſold by Meſſrs. Robson, in Bond-ſtreet; Dodsley, Pall-Mall; Becket, Adelphi, Strand; Taylor, near Great Turn-Stile, Holbourn; Richardson and Urquhart, Royal Exchange; and by the Author, No. 3, Poland-Street, Oxford Road, near the Pantheon.
MDCCLXXV.


G.R

TO THE

PRESIDENT AND MEMBERS,

OF THE

ROYAL ACADEMY,

For PAINTING, SCULPTURE, and ARCHITECTURE;

INSTITUTED AT LONDON,

By, and under the Auſpices of, his moſt gracious MAJESTY

GEORGE THE THIRD.

Gentlemen,

IF the unremited labour and aſſiduity, with which I have proſecuted the ſtudy of the Science and Art of Perſpective, have thrown any new Light on that moſt neceſſary branch of the Polite Arts, ſo as to render its Principles clearer and better underſtood, in Theory, more eaſily applicable in Practice, more generally uſeful and ſubſervient to the Arts, of which it muſt be allowed to be the foundation, 'tis what I have chiefly aimed at, and preſume, this Work may not be wholly undeſerving your Patronage and Encouragement; although it may not merit your entire Approbation. Such as it is, Gentlemen, I ſubmit to your Candour, and claim your Protection.

The propriety of dedicating ſuch a Work to Gentlemen, who are, undoubtedly, the moſt competent Judges of it, will plead an excuſe for my preſumption; if, on an accurate examination of its Contents, it be found, that I have rendered an apparently intricate Science more familiar, and better adapted to the capacities of young Students, in ſo eſſential a part of their Studies; the neglect of which, amongſt the riſing Artiſts, is much to be lamented. Perſpective ſeems to be looked on as an Appendage, only, which may be diſpenſed with, inſtead of the firſt requiſite; in which, the Student, who would be a Candidate for Fame, ſhould be well grounded.

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It is with reluctance I add, but it is too obvious, that, in many fine Pieces, in respect of Design, Composition, Drawing, or Colouring, there seems to be a want of a just Idea of Perspective. Excuse me, Gentlemen, I do not mean to depreciate the Merits of such experienced Artists, in their several Performances; for, in my opinion, there only wants a thorough knowledge of Perspective, to render the present Age as famous as any former Æra; which, now, is established on the most permanent and infallible Principles, whereby, the trouble of projecting Objects, perspectively, is, in the process, greatly abridged and facilitated.

I am,

Gentlemen,

with great respect,

Your most obedient,

and obliged humble Servant,

THOMAS MALTON.

BOOK I.

OPTICS or VISION.

SECTION I.

Of LIGHT and COLOUR.

To treat on any Science, properly, it is necessary to begin with the Elements of it, or first Principles; which is the reason, as I suppose, that some writers, on Perspective, have prefaced their Works with a chapter on Optics and Vision. Some of whom, not daring to advance any thing of their own, on the subject, have favoured the world with extracts from Sir Isaac Newton's and Smith's Optics; which, indeed, is very little to the purpose, as the much greater part is no way subservient to Perspective; considering it not merely optical, but purely a mathematical Science, supported on the firm Basis of Geometry.

The ingenious Mr. Hamilton, in his compleat body of Stereography (a work which does great credit to the Author) to which I acknowledge myself indebted, has given some judicious observations on the subject of Vision; and some, which are rather exceptionable. But, the Theory of Colours (if that can be called a Theory, which is so little known, and nothing in it demonstrable) he, very wisely, declines enlarging on; as not being subservient to Perspective, nor indeed to Painting: I mean the Theory of the Prism, which, Sir Isaac Newton and others have so copiously expatiated on.

I would by no means have it thought that I intend, or have the least desire, to derogate from the character of so great a Man, whose name I highly revere; but I am persuaded, that the pursuits of the greatest Men are, sometimes, in themselves, trivial and merely amusing; when the character of a great Man is established, it gives a sanction of consequence to his most insignificant amusements; for, why may not the most profound reasoners relax from their intense studies, and amuse themselves, sometimes, with trifling matters? by which means things of great utility have been brought to light; as must have been the case in respect of the Prism: nor do a find that he has perfected what he had in view, but left his observations for others to improve on, having (in his own Words) no inclination to take it up again, and pursue it further, as he had intended.

Had the Theory of Colours, as deduced from the Prism, been amongst the first and chief of this great Man's pursuits, I am much in doubt, if the reputation he has acquired had ever been established, at least on that Basis; things of infinitely more importance to the Community, fixed his credit (most deservedly) on the highest pinnacle of Fame; for, what useful and necessary knowledge has been communicated to mankind, by this acquisition to the Science of Optics? which (with such, apparently wonderous, sagacity and penetration) he has explored and given to the world.

To define, with any degree of precision and perspicuity, what Light is, is not possible; seeing we cannot comprehend, and enter into an accurate disquisition of Fire; which, is the only cause we can conceive of Light. From the different and obstinate refrangibility of the different Colours produced by a Prism, when applied to the Sun's Beams, it is concluded, that Light is, in its nature, heterogeneous; that it is composed of Particles, of different qualities, which produce very different effects on Objects. I cannot, from any thing I have yet seen or read on the Subject, give my assent to that opinion; for, I believe, that, what we call Light (the vivid glow occasioned by any luminous body) is perfectly homogenial;

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and that it is not composed of Particles; which implies that it is material, a Body; for, how else can it be a composition of Particles? which, by means of the Prism, are supposed to be dissevered, and separated from each other.

But, what reason can be assigned, that a Prism should have this most extraordinary power of separating them? owing to its form, only *, not the matter of which it is composed: But it is not owing to either, as it is manifest by Experiment. Allowing the rays of Light to be thus sifted or separated, what use is made of it, or what further knowledge is deduced from it?

'Tis obvious, that the same Object, having its surfaces differently disposed, or situated, exhibits different Colours, according as its surfaces are situated to the Light; that is, to that quarter, from which any thing luminous causes an illumination of them; and those parts, which are most directly opposed to it, are more intense in Colour; whilst the opposite faces (the Light being obstructed by the Object) are almost deprived of Colour; which they would be entirely, but for other Objects in vicinity with and opposed to them: which other Objects being strongly illumined, are said to reflect Light to them. Hence, a conclusion is drawn, that Objects have, in themselves, no inherent Colour; but that, all the sense we have of different Colours, in the same or in different Objects, is entirely owing to their different qualities and modifications (or rather, to the construction of their Surfaces, simply) by which they are fitted and disposed to reflect different coloured Rays, some more copiously than others. But, it does not necessarily follow, that they have no inherent Colour; for although Objects, when opposed to the Sun (that is, when nothing opake intervenes) exhibit very different, and more brilliant colours than otherwise; yet, if its Rays are obstructed, instantaneously (the Light, around them, being then supposed stagnated) they still exhibit Colour, and of the same kind, or hue, though less vivid, and greatly inferior in the degree of it.

The chief Argument advanced in favour, and the only reason assigned in support of their Hypothesis is, that Objects exhibit no Colour, when they are entirely deprived of Light; which argument, cannot be denied; because, no Object, in such Case, is perceptible, by means of Vision; without Light our Optics are of no use at all. It is the same if we close our eyes, when Objects are fully illumined; but, certainly, the Colour, as well as the Object, remains, whether, by shuting our Eyes, we perceive it or not. I look on this as a specious and subtle, not to call it a sophistical way of reasoning; since without Light, there can be no Vision; and, consequently the sense of Colour, more than of the Figure, Magnitude, or Situation of Objects, cannot be communicated. But, if Light be considered as a Medium existing of itself, without the Luminary, how can there be a deprivation of it, if it fills the whole surrounding space, as Air does? which, seeing that, Air is also considered as a material Body, is somewhat repugnant to the established, and universally received Maxim, that two Bodies cannot fill or occupy the same space, at the same time. And, if Light be a Medium, of what use is it, without the Luminary? or, how can there be a perpetual emission, from the Luminary, the whole space being, every where, filled with Light? and, if it be material, what becomes of it? To suppose that it is absorbed, by the Earth and other Planets, would be ridiculous in the higest degree; because, the whole surfaces of all the Planets bear no sensible proportion to the immense space between them, in which there is no obstruction to the progression of Light, flowing from the Luminary in all directions.

That Light should exist in darkness (notwithstanding the scriptures say, that Light was created before the Sun) is not only a direct contradiction in terms, but to reason and common sense, and the nature of things. Also, that Objects exhibit no Colour,

* A Prism, as defined by Euclid (Def. 13. 11.), is a Solid, contained by Planes, of which, two opposite are equal, similar, and parallel; all the rest are Parallellograms; so that, any right angled Parallelopiped is a Prism, by which no effect of colour is produced. Now, as it depends, solely, on the inclination of one Plane to another, an acute angled Pyramid (in which all the planes are triangles) or frustum of a Pyramid, is the same, in respect of colour, as a Prism.

Colour, when deprived of Light is certain; because they are no longer visible; but, since every other quality remains, it is most probable (Colour being perceptible by sight only) were they objects of Vision, in total Darkness, Colour would remain likewise: For, though Colour cannot be perceived without Light, yet it is not the efficient cause of different Colours, in Objects composed of different materials.

In treating on Colours, the learned Boyle gives us a wonderful account of a blind man's distinguishing Colours by the touch; which, can only be, by the asperity or roughness of the surface of the coloured Body; and which, is construed as favouring their Opinion. It may be possible, that a difference in Colours, artificially laid on the surfaces of Bodies, may be felt; or, that the qualities of dies or stains may also affect Bodies so, as to make them sensible to the touch. But, will any person venture to say, that, if Bodies, stained or otherwise coloured, were polished or made equally smooth, the difference, in Colour, could be perceived by feeling; or, that the natural Colours of Wood, Metals, or Stones, nearly of the same qualities, except Colour, when equally polished, could be distinguished by that mode of Perception? I affirm they could not; nor can I give any credit to such assertions. Then, since that, Bodies, equally hard and equally polished, exhibit different Colours, I see no reason for supposing, that the cause of their differing in Colour is owing to the different texture or construction of their Surfaces; or, that it is possible to distinguish them by their asperity; for, Bodies that are of equal hardness, and being equally polished, must have Surfaces alike, except in Figure and Colour.

Light, however propagated, has, I presume, the same properties and influence, and, consequently, acts the same on Bodies. I would ask then, what is the reason, why Blue and Green are scarce distinguishable by the light of Fire? If the action and reaction of Light, on the surfaces of Bodies, be uniform, by a certain law, according as they are fitted and disposed to reflect any particular species of colour-making Rays, whence arises the difference in this case? I should be glad to hear or read an ingenious disquisition of that point; as, I do not doubt, there are Men of sagacity and penetration sufficient to set it in the clearest light, and make it as evident, to the understanding, as any common Case in any Science whatever.

To return to the Prism. It is certainly true (and 'tis a curious and entertaining Experiment) that, through an acute angled Prism, we perceive Objects very differently coloured from what they really are, by Nature; also, by applying the Prism, properly, to the Sun's Beams, there is exhibited a curious and most extraordinary Phænomenon, viz. a diversity of Colours, the most lively and beautiful that can be conceived; and, the more uncoloured the Surface is, on which they fall, the more vivid they appear.

Now it is certain, that, here is a perception of Colour, on the surfaces of Bodies, which is not inherent; and so is there when the Sun's Beams, only, fall on them: yet, we cannot from thence conclude, that they have no inherent Colour. The Colours which the Prism exhibits are Red, Orange, Yellow, Green, Blue, Indico, and Violet or Purple. But why is there, here, made a distinction of seven Colours, when, in reality, there are not above three or four simple Colours in Nature? (unless Black and White may be called Colours) viz. Red, Yellow, Blue; and Green, which may be compounded of Blue and Yellow. Indico is Blue, Orange and Purple are Compounds, or mixed Colours. Now, if any Philosopher, or Artist whatever, can, from these three or four simple and brilliant Colours, produce all the variety which we see in Nature; or, did the Prism exhibit, distinctly, all the simple Colours which are in Nature, only, without mixture, I should then be better disposed to give credit to their Theory: or, when I can have conviction, that, a mixture of all these Colours, together, in any Ratio, will produce a perfect White (as Snow) I may then be a perfect Proselyte; till then, I am persuaded that I must dissent from their Opinions.

I shall, next, enquire, how a Prism, particularly, has the wonderful property of separating the different coloured Rays of Light. The Prism is a Body of Glass, or it may be of Chrystal, or other pellucid, uncoloured Stone;

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is the property then in Glafs, or Stone? No; in the Figure that is made of it, only; amazing! that the difpofal of the fame Matter into different forms, fhould produce fuch very different effects. Glafs, difpofed into a portion of a convex Sphere, will make Objects, feen through it, appear magnified; and oppofed to the Sun's Beams, will collect real Fire; aftonifhing indeed! If the Surfaces be concave, Objects, feen through them, appear lefs than they really are; in the forms of Prifms it has various effects; but, we do not fee the effect of exhibiting Colours through a right angled Prifm; I mean, a four fided one, whofe oppofite faces are parallel, and the Angles right ones; neither has it that effect through the right Angle, of a right angled, triangular Prifm, but, only through the acute Angles. The caufe, then, of exhibiting Colour, is not in the Matter, itfelf, or Figure, nor in the Surfaces, fimply, but in the inclination of the Surfaces to each other; for, being parallel, or at right angles, or nearly fo, they do not produce that effect; and, we may as well afk, why convex Surfaces collect Fire, rather than plane or other parallel Surfaces, as, why a Prifm, whofe Planes are inclined to each other, fhould have the property of exhibiting the different Colours I have mentioned, rather than one that is right angled; and I am perfuaded, that, with all the fagacity Man is endowed with, he will never be able to account, truly, for either.

Yet, I do not condemn all enquiries into the caufes of the various effects which we perceive in Nature, but think the purfuit rational, and truly commendable, when it is founded on certain Data, and real Hypothefis; and, the refult of our refearches productive of real utility, deducible from it, as in fome other branches of Optics. In the Theory of the Colours produced by a Prifm, there is none yet difcovered, and I do believe there never will; it has not the leaft apparent tendency to benefit mankind accruing from it. 'Tis afferted, that the perfection of Telefcopes is owing to the Theory of the Prifm. Now, as I am not converfant in the mechanical conftruction of Lenfes, and in their application to Telefcopes, I cannot fay how far it may have been of ufe in that refpect; but certain I am, that it is of no ufe to a Painter, to compound his Colours and form a Theory thereof; by means of which, he may fooner, and with certainty, arrive at perfection in his Art: and I muft needs fay, that the attempt made by Brook Taylor, in the Appendix to his fecond part, does not fhew the Doctor's judgment, in that, to be of a piece with the reft of the Work.

The Sun's Beams paffing through Glafs, whofe furfaces are neither parallel nor perpendicular to each other, or nearly fo, exhibit the various Colours, fpoken of above, in fome degree, according to the inclination of the oppofite Surfaces, and purity or clearnefs of the Glafs; through the thick part, near the knob, in the middle of a table of Crown Glafs, I have often obferved the Phænomenon, in a window of thofe Squares: but, it is alfo obfervable, that the effect ceafes when the Sun ceafes to fhine on them. It is alfo certain, that Objects appear coloured, otherwife than what Nature affigned them, when we look through Mediums denfer than Air; whofe furfaces, through which we look, are inclined to each other; and confequently, there will be the effect of Colour produced, in fome degree, which is not natural to Objects, in looking through Object-Glaffes or Lenfes, with which Telefcopes are conftructed; feeing that, their furfaces are inclined, at the edges. The bufinefs, therefore, of an Optician, in this refpect, is, fo to contrive and difpofe his Glaffes, as to diveft the Object, as much as poffible, of the borrowed Colours which do not belong to them, naturally; and I could almoft affirm, that the perfection, fpoken of, has been found out by repeated trials, not from any certain eftablifhed law, deduced from the Theory of the prifmatic Colours: for, if a certain Theory had been eftablifhed, what hindered the immediate perfection of them, as foon as the caufe of the imperfection was known, and certain laws eftablifhed, whereby, the unnatural effect of Colour might be removed from the field of View?

From what has been obferved, refpecting the effect of Colours produced by the Prifm (unlefs it can be proved that the particles of Air, through which Light muft neceffarily

necessarily pass, are Prisms) to infer, that Nature has given no inherent Colour to Objects, is bold and assuming; it is also groundless, seeing that, the Colour, which is natural to each, remains when the Sun does not shine on them, though it differs greatly in the degree of it. We may with as much reason conclude, that Objects are much larger than we perceive them to be with the naked Eye, because they appear so, when view'd through Glass whose surfaces are convex; and, we may as well enquire how Vegetables, as Flowers, Fruit, &c. spring out of the Earth, and are adorned with all that beautiful variety of Colours which we see in Nature; by what means they are continually varying in their Colours, from their first formation, in embryo, to maturity; as why, the Prism exhibits the various Colours of the Rainbow, to which they are, in a great measure, similar.

Colours produced by alkaline Salts, &c, and mixtures of different Fluids, with all such like chymical operations, I look on, equally, as entertaining Experiments; but no way productive of any advantage, accruing from it, to the Theory of Colours necessary for a Painter to know, in order to reduce it to real use, in practice.

I shall not, therefore, trespass any longer on the readers Time, as it is entirely foreign to the purpose of Perspective; but, confine my further observations to that part of Optics relative to Vision, only. In respect of direct Vision, it is certainly of use and subservient to Perspective; and essentially necessary to give a clear Idea of the Principles on which it exists, both in Theory and Practice.

Colour, whether inherent or incident to Bodies, is not material towards a perspective Description or Delineation; 'tis the Figure of the Object, only, it contemplates.

All Bodies, whatever, become the objects of Perception from their Figure or Colour. Figure is more particularly inseparable from our Ideas of Matter, than Colour; it being impossible, in the nature of things, to have any notion of Extention, abstracted from the Idea of Figure, which limits the Space Bodies occupy or fill. It is the same with or without Light, and may be perceived by the sense of feeling only. From the known external Figure and situations of Objects, may be delineated, by mathematical rules, a true perspective Representation, on a Plane; which, by the help of Light and Shade, will raise a perfect Idea in the Mind, of the figure of the Object, and the different forms and positions of its Surfaces, in respect of each other, without the assistance of Colour; but, to exhibit a true and natural Picture of Objects, and create a perfect Idea in every circumstance, Colour is absolutely necessary; and is the last degree of perfection, with which the omniscient Creator has embellished Nature, that can be given to a Picture.

Light, considered as a Medium by which Vision is convey'd to the Eye, is of too refined a nature for my speculations. But, as the great author of Nature has given a portion of reason to every human Being, we have certainly a right to make use of that reason; and, if we will exert it, properly, it is possible, nay certain, that one person may penetrate as deeply into the mysteries of Nature as another, tho' not bless'd with quite so much learning.

We are told by several great and profound Philosophers (for to be a Mathematician is not necessary) that we perceive Objects, only by means of Rays of Light, reflected from every point in their Surfaces to the Eye; which enter there, and form an Image, or Picture of the Objects perceived, on the Retina, or fine Membrane which surrounds all the back part of the Eye, internally. The Retina is said to proceed directly from the Optic Nerve, which dilates or spreads itself as before-mentioned; and, the impression being made thereon, we are further told, is conveyed, by the Optic Nerve, to the Brain or seat of Perception.

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But, why do they ſtop here? I expected and ſhould be glad to be convey'd into the inmoſt receſſes of the Brain, and be ſhewn or told, how the Image of the Object, on the Retina, is there perceived; for I muſt own it is aſtoniſhing! that, from what is perceiv'd or felt within, we ſhould have a true Idea of the Figure, Colour and Magnitude, Situation and Diſtance of Objects, which are external, or ſituate without the Eye.

Now, after all this parade, and pompous diſplay of great ſagacity and deep penetration, what does the Sum total amount to? Why, that the Object is perceived, i. e. the Mind is ſenſible of the exiſtence of ſuch Objects as it perceives; and, that the Viſion of them is convey'd in right or direct Lines from the Object to the Eye, or from that wonderous Organ to the Object; but how, or in what manner, remains as much a myſtery as before. So that, after all which has been ſaid on the Subject, and, allowing the Image, formed on the Retina (inverted or otherwiſe) as perfect as they pleaſe, what nearer are we? where is the Perception of that Image or Picture of the Object? how the Mind or Soul perceives the Image on the Retina, any more than the real Object, we are as much at a loſs to account for as ever.

In vain, therefore, does Man torment himſelf, in endeavouring to explore the hidden myſteries of Nature, which are for ever hid from our reſearches. Let us then purſue what is within our reach, and not an airy Phantom, which will ever elude the purſuit. In mathematical Sciences we have ſome certain Data, whereon to frame an Hypotheſis; and although we muſt not expect perfection, except in Theory, yet, we can diſcover ſo much of Truth as gives us ſufficient encouragement to purſue it, and makes ample amends for the imperfection of human nature.

I do not intend, nor ſhall I attempt to explain the nature of Viſion, or enquire into the true cauſe of it; being fully convinced of the inſufficiency of our reaſoning faculties, for ſuch a diſquiſition. It is ſufficient, for the ſcience of Perſpective, to know, or even to ſuppoſe that it is convey'd to the Eye, the ſeat of Sight, in Right Lines. But, whether it be by means of Rays of Light, reflected from all parts of Objects to the Eye (as is the general Opinion) or whether the Allwiſe former of the Eye has given a power to that truly wonderful and amazing Organ, to convey to the Mind, by Viſion, the perception of Objects by other means, I do not mean to make the ſubject of my enquiry. Yet, I muſt acknowledge, that I have ſtrong objections againſt the general Opinion, as it is now received and almoſt univerſally aſſented to; viz. that the perception we have of external Objects, from Viſion, is by means of Rays of Light, reflected from all parts of their Surfaces to the Eye; and, that thoſe Rays are material or compoſed of Matter.

But, as it is of no conſequence, in Perſpective, by what means Viſion is performed, ſo it be convey'd in Right Lines, which may paſs for an Axiom; ſeeing that, the refraction of the Rays (if there be any) in the Air which ſurrounds us, is ſo very little, in Objects that are at an immenſe diſtance beyond the whole Atmoſphere, it cannot be ſenſible at any Diſtance we can perceive Objects, within it, ſuppoſing the refraction uniform or regularly curved, between the Object and the Eye. But I rather ſuppoſe the refraction of the Rays to be at the common Surface of two Mediums, only; at entering any other Medium denſer or rarer than that through which they firſt paſs, and thence proceed again in Right Lines; conſequently, there is no refraction at all within the Medium; either in Air or Water, I ſuppoſe Viſion to be convey'd in Right Lines, if the Object and the Eye are both in the ſame Medium.

At the ſame time, I would not have it thought, that I ſuppoſe any Rays to paſs, by means of reflection, from opake Bodies; and in luminous ones, the Light, which proceeds

proceeds from them, filling a concave Sphere, as far as they can be seen, I do not suppose to proceed in Rays, nor in Planes (as a modern Author has supposed) the Hypothesis is absurd; seeing, that the whole Space is illumined in every part, it is impossible, that from every Point in the Surface of the luminous Body, Rays can proceed to an immense distance, filling all the Space between.

Suppose every Point in the Surface of the Sun to emit a Ray of Light, proceeding in Right Lines from the Center; it is evident, that every Ray, at the distance of the semi-Diameter, will fill a Space four-fold of its first dimensions, viz. in a duplicate Ratio of the Distance; at the distance of the whole Diameter, it must fill a Space nine times as large; i. e. it increases in proportion to the Squares of the Distance; what, then, will be the magnitude of a Single Ray of Light, proceeding from the Sun to Jupiter or Saturn, before it reaches those Planets *? or how can the whole of the Space it occupies be filled (as it certainly is) with Light, which proceeded from a single Point, in the Luminary, only?

Can one individual Ray be shattered into thousands, into millions, lying close to each other? what direction do they proceed in, from the Luminary? not in the direction of a Right Line from the Center, I presume.

I know it may and will be alledged that it expands as it proceeds, still filling the whole Space. I presume then, it cannot proceed in Rays, or Right Lines from the Center of the Luminary, as it is imagined. Air may be rarified and expanded to a very great degree; also, Water and other Fluids are expanded, as in steam, by Fire, or, in Mists and Vapours, by Exhalation, till it floats on Air. But can one drop, or the minutest particle of Water be expanded infinitely? as a Ray of Light, emitted from one single Point in the Luminary, must, in this Case, necessarily be; else, how could we see those Stars, whose distances are, to all sence, infinite?

These things, which may amuse and pass for orthodox with the generality of mankind, cannot pass with a thinking Person. Therefore, since all which can be said about it is, at best, but an ingenious conjecture, and since it is in no wise subservient to Perspective, I shall at present take my leave of the Subject, to pursue that part or branch of Optics, which is essentially so, respecting direct Vision; and after that, make a separate Chapter or Section, of my objections to the received Opinion of the cause of Vision.

* I find by calculation (from the Table in Harris's Use of the Globes, which, I suppose are as much to be depended on as any other) that the distance of the Earth, from the Sun's Center, is 212 semi-diameters of the Sun, nearly. Wherefore, if, instead of a Point, we suppose a certain quantity of Light to flow from one square Inch in the surface of the Sun; each side of that square Inch will, at that distance, be increased to 212 inches, or 17 feet 8 inches; the square of which is 44944 square inches. Consequently, a quadrangular Pyramid, whose Base is 17 feet 8 inches square, and a its altitude, the distance of the Sun from the Earth, is filled with Light, from one square Inch, only; and consequently, each single Ray, proceeding from every Point in its Surface, will, at that distance, be multiplied 44944 times.

At the distance of Saturn (by the same Tables) a square Inch, and consequently a single Ray, will be encreased or multiplied 4,149,369 times; its distance being 2037 semi-diameters of the Sun, nearly; consequently, a quadrangular Pyramid, whose Base is 169 feet 9 inches, on each side, and its altitude 777 millions of Miles, is filled with Light, flowing from every square Inch on the surface of the Sun; an amazing circumstance indeed! if it is supposed, or considered to be a material Body. But, how far Light may proceed, from the Sun, beyond the realms of Saturn, is not yet, I presume, determined.

N. B. If a single Ray be divided into so many thousands before it arrives at the Earth; but one, which is the Axis of the Pyramid, can proceed in the direction of a Right Line from the Center of the Sun.

SECT.

S E C T I O N II.

Of the ſtructure of the Eye, its parts deſcribed, and a ſhort Introduction to the nature of Vision.

THE Conſtruction or Figure of an entire human Eye, within its Sockit, is globular, of ſomewhat more than an inch in Diameter.

Plate I. Fig. 1. Let HIKL be a vertical Section through the center of the Eye, which, in this Figure, is conſiderably larger than Life; in order, that the ſeveral parts may be deſcribed with greater accuracy.

The external part, from H to I, is called the Cornea, or Horny Coat. It is perfectly tranſparent, and is ſomewhat more convex than the part which is within the Sockit, by means of a fine, clear aqueous or watry Humour, between the Cornea and the Iris (the coloured circle within.) Immediately behind the Iris, (ghik,) through which there is an Aperture, in the middle, to let in Light (or Viſion by means of Light) is a whitiſh Subſtance (MN) like a ſtrong Jelly, or cold Glue, of a moderate conſiſtence. It is as clear and pellucid as Chryſtal; from which, it is called the Chryſtalline Humour. It is, in every reſpect of the ſame nature and uſe as a double convex, microſcopic Lens; and is ſaid to be more convex on one Side (the inward) than the other. Its uſe is to contract the Rays of Light, by which Viſion is convey'd, to a Focus, beyond this Humour, in the Center of the Eye, at E; from whence, they become diverging, and fall on the bottom or back part of the Eye, at *acb*.

The Chryſtalline Humour, (MN,) is ſaid, by thoſe who are acquainted with and accuſtomed to diſſections of the Eye, to be furniſhed with muſcular Fibres; by means of which, it is made either more or leſs convex; or, by contracting them, be brought forward or otherwiſe as occaſion requires; in order, to render the Images, or Pictures of Objects perfect, in the bottom of the Eye, as the Objects are nearer to or further from the Eye; all which ſeems conſonant to Reaſon.

The Aperture or Pupil, (hi,) is alſo, by ſome ſuch means, expanded or contracted, as there is occaſion for more or leſs Light to enter. I have obſerved, in the eye of a Child, in a Room juſt light enough to diſcover it, the Pupil enlarged to three tenths of an inch in diameter; and immediately, on bringing into a full Light, it was contracted to one tenth or leſs; amazing Structure! and this is performed involuntarily, whether we will or no. There can hardly be any Perſon who has not made the experiment; on going into a darkiſh Room, he can ſcarce, at firſt, diſtinguiſh any thing; when preſently, he will begin, after the Aperture in the Iris is open'd to a proper dimenſion, to diſcern Objects diſtinctly; and again, on going into a full Light, he cannot bear to look on any illumined Object, till the Pupil is contracted; it is even painful to the Eye, at firſt, going out of a dark Room into a ſtrong Light, eſpecially if the Sun ſhines out.

All the back part of the Eye, within, is lined with a fine Membrane of a moſt curious and delicate texture; called the Retina, from its reſemblance of net-work, The large cavity, VV, which is bounded by the Retina, backward, and the Criſtalline Humour.

Humour, before (which is contained within it) is filled with a glutinous Fluid like the white of an Egg; being perfectly pellucid and uncoloured, it is called the vitreous or glassy Humour; in which, the Rays converge, at E, the center of the Eye; and, crossing each other, in that point of contraction, they fall diverging on the Retina, and form the Images of Objects thereon; the sensation, of which, is supposed to be communicated, by the Optic Nerve (which is connected with the Retina) to the Sensorium in the Brain.

As the manner in which the Rays of Light are refracted, in passing through the various humours of the Eye, is entirely conjectural; seeing that, through a small pin hole, applied close to the Eye, at q, there may be perfect Vision, of an Object of any magnitude. And, this Essay not being intended as a Treatise on Optics, I shall not insist on any thing; but, shall only suppose the Visual Rays (when the Eye is naked) to converge and cross each other in the Center of the Eye, at E (without considering their Refractions) from whence, I suppose, they diverge in equal Angles, falling on the Retina, at *a* c *b*, in the back part of the Eye.

DEFINITIONS.

1. The Axe of the Eye is a Right Line passing through the Center of the Eye and the Center of the Aperture or Pupil. As cEC.
   When the Pupil is direct, the point c, where the Axe cuts the Retina, is the Center of the Retina, the Seat of distinct Vision.

2. Visual Ray. If AB be supposed an Object of Sight; the Right Lines AE, BE &c. from all parts of the Object to the Eye, or to its Center, E, under which, or by the means of which, the Object is seen, or supposed to be seen, are called Visual Rays.

3. Optic Angle. By Optic Angle may be understood, either a plane or solid Angle.
   If the Object AB be considered as having no dimensions but length, only; i. e. if it be a Right Line, simply; the two Visual Rays EA, EB, from the Eye to each extreme, form a Plane Angle, AEB; which is the Optic Angle; under which, the Line AB is seen. And, AED or DEB is the Optic Angle of the Segment AD or DB.

4. Cone, or Pyramid of Rays. If the Object of Sight be either globular, or circular; as ABG; the Visual Rays, EA, EB, &c. from the Eye to every part of the extremes of its Surface, being towards the Eye, form a Figure resembling a Cone, which is called the Optic Cone. Fig. 2.
   But, when the Object is either a right lined Solid, or Plane Figure, as CD; the Visual Rays EF, EC, EH, &c. form a solid Angle, composed of several plane Angles, FEC, CEH, &c. which, being of a pyramidal form, is called a Pyramid of Rays.

The Visual Rays crossing each other, in E, and, from thence, become diverging to the Retina, each opposite plane Angle being equal,* there is formed a similar Pyramid, cEd, which is called the opposite Pyramid; and E is their common Vertex. * 2. 1. El.

The Figure, a b or c d, which is the Base of the opposite Cone or Pyramid of Rays, is the Image or Picture of the Object, AB or CD, on the Retina.

The small Characters, corresponding with the Capitals, shew in what manner the Image is supposed to be inverted.

Plate I. Fig. 1.

Let AB be ſuppoſed an Object, direct before the Eye, that is, perpendicular to its Axe, EC. It is ſuppoſed to be ſeen, by means of the Viſual Rays EA, EB, EC, &c. from every part or point in the Object, towards the Eye, whether it be Line, Plane, or Solid; which paſs through the Criſtalline Humour, to the Center, or point of contraction, E; from whence they proceed, diverging, to the back part of the Eye, where they fall on the Retina, and form the Picture, *a c b*, of the Object AB.

The Image of the extreme point A, by means of the Viſual Ray AE, falls on the Retina at *a*, and the other extreme, B, is repreſented at *b*; and CE, the Axe of the Eye, conveys the Image of the point C to c, in the Center of the Retina. From which it is evident, that the Picture of an Object, formed on the Retina, is neceſſarily inverted.

Nor is there any thing ſurprizing in this, as ſome may imagine. For, ſuppoſe A the top, and B, the bottom of the Object. It may be imagined, from the inverted poſition of the Image, *acb*, that the Object muſt neceſſarily appear upſide down. But, if we conſider, that the ſenſation of the Point A, being perceived or felt at *a*, is by the direction of the Viſual Ray AE, which determines its place in the Object; the Mind, by long experience, having acquired a habit of determining that part of an Object, perceived at *a*, to be above, and that at *b*, below; and which, is no matter of ſurprize at all; no more than, that, by the ſame experience, we have acquired an Idea of the diſtance of any Object perceived.

If a Perſon, who was born blind, could, when grown to maturity, be made to ſee, he would have no Idea at all of Diſtance, or the ſituations of Objects; which way was up, or which down; and would as ſoon attempt to lay hold of the moſt diſtant Object, which he perceived by the ſenſe of ſeeing, only, as others which were near him; it being impoſſible to diſtinguiſh, merely by Sight, whether the Point perceived at c, on the Retina, be at *C* or C, or at any diſtance beyond C, in the direction of EC. But, by cuſtom and a familiarity with Objects, from our infancy, we have acquired the Art of ſeeing; and by comparing Objects, according to their known magnitudes or diſtinctneſs of parts; or, according to the length of ground, which we imagine to lie between us and the Objects, we judge them to be at ſuch or ſuch a Diſtance. Whereas, when we look at the Heavens, one Star appears as far off as another; nor can we form the leaſt Idea of their diſtance from ſight, ſimply; for, a Star may as well be ten thouſand millions of Miles off as ten Miles, its Magnitude being in Proportion to its Diſtance. (See Fig. 7. A, B and C.)

The Art of Seeing is acquired, regularly and progreſſively, as all other Arts or Knowledge whatever. Do not we ſee a Child attempt to catch the Moon or other ſtriking Objects, though at an immenſe Diſtance? but, growing up in a familiarity with them, by common and frequent experience, they become ſo intimately connected with our Ideas, that we form a judgement of their Magnitudes and Diſtances, inſtantaneouſly, at firſt ſight; and alſo of their Situations, in reſpect of themſelves and of each other.

Now, the judgement we form of Diſtance, &c. being grown up with us from a Child, is ſo little attended to, that, at firſt thought of theſe things, we ſuppoſe it to be merely from Sight; than which, no Senſe is more deluſive and uncertain. The deceptions of Viſion are many and frequent; as, when we look at Objects through refracting Mediums, i. e. through any tranſparent Medium whatever, ſolid or fluid; according to the different denſity of the Medium, or to its Figure, &c. we frequently imagine an Object to be nearer or farther off, larger or ſmaller than it really is; or to be where it really is not; nay, it is poſſible, by Light and Shade, judiciouſly diſpoſed, and the aſſiſtance of Perſpective, to deceive both the

Eye

Eye and Judgement; in imagining a Figure described or projected on a Plane, to be a real solid Object.

That Vision is conveyed in Right Lines to the Eye, I presume no person will attempt to dispute (I would be understood to mean, of such Objects as are situate on or near the Earth; and when no other Medium, but Air, is between the Object and the Eye) I shall, therefore, give it as a general Axiom; which, nevertheless, may be illustrated or occularly demonstrated, by straining a fine silken thread, to a Right Line, from any Point, as *F*, of an Object *(DF)* to the Eye.

Then, if any other Object, as QR, be interposed, as soon as it touches the Thread E*F*, they will appear to be in contact; and if it be so interposed, as to hide the Point *F*, the Right Line or Thread, which may represent a Visual Ray, will be refracted, or broken into two Right Lines ER, R*F* forming an Angle, ER*F*.

Hence, it is evident, that the Points F and *F*; also, the Point C or *C* being in the same Right Line with *C* 2, will perfectly coincide with each other, to the Eye at E.

SECTION III.

Containing a brief THEORY of direct VISION.

THEOREM. I.

OBJECTS appear to have that Proportion to each other, respectively, as the Angles under which they are seen.

Let *a* c *b* represent a section of the Retina, a portion of a concave Sphere; and, let AB be an Object direct before the Eye; divided, in any Ratio, in the Points D and F.

DEM. It is evident that the Object AB, by means of the Visual Rays EA, EB, is seen under the Angle AEB; and the several Parts of AB, as AF, FD and DB, are seen under the several Angles AEF, &c. Fig. 1.

But, the opposite Angles *a* E f, f E d and d E *b*, are, respectively equal to the Angles AEF, FED, and DEB — — — — 2. 1. El.

And, the Images *a* f, f d and d *b*, on the Retina, of the Originals, AF, FD and DB, are the measures of those Angles, respectively (Th. P. Ang.*) the Retina being supposed a portion of a Sphere; each part is, therefore, equally distant from the Center.

But, the Ark *a* f is to the Angle *a* E f, as the Ark f d is to the Angle f E d, and as d *b* to d E *b*. — — — — — 19. 6. El.

Therefore, the Images or Pictures on the Retina, and consequently, the apparent magnitudes of Objects are in the same ratio, or proportion, as the Angles they subtend at the Eye. Q. E. D.

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* (Th. P. Ang.) refers to the Theory of Plane Angles, in my Treatise of Geometry.

Nor

Plate I. Nor is there occasion to have recourse to the Images on the Retina; but, allowing the Visual Rays EA, EB, &c. to be Right Lines, and an ark of a Circle, of any radius, being drawn, cuting those Rays; the portions of the Ark, l m n o, intercepted between the Visual Rays, as l m, m n, &c. measure the several Angles AEF, FED, &c. respectively; and consequently, the parts AF, FD, &c. of the Object AB, will appear to the Eye, at E, in proportion to the Arks l m, m n, &c.

All remote Objects appear equally distant from the Eye; as the Planets, Stars, &c. appear as if they were all in the same concave Sphere; of which every Eye is the Center. So likewise, near Objects, when we stand in the line of direction of two, three, or more, appear to touch each other; nor could we judge, merely from sight, which was farthest off, or which the largest.

For, suppose the Visual Rays EF, ED, EB, produced to *F*, *D* and B; any other Objects, as *EF* and *BD*, being in the same direction with FD, and DB, and touching the same Visual Rays, EF, ED, EB, will coincide with FD and DB; the less will hide the greater from Sight, and appear to be of equal Magnitude.

2. I am somewhat surprized at what Mr. Hamilton has advanced in Art. 6, Sect. 1. where he supposes, that Objects do not appear in proportion to the Angles they subtend, but in proportion to the Tangent of half the Angle; and imagines the Retina to be, nearly, a Plane, in the center of the Seat of Vision.

How Mr. Hamilton could conceive and account for such a strange and unwarrantable Opinion I cannot imagine. If, as he has truely asserted, the space on the Retina, so far as there is distinct Vision, does not exceed an Angle of one or two Degrees, on every side of the Axe; which is as much as to say, it does not exceed an Angle of two or four degrees; and I am fully convinced, that it does not exceed one Degree; what difference can it make, whether that small portion of the Retina be nearly a Plane or a portion of a Sphere? for, he certainly could not suppose a large portion of the Retina to assume that Form.

Now, if the Retina does assume the form of a Plane, of the space *a b*, then it is certain that the Images, *af*, *fd*, and *db*, on that Plane, (of which, the Right Line *a b* may be supposed a Section) are not in proportion to the Angles AEF, FED, and DEB, equal *a*E*f*, *f*E *d*, and *d*E *b*, respectively.

DEM. For, the Tangent c f or c d is not in proportion to c a or c b as the Angle c E f is to c E a; because, c f is equal f a (AF being equal FC) therefore, the Angle c E f is not equal f E a, or c E a double c E f. — 3. 6. El.

But, *a b* is parallel to a b; therefore, *af*, *fc* and *c b* are in the same Ratio, as a f, f c and c b. — — — — Cor. to 6. 6. El.

Thus it is clear, if the Retina does take the form of a Plane, that Objects do not appear in proportion to the Angles they subtend; but, as it is the only instance in which I ever knew it contested, so, I am persuaded it will never be universally adopted, on that Hypothesis. However, as it is merely conjectural, it seems to me more rational, to suppose the Retina to retain its original spherical Form, which the all-wise Creator has given it, than, that it should, continually, at every motion of the Eye, be changing to that of a Plane.

Wherefore, on the presumption that the internal Eye is truely spherical, it is very reasonable to suppose, that, Objects appear, exactly, in proportion to the Angles under which they are seen; and not in proportion to the Tangent of half the Angle, as Mr. Hamilton asserts and has endeavoured to demonstrate, in 7 and 8 of the same Section; which cannot be as himself has stated it, but on the supposition

tion of the Retina assuming the form of a Plane, which is absurd to suppose. Or granting it really so; the portion of the Retina, which the compass of an Angle of one or two Degrees, at most, takes up, is so very small, that it is impossible to distinguish it, in a Sphere of an inch in Diameter, from a Plane. As the 360th part of a Circle, of a small radius, is not distinguishable from a Right Line; consequently, there could be no difference in the apparent magnitude of an Object, in either Case, which is seen under so small an Angle. As, the small portion at d, on the Retina, subtending an Angle, at E, of about five Degrees, is not distinguishable from a Right Line.

3. It is also evident, if the Visual Rays are refracted in passing through the aqueous Humour, before they enter the Pupil (unless that Refraction be rectified again at the Crystalline Humour or elsewhere, and, by that means, proceed in the same Direction to the point of convergency, in the center of the Eye) that Objects cannot appear exactly in proportion to the Angles they subtend. Nor have I any conception how a Visual Ray, AE, falling remote on the Cornea, at *e*, can be refracted so as to pass in the same direction through the Aperture, h i, to the point of convergency, at E, and from thence to the Retina.

For, suppose a Visual Ray, A*e*, from any Point A (seen very oblique) falls perpendicularly on the Cornea, at *e*; if it pass at all through the Pupil, h i, it must pass in the direction *eg*; where it falls on the Crystalline, at *g*, and where, it is possible it may again be refracted in the direction *gb*, to the Retina; making an equal Angle with the Axe, EC, as the original Ray, A e; in which Case, the Image of the Point A is seen at *b*, in its proper position.

And since no one can assert that it is not so, I shall therefore conclnde that it is, at best, but Conjecture; since, as I have already observed, the whole System of Rays will pass through the finest Pin Hole, at q, without the Eye; and from thence diverge to the Retina (for how can it be otherwise? seeing, it is manifest they must all pass through the same Point q, it is therefore their common point of convergency) forming on the Retina an Image of the Object, in every respect the same (except in Hue, for want of sufficient Light) and of the same Magnitude as it appears to the naked Eye. Which single circumstance explodes, in my opinion, the notion of Refraction at the Cornea and through the various Humours, in the passage of the Rays to the Retina. I shall therefore suppose, that, when the Eye is naked, the point of convergency is in its Center; and, on that supposition, infer, that Objects do appear in proportion to the Angles under which they are seen. Of which I will give an easy and familiar Example.

4. Let AB, CD, and FG be three Globes, at different Distances from the Eye, at E, and of different Magnitudes; seen under the Angles AEB, CED, and FEG, respectively.* Fig. 3.

Now, if the Distance of the Globe CD be equal to twice the Distance of AB, and FG be double of CD, four times the Distance of AB; then will the Angles AEB, CED, and FEG, which they subtend at the Eye, if the Diameters of the Globes are in the same proportion as their Distances, be equal; and consequently, their apparent Magnitudes, to the Eye at E, are equal.

* The full Diameter (FG) of a Globe cannot be seen; for, the Visual Rays being Tangents to the Globe, from the same Point E, as E*f*, E*g*, are all equal (Cor. 2. 16. 3. El.) and, by reason of the convexity, towards the Eye, they cannot touch the two extremes of the same Diameter; unless the Distance be infinite; and then, the Visual Rays are considered as being parallel. *fg* is, therefore, the apparent Diameter of the Globe FG.

Plate I. Draw ES, to the Center of the Globe FG; draw C*D* perpendicular to ES, and equal to CD, the diameter of the Globe CD; alſo, draw B*A* perpendicular to ES, and equal to the Diameter of the Globe AB.

DEM. It is evident, that the Viſual Rays FE and GE, from each extreme of the Diameter of the Globe FG, will paſs through the extremes of C*D* and B*A*.

For, in the ſimilar Triangles FEG, *D*EC and BE*A*; FG : SE :: *DC* (equal CD) : KE, and, as B*A* : LE. — — — — — 6. 6. El.

Then, ſince they touch the ſame Viſual Rays FE and GE; conſequently, they are all ſeen under the ſame Angle, FEG. — — by Theorem

And the Diameters AB, CD and FG, being in proportion to their central Diſtances, they are, therefore, all ſeen under equal Angles, AEB, CED and FEG.

Conſequently, the Images, a b, b c and c d, on the Retina, ſubtending equal Angles, are equal; and therefore, the appearances of the three Globes AB, CD, and FG are equal. Q. E. D.

If the Globe CD be ſuppoſed to be moved to M, its central Diſtance EM being equal ES; and, being, in Diameter, equal half FG; it will ſubtend an Angle GEH, equal to CEI, half CED, equal FEG; as the Arks 1 2 3 4, which meaſure thoſe Angles, exemplifies.

Conſequently, the Image h c, of the Globe GH, at that Diſtance; is equal to half the Image c d, of the Globe CD of the ſame Magnitude, and ſeen at half the Diſtance; for, they are in the ſame ratio as the portions of the Ark 3 0 to 3 4.

5. Yet it is manifeſt, that two or more unequal Objects, at an equal Diſtance from the Eye, and ſeen direct, do not appear exactly of the proportion of their reſpective Magnitudes; likewiſe, equal parts of the ſame Object, on the ſame ſide of that Point where it is cut by the Axe of the Eye, do not appear equal.

Let AB be an Object, of length ſimply, biſected by the Axe EC, to which it is

Fig. 1. perpendicular, in the Point C; and ſuppoſe AC and CB again biſected, in F and G.

Then, the parts AF, FC, &c. are equal, and AB is double FG; but, they do not appear of that proportion, at E.

DEM. Now, becauſe AF is equal to FC, the Angles AEF, FEC, under which they are ſeen, are unequal — — — — — 6. 6. El.

For, if the Angle AEC be biſected, the Right Line EO, biſecting it, will cut AC, in the Point O, in the ratio of EA to EC.

But, EA is greater than EC; for, EC is perpendicular to AC.—Cor. to 12. 1. El.

Wherefore, AO is greater than OC, and the Angles AEO, OEC, under which they are ſeen, are equal; conſequently, the equal parts AF, FC, are ſeen under unequal Angles; therefore, they appear unequal, by the Theorem.

And, becauſe the Angle CEF is greater than FEA, the equal part CF will appear greater than FA.

After the ſame manner it may be proved, that the equal part CG appears greater than GB. Conſequently, CG + CF, i. e. FG, appears greater than half AB. Q. E. D.

COR. Hence, it is evident, that if AB be produced, and the equal diviſions, CG, GB, BP, are continued, they will appear continually leſs, being ſeen under leſs Angles; for the Angle BEP is leſs than BEG, which, is leſs than GEC; as the meaſures, p o, o r, and r *C*, of thoſe Angles, ſufficiently evinces.

N. B. The farther any two or more Objects are from the Eye, the nearer they will appear in the proportion of their reſpective Magnitudes.

Becauſe they are ſeen under leſs Angles; and, the Tangents of ſmall Angles deviate leſs from the ratio of the Angles, than the Tangents of greater Angles.

6. As

6. As Objects of different Magnitudes, at equal Distances, do not appear in the proportion of their respective Magnitudes, so neither do Objects of equal Magnitude, seen at different Distances, appear exactly in the ratio of their several Distances.

Let AB be an Object at any Distance (E d) from the Eye, at E; and, let CD be another Object, parallel to AB, of equal Magnitude, as to length simply; also, let CD be at twice the distance of AB (ED double E d.) Fig. 4.

By means of the Visual Rays EA, EB, &c. those Objects are seen under the Angles AEB and CED. I say, the Angle AEB is not double of the Angle CED.

Draw EG parallel to a Right Line (AC) drawn through the extremes, A and C, of the Lines or Objects AB and CD; likewise, draw EA, EC, ED and EB.

DEM. Then because AB is parallel to CD, EF : FG :: Ed : dD; and EF : EG :: Ed : ED — — — — } - 2 and 4. 6. El.
But, Ed is equal to half ED (Hyp.) conf. EF is equal to FG.
And, because FG is parallel to AC, and AF to CG, AF = CG. — 15. 1. El.
Now, because AF is parallel to CG, the Triangles EcF, ECG are similar, consequently, as EF : EG :: Fc : GC — — — — 4. 6. El.
But, EF is equal to half EG; wherefore, Fc is equal to half GC.
Consequently, AF (equal CG) is bisected in c; i. e. Ac is equal to cF.
Now, since Ac is equal cF, the Angle AEc is not equal cEF.
For, if the Angle AEF had been bisected by EC, AF would be cut, by EC, in the ratio of EA to EF; i. e. Ac would be to cF :: EA : EF — 3. 6. El.
But, the Angle EFA is obtuse, consequently, EA is longer than EF — 12. 1.
But, it has been proved, that objects appear to the Eye in proportion to the Angles under which they are seen — — — — by Theoerm
Conf. Ac does not appear equal to cF; because Ac is = cF; by the foregoing.
But, CG, being seen under the same Angle (CEG) as cF, appears to the Eye, at E, equal to cF. Wherefore, CG, equal AF, appears greater than half AF; notwithstanding its distance, ED, is double that of AF.

After the same manner, the remaining part, GD, equal FB, may be proved to appear equal to half FB.

For, let the Angle FEB be bisected by ED, cuting BF and DG, perpendicularly; the Triangle BEF is Isosceles; i. e. EB = EF — 9. 1. El. Cor. 3.
Consequently, Bd = dF; and they are seen under equal Angles, BEd = dEF.
Wherefore, DG, being seen under the same Angle (DEG) as dF, will appear equal to dF, half BF. — — — — — Art. 4.
But, if equals be added to unequals the sums are unequal. — Ax. 8. El.
Wherefore, the Angle CEG+GED, i. e. CED, is greater than AEc+dEB; and consequently, CD (equal AB) appears greater than half AB, though seen at twice its Distance. Q. E. D.

Again; let IL be another Object, equal and parallel to AB, and, at three times the Distance; IL will appear greater than one third of AB.

Produce IL, and the Perpendicular, ED, to M; then EM is its Distance.
Draw EI and EL; also, produce AC and EG, to I and H.

It may be proved, nearly after the same manner as the former, that the Angle AEF is not trisected by the Right Line EI.
Also, that the Angle IEL is more than one third of AEB.

DEM.

Plate I. Fig. 4.

DEM. For, becauſe AF and CG are both parallel to IH, the Triangles EiF, EnG and EIH are ſimilar; conſequently, Fi : Gn, or Hi,::EF:EG or EH - 4. 6. El.
that is, Fi : FA (equal HI) : : EF : EH;
Conſequently, if HI was produced, making IK equal HI, and EK drawn cuting AF in k; then Fk : HK : : EF : EH.
But, EH=3EF; wherefore Fk=one third of HK; and Fi of HI.
But, FA=HI, equal $\frac{1}{2}$ HK; wherefore Fi=$\frac{1}{3}$ of HI, and Fk = two thirds.
Conſ. FA is triſected, in i and k; i. e. the parts Fi, i k and kA are equal.
But, becauſe, Fi is equal ik and i k=kA; the Angle FEi is greater than iEk, and iEk greater than kEA — — — — 3. 6. El.
For, if the Angles FEk and iEA were biſected by the Right Lines Ei and Ek, Fi would have that ratio to ik as EF to Ek; and, i k would be to kA : : Ei : EA.
But, EF is leſs than Ek, and Ei than EA — — Cor. to 12. 1. El.
And Fi is equal to i k, equal kA — — — — proved above
Wherefore, the Angle FEi, i. e. HEI, is greater than i e k;
and i e k is greater than kEA; — — by the 3d of El. 6. as above
Conſequently, the equal parts, Fi, i k and kA, do not appear equal - Art. 5.
Therefore, H I, ſeen under the ſame Angle (IEH) as Fi, appears to the Eye, at E, in a greater proportion than their reſpective Diſtances, EM to Ed, or EH to EF.

Let the remaining part, HL (equal FB) be equal to two thirds of HM; it does not appear in that proportion, to the Eye at E.

DEM. Becauſe, the Angle HEL is not two thirds of HEM, for LEM is more than one third; as above.
Conſ. HL appears in a leſs proportion to FB, than their Diſtances, Ed to EM
But, the ratio of the Angle LEH to HEI is not as LH to HI, or lF to Fi
For, becauſe they are both on the ſame ſide of the Perpendicular EM;
if LH was equal HI, the Angle LEH, being neareſt the Perpendicular, would be greater than HEI — — — — — — 3. 6. El.
For, if they were equal, LH would be to HI : : EL : EI.
Therefore, the ratio of the Angle LEH to HEI is greater than LH to HI.
Conſequently, the part HL, of the Object IL, appears greater in ptoportion to its Diſtance, EM, than HI; but, it appears leſs in proportion to BF; ſeeing, the Angle HEL, i. e. FEl, is not one third of FEB, or two thirds of HEM.
But, the Angle HEL being nearer the Perp. EM, has a leſs ratio to FEB, than HEI to AEF; i. e. the deficiency of the Angle FEl, or HEL, to one third of FEB, is leſs than the exceſs of FEi, or HEI, to AEF.
Conſequently, LH appears nearer to the proportion of BF, than HI to AF.
Therefore, the whole Angle IEL is more than one third of AEB;
and conſequently, IL appears greater than one third of AB;
i. e. IL appears in proportion to AB, greater than E d to EM, their reſpective Diſtances.

Thus, I have made it appear, to Demonſtration, that Objects do not appear in that proportion to each other, as their reſpective Diſtances; in which I have been more particular, becauſe, it is expreſsly ſaid, by a certain Author, Page 13, that theſe Pictures (i. e. the appearances of Objects) will be to each other, as their ſeveral Diſtances are to each other.* Rohault ſays, nearly ſo; Page 243.

* The Appearances, i. e. Angles, being in proportion to the Diſtances is, literally, abſurd; becauſe, as the diſtance of an Object is increaſed, the Angle, under which it is ſeen, decreaſes; which kind of reciprocal Proportion is always to be underſtood in this Caſe.

2. If

7. If the Eye be removed from E to *E*, equal GH, and *E*I, *E*L be drawn, AB will be cut in the same Points, c and d, as by the Visual Rays EC, ED;

But, IL will not appear of the same proportion to AB, at the Point *E*, as CD, at E, the Objects AB, CD and IL being equal.

DEM. 1. Because AB, CD and IL are parallel amongst themselves, the Triangles EcF, ECG, also *E*cF and *E*IH are similar;
wherefore, as EF : EG :: Fc : GC; and, as *E*F : *E*H :: Fc : HI } 2 and 4. 6. El.
But, EF = half EG, and *E*F = half *E*H, by the Hypothesis;
consequently, Fc = half GC, and also equal to half HI, equal GC.
By the same reasoning, F d may be proved equal to half GD, or HL.
Therefore, AB is cut in the same points, c and d, by the Visual Rays EC, *E*I, &c. from both Points of View, E and *E*. Q. E. D.

2. CD appears to the Eye, at E, in proportion to AB, as the Angle CED to AEB; and IL appears, at *E*, as the Angle I*E*L to A*E*B.
Let *E*H be drawn parallel to AI.

DEM. The ratio of the Angle GEC to GEA, is greater than H*E*I to H*E*A.
For, the Angle GEA is greater than H*E*A (equal GEC) }
i. e. the Angle FEA is greater than F*E*A (equal FEc) } — 10. 1. El.
wherefore, the ratio of FEc to FEA is greater than that of F*E*c to F*E*A.
For, draw Eo parallel to *E*c; then, the Angle FEo = F*E*c, or H*E*I - 4. 1.
consequently, the ratio of FEo to FEc = to the ratio of F*E*c to F*E*A;
because, the Angles on both Sides are equal, respectively.
But Fc is equal to cA (proved above) wherefore, Fo = oc.
And, it has been proved, that equal Divisions, on the same side of a Perpendicular, from the Eye, to a Right Line, subtends a less Angle, the farther they are from the Perpendicular. — — — — — Art. 5.
Therefore, the Angle FEo to FEc, i. e. H*E*I to H*E*A, has a less Ratio than FEc, or GEC, to GEA; for they are nearer to equality.
Consequently, HI appears to the Eye, at *E*, less in proportion to AB, than GC, in the point of View, E.
After the same manner, HL may be proved to appear less in proportion to FB at the Point *E*, than GD at the Point E.
Consequently, the whole, IL, appears less, in proportion to AB, at *E*, than CD at the Point E; i. e. the Angle I*E*L is less, in proportion to A*E*B, than CED, at half the Distance from AB, to AEB; the Subtenses CD and IL being equal. Q. E. D.

Hence it is manifest, that, notwithstanding the sections of the Visual Rays *E*I and EC, *E*L and ED, with AB, are the same; the Distances, on both sides of AB, being proportional; i. e. the Distance of the Eye on one side of AB, and of the Object on the other; yet, the appearances of the Objects, to each other, are varied, considerably, as the Distance is greater or less.

For, two Objects of equal height, at a great Distance from each other, and parallel between themselves, and the Eye at the same Distance from one of them, and in a Right Line with both, will appear, in proportion to each other, nearly as their Distances; i. e the farthest off will appear half the height and half the width of the nearest, being at twice the Distance, and, one third in height and width at three times the Distance, nearly.

As IH appears, at *E*, nearly equal half AF,* the distance of the Eye from IH, being double the Distance of AF.

* Nearly equal, in this place, means somewhat more; for, IH appears more than half AF, the two Distances *E*F and FH being equal.

Plate I. Fig. 4. But CG (equal IH) appears much greater than half AF, at the point E; notwithſtanding the ſection of the Viſual Rays EI and EC, in the point c, is the ſame; and, Fc is equal cA.

If the Eye be at three times the Diſtance from one, of two equal and parallel Objects, (and alike poſited, to the Eye) as from the other, the fartheſt off will be repreſented, on a Plane parallel to them, one third in height and width of the neareſt.

As, at E, the Diſtance of the Eye, from AF, is one third of the Diſtance from HI; and the portion Fi, of FA, where it is cut by the Viſual Ray EI, is equal to one third of FA.

But, I have already proved that it does not appear in that proportion, but in proportion to the Angle HEI, which is conſiderably more than one third of AEF; as the portion F*i*, on the Ark *ai*d, manifeſtly evinces.

And, it is alſo manifeſt, that the Diſtance muſt be, to all ſenſe, infinite, before they can appear in equal ratio to their Diſtances; i. e. the Viſual Rays, EA, EI and EH, muſt be parallel or nearly ſo.

THEOREM. II.

Parallel Right Lines, however ſituated, being produced, appear to approach towards each other; and, if produced infinitely, they will appear to meet in a Point at an infinite Diſtance.*

Fig. 5. Let IF and HG be too parallel Lines; and E, an Eye, ſituated any how between them.

Let K, B, D, and G repreſent various Diſtances in the Line HG, and let the Lines IK, AB, &c. be drawn perpendicular to HG, cuting the other Line (alſo perpendicularly) in the points I, A, C and F.

Now, if the Viſual Rays, or Right Lines EI, EK, EA, &c. be drawn, the ſpace between the Lines AF and BG at the ſeveral Diſtances ES, EO, &c. will appear, to the Eye at E, under the ſeveral Angles IEK, AEB, &c. each of which, as the Diſtance is further from the Eye, is leſs than the other.

For, the Space between the parallel Lines (meaſured by the Perpendiculars AB, CD, &c.) is every where equal. — — — — Def. 7. Geo.

Dem. The Space, between thoſe parallel Lines, at IK, is ſeen under the Angle IEK; and the ſpace between them, at AB, under the Angle AEB; which, it is evident is leſs than IEK, nearly in proportion to the Diſtances ES to EO; the Space at CD ſubtends the Angle CED, and FG the Angle FEG; each of which is leſs than the former; for their Subtenſes, AB, CD, &c. are equal; and the Lines EA, EB; EC, ED, &c. forming or containing the Angle, are continually longer; wherefore, the Angle FEG is leſs than CED, which is leſs than AEB, &c. C. 14. 1. El.

And thoſe Angles diminiſh nearly as the Tangents OA, OC, OF, when the Angles are ſmall.†

* It is ſaid in Smith's Optics (Vol. I. Art. 156, Page 58) that parallel Lines, ſeen obliquely, appear to converge more and more as they are farther Extended from the Eye; which, will ever be the caſe however the Eye is ſituated, in reſpect of the parallel Lines. For, being ſeen obliquely, means nothing more than when we look towards either of their extremes; as no part of both Lines can be ſeen direct according to his Diagram, the Eye being ſituate between them; or, being ſituated without them, they are ſeen direct only where a Right Line, from the Eye, cuts them both at Right Angles; on either ſide of the Perpendicular they are neceſſarily ſeen oblique.

† When the Angle, AEB or CED, under which any Object, as AB, is ſeen, does not exceed 20 or 30 Degrees, at the moſt, and that Object is removed to the ſeveral Diſtances, from EO, to EP, EQ, &c. the

From which it is manifeſt, that at a greater Diſtance the Angles will be ſtill leſs; conſequently, it will, at laſt, become inſenſible, and the interval, between the Lines, loſt to Sight. Q. E. D.

Hence, the Lines, IF and HG, and, conſequently, the ſpace between them, is ſaid to vaniſh.

2. Let ER be drawn, from the Eye, parallel to IF and HG; thoſe Lines will appear to approach, continually, towards ER; and being produced, infinitely, they will all appear to meet in the ſame Point; however ER may be nearer to one than the other; for, the further any parallel Line is from ER (which may be called its Radial) the more ſudden is its apparent approach to that Radial.

The Space IA, of the one, appears to advance from *h* to *i*; an equal Space KB of the other, advances from *p* to *o*; which is leſs than the other, in proportion to the Angle KEB to IEA; i. e. as the Ark *po* to *hi*. The Space from A to C appears to advance from *i* to *k*; an equal Space, BD, in the other from *o* to *n*. From C to F it advances from *k* to *l*, an equal Space, DG, from *n* to *m*; in the proportion of the Arks *kl* to *mn*.

It is evident, that if thoſe Lines were produced longer, they would ſtill appear to approach nearer to the point e, as EL, EM, evinces. And, being infinitely produced, they would at laſt appear to terminate in the Point e and be loſt to ſight.

For, it is manifeſt, that the further any Points L and M (conſidered as being in the parallel Lines IF and GH) are diſtant, the nearer the Viſual Rays EL, EM are to a coincidence with ER; which, it is evident, muſt be infinite before they can coincide perfectly; in which caſe, EL, EM, and ER will be the ſame; and they will all appear to unite in the Point S, or R.

3. Hence, it is eaſy to account for the appearance of a horizontal Plane, or continued level Surface; which, being below the Eye, will gradually appear to aſcend; and, being above the Eye it will appear to deſcend; and if they were produced infinitely, they would appear to meet in a Right Line, on a level with the Eye.

Let the Right Line HG be a ſection of a horizontal Plane, below the Eye, at E; and let IF be a ſection of one above the Eye.

Alſo, let IK be a ſection of a Plane direct before the Eye * (at E) perpendicular to the other two,

Now if the Viſual Rays, EA, EB, EC, &c. be drawn, they muſt neceſſarily cut the Plane of which IK is a Section; in a, b, c, &c.

It is evident that the ſpace KB will appear to riſe, towards S, on the Plane IK, from K, its interſection, to b, where the Viſual Ray EB cuts that Plane; the ſpace KD will riſe to d, and KG to g.

For the ſame reaſon, the ſpace IA will appear to deſcend from I to a; IC from I to c; and IF, from I to f; each Plane appearing gradually to approach towards S, where a Perpendicular, ES, to that Plane, cuts it.

And, if the Planes were produced infinitely, they would at laſt vaniſh, or be loſt to ſight, in a Right Line paſſing through the Point S, parallel to the interſection of the other two Planes, with the Plane IK.

the difference between the Tangents OA, OC, &c. deviates but little from the Arks *OCq*, *OC*, &c. and that ſtill leſs as the Angle is leſs; as PC, *PF*2, &c.

* By being direct before the Eye, I would be underſtood to mean, in a vertical Poſition; i. e. perpendicular to the Horizon; and when a Right Line, from the Eye, parallel to the Horizon, would fall within its Bounds, cuting the Plane perpendicularly.

For, if the Eye be not in the Plane, or in a continuation of it, the Eye, being conſidered as a Point, can have but one poſition, either to a Plane or to a Right Line, being produced; however the Plane may be ſituated, in reſpect of the Horizon.

And

Plate I. Fig. 5. And this is evidently the case however a Plane be situated; whether horizontal, vertical or inclined, it is still the same; for, a Plane is still a Plane in all positions, and has no properties peculiar to any position, in respect of the Horizon.

Therefore, there may be drawn this Conclusion, that every Plane, in which the Eye is not situate, will appear to approach towards, and at length to meet, another Plane, passing through the Eye, parallel to the former.

4. Let AB, CD, and FG represent three Objects of equal height, at the several Distances, EO, EP, and EQ, from the Eye, at E.

The Visual Rays EA, EB, EC, &c. being drawn, those Objects will appear in proportion to the Angles AEB, CED, and FEG.

For, the portions of an ark of a Circle, whose Center is E, intercepted between the Visual Rays, are the measures of each Angle, respectively, and determines the apparent proportion of the Objects AB, CD, and FG; and, whatever number of Degrees those Arks contain, the Angles which they subtend are in the same Ratio.*

* 19. 6. El.

Wherefore, since, in Perspective, the Representation is always on a Plane, and not on a spherical Surface; suppose IK a section of a Plane, in a direct position; the Spaces, a b, c d, and f g, between the Points where the Visual Rays cut and pass through that Plane, considered as a Picture, are the proportions of the Representations of the Objects, AB, CD, and FG; but their apparent Magnitudes are the portions *io*, *kn*, and *lm* of the Ark *hep*.

For, let HG represent, a section of the level Ground, or any other horizontal Plane; and AB, CD, and FG, Objects perpendicular thereto.

If IK be supposed a section of a vertical Plane, the foot or seat B, of the Object AB, will appear, on that Plane, at b; and consequently, Kb will represent the space of Ground, KB, between the Picture and the Object AB; i. e. it will appear to rise so high on the Picture; Kd will represent the Space of KD, and Kg of KG. So likewise, a, c, and f shew how much the tops of those Objects appear to descend, on the Picture.

5. Now, if the position of the Picture be inclined to the Horizon, as *a*K, the Objects and the Eye remaining as before; the Representations *ab*, *cd*, and *fg*, on that Plane, are very different from the proportions of a b, c d, &c. on the Plane IK.

But, the apparent proportions, of both, are the same; viz. the portions on the Ark *hep*, which measure the Angles AEB, CED and FEG.

Nor, is it possible to be otherwise; seeing that the Objects, AB, CD, &c. remain the same, in respect of each other; and the situation of the Eye, at E, is not varied; consequently, the Angles AEB, CED, &c. are not altered, and the Objects AB, CD, &c. the Subtenses of these Angles, must, necessarily, appear the same, represented on any Surface whatever, and in any Position, cuting the Visual Rays EA, EB, &c.

N. B. The Proportions *CD*, and *FG*, &c. on AB, are the same, as c d and f g on IK (AB being parallel to IK); wherefore, the Triangles AEB, aEb, also CED, *CED*, and cEd, &c. are similar; and therefore, as f g : c d, or to a b, :: *FG* : *CD*, or to AB. — — — — — 4. 6. El.

Consequently, Visual Rays, being cut by parallel Planes, in any position; are not only themselves cut in equal Ratio, but also the Sections, or the Projections on the Planes, by their sections with the Rays.

And, their Proportions are as their Distances ES, EO, EP; i. e, as *fg* : ES :: *FG* : EO :: *F2G2* : EP and, as FG : EQ;* and so of any other.

* 6. 6. El.

Hence, it is manifest, that the Sections of any system of Rays, by parallel Planes, are similar.

PROBLEM.

PROBLEM.

A Right Line, AB, being obliquely situated in respect of the Eye, at E, to determine the Point D, to which if the Axe of the Eye be directed, the two extremes, A and B, will appear alike distinct; so that AD and DB shall be seen under equal Angles. Fig. 6.

Without drawing the Visual Rays.

Make AD to AB, as the distance of the Point A, from the Eye, E, is to the distance of the same Point A, added to the distance of B. — Pr. 32. El.
Then, AD : AB :: EA : EA + EB; and, AD : DB :: EA : EB.

Dem. The Visual Rays EA, ED, EB being drawn, the Angle AEB is bisected by ED; wherefore, AD and DB are seen under equal Angles, at the Point E.
For, if the Optic Angle AEB be bisected by the Right Line ED;
then is AD : DB :: EA : EB. — — — — — 3. 6. El.

2. If the Axe of the Eye be directed to C, the middle Point of AB, then will the Point B be more distinctly seen; because its Image falls nearer to the Image of C than the Point A. And, if the Ark cf be made equal ca and Ef be drawn, meeting AB produced in F; AC will then be to CF as EC to EF; and, notwithstanding the difference between AC and CF, their Images on the Retina, and consequently their apparent Proportions, are equal, being seen under equal Angles AEC equal CEF.

Now if those Visual Rays are cut by a Plane in the position IK; the part ac, the greater portion of af, will represent AC, the lesser Segment of AF; and cf, the lesser portion, represents CF, the greater Segment;
For, since the Optic Angle, AEF, is bisected, by EC; ac : cf :: Ea : Ef; - 3. 6.
also, as AC : CF :: EA : EF; consequently, ac, cf represents AC, CF.

3. If the Right Line AB be produced, at the extreme A, cuting the Plane IK, at I, the whole indefinite representation of that Line; on the Plane IK, is the section IK, of a Plane passing through the Line AB and the Eye, E, and terminated by a parallel Line, EK, in that Plane.

For, because EK is parallel to AB, they will appear to meet in a Point at an infinite Distance;* which Point is represented by the Point K, on the Plane IK; for, seeing that the Line EK passes through the Eye, E, its whole appearance is but a Point; consequently, the Point K represents the whole Line EK, produced infinitely; and consequently, AB, infinitely produced, will appear to meet EK in that Point.

* Theo. 2.

4. If the Line AB be produced on the other Side of the Plane IK, and any Point, G, be assumed, between the Eye and the Plane; its representation, on that Plane, is the Point g, in which the Right Line EG produced, cuts KI produced; the part Ig, beyond the Intersection I, represents IG, the portion of the Line, AB, lying on the side of the Plane, IK, towards the Eye; as Ia or Ib represents the part IA or IB on the other Side; and, if the Angle AEB be equal AEG, the portion AG appears equal to AB. All which is so very obvious, from inspection of the Diagram, it is needless to say more about it.

5. After the same manner as the apparent magnitudes of Objects are determined, so are their apparent Distances, or Bearings, in respect of each other; viz. by the Degrees on the Ark of a Circle, which measure the Angles they make at the Eye.

Plate I. Fig. 7. Let C, D, and F, be three Objects, and E the place of the Eye.

Draw the Visual Rays CE, DE, FE; then, the Angles DEC, CEF are the Optic Angles of their apparent Distances, or Bearings at the Station E.

Notwithstanding their real Distances from each other, CF is nearly double that of CD, yet, the Angle CEF, is much less than the Angle DEC; as it is evident, from the portions, c f and c d, of the Ark d f.

If *f d* be supposed the section of a Plane, the appearances, or places of those Objects thereon, are at *f*, *c*, and *d*.

6. Suppose C, D, and F to be Stars, in the unbounded Expanse of the Heavens, at an immense distance from each other and from the Eye, at E.

It is impossible to form an Idea of their real Distances or Situations, in respect of each other; for, if the Star C was at B, and F at G, or any where else in the direction EG, their apparent places are still at c and f, in an imaginary Arch of the Heavens, as it appears to us, equally distant in every part.

Now, since the whole Diameter of the Earth's Orbit is not sufficient to make any sensible difference in their Bearings, and, consequently, of their apparent Places, in the Arch d f, there cannot be any positive Idea formed of their real Distances; for the portions f c and c d, of the Arch f d, are the measures of their apparent Distances, only; i. e. of the Angles dEc, cEf, or the Originals of these Angles DEC, CEF under which they are seen.

7. Hence, is constructed the Celestial Globe or Sphere, which is a true Picture of the Heavens, of Stars, &c. divided into various Figures called Constellations.

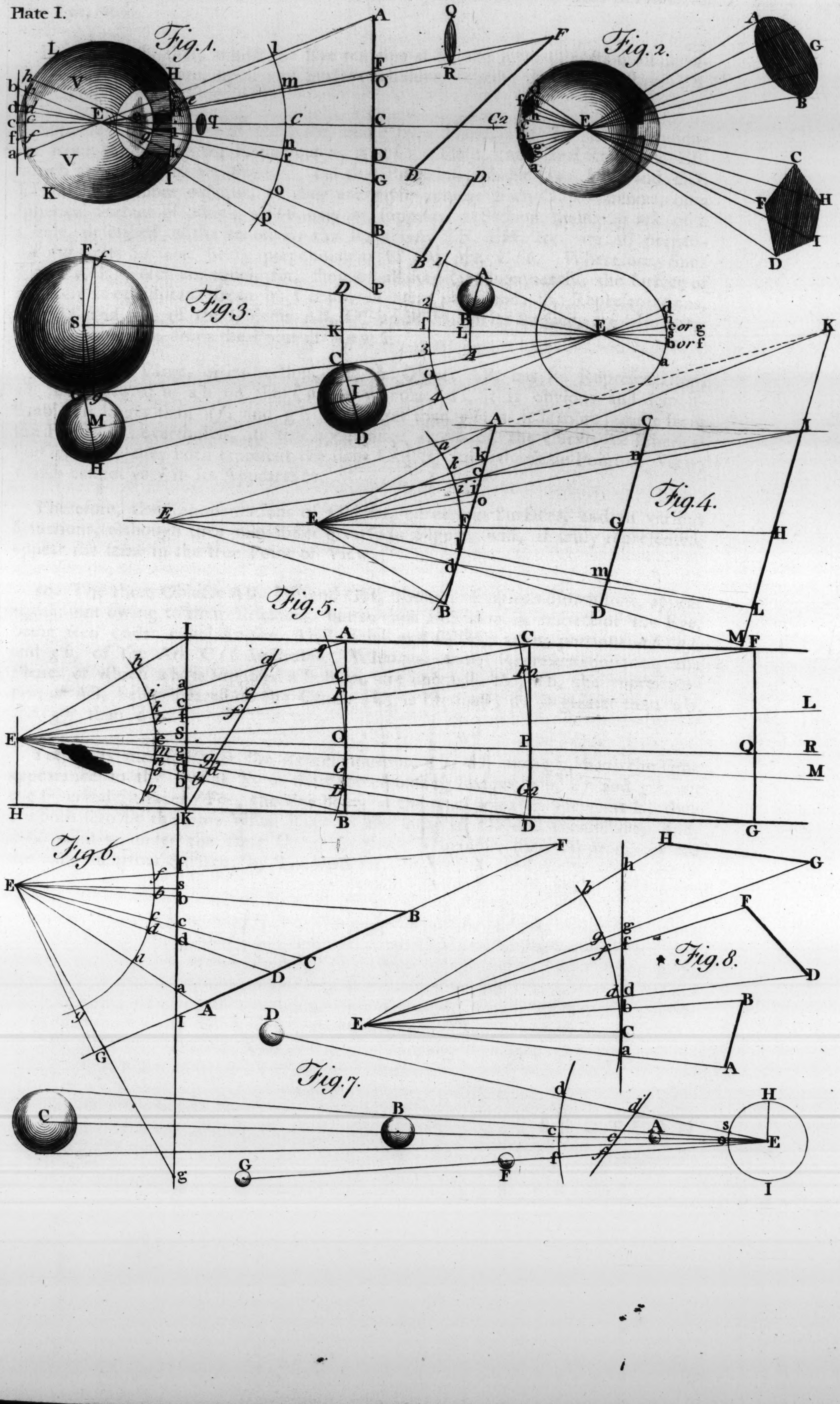
If d c f be supposed a portion of a Sphere; or, let HI be an entire Globe.

It is evident, that from its Center, E, which is also the Center of the Arch d f (E being supposed the Eye of a Spectator) the Star C will appear on the Globe at o, and D at s, &c. in the same position, situation and distance, in proportion to the Radius, EH or Eo, to Ec, as in the imaginary Arch of the Heavens, d f. So that, whether the real Star be at A, B, or C, or any where else in the Right Line EC; its apparent or representative Place, on the Globe HI, will still be at o; and consequently, can make no difference, in its true Place thereon; but its apparent Magnitude will be in proportion to its Distance, nearly.

8. From what has been advanced in this Section, it is evident that there is a manifest difference between the Representation of an Object on a Plane and its true Appearance; which Difference is the greater, as the Eye is nearer to the Object. For, since the Visual Rays must necessarily cut any Plane, on which they fall, more and more oblique, the farther their Intersections are from that Point, in which a perpendicular Line from the Eye would cut the Plane; consequently, the Representations of Objects, on a Plane, cannot be in proportion to their true Appearance, but deviate continually, more and more, as they fall farther from the Point described. Whereas, on a spherical Surface, which is every where equally distant from its Center, every Right Line, and in every Direction, which tends to its Center, is perpendicular to its Surface, and none of them can be said to cut it oblique; consequently, the true Representations, and also the true Appearances of Objects, can only be depictured on the Surface of a Sphere; that is, the Representation on its Surface and the true Appearance of the Object, from the Center, as the Point of View, are the same.

9. Let

Plate I.



9. Let AB, DF and GH be three Objects, promiscuously situated in respect of the Eye, at E. Fig. 8.

It is manifest, that, whilst the Eye remains at E, the three Objects must necessarily appear the same, upon any Surface whatever, cuting the Visual Rays EA, EB, &c.

Suppose a h a section of the Rays, made by a Plane, and EC a Perpendicular from the Eye to the Plane, cuting it at C. Then, the Visual Rays EA, EB, which are nearest to the Point C, cut that Plane less oblique than EF; and EG, EH are still more oblique, as they are more remote from G. Whereas, on a spherical Surface of which a*dfh* may be supposed a Section, being an ark of a Circle, described on the center E, the Rays EA, EF, EH, &c. are all perpendicular to its Surface, being perpendicular to the Ark C*fh*. Wherefore, since E a, E*d*, E*g*, &c. are equal, for, they are all Radii; consequently, the surface of a Sphere is equidistant from its Center, in every part; and, the Representations, a b, *df*, and *g h*, of the Objects AB, DF and GH, on its Surface, are also their true Appearances, from the Point of View E.

But, on the Plane, or its Section, a h, the Object AB, has its Representation, a b, nearly equal to a b on the Curve; whereas, d f, it is obvious and demonstrable, is larger than *df*; and g h still larger than *g h*, as it is more remote from the Point C; nevertheless, its true appearance is *g h* on the Curve, or spherical Surface; for, they both represent the same Object, from the same Point of View, which cannot vary in its Appearance.

Therefore, the Representations of Objects, on various Surfaces, and in various Situations, although they may differ greatly in Figure, will, if truly represented, appear the same in the true Point of View.

10. The three Objects AB, DF and GH, though of various dimensions, appear equal; not owing to their Distances, but to their Positions in respect of the Eye, being seen under equal Angles, AEB, DEF and GEH; as the portions *a b*, *df*, and *g h*, of the Ark C*fh* indicates. Whereas, their Representations on the Plane, of which a h is supposed a Section, are unequal; for, a b, the representation of AB, being nearest to the Center, C, is the least; d f is greater than a b, and g h than d f.

Yet, it is manifest that the Representations, a b, d f, and g h, have the same appearance to the Eye at E, as their corresponding Images, a b, *df* and *g h*, on the spherical Surface. For, the Eye being in the true point of View, at E, they are both seen by the same Visual Rays as the original Objects themselves; and, consequently, under the same Optic Angles. Wherefore their Appearances are the same on either Surface; by Theorem 1st.

SECTION IV.

Containing some objections to the received Opinion of the Cause of VISION.

IN the last Section I have explained, demonstrated, and briefly illustrated the whole Theory of Direct Vision; or so much, at least, as is essential, to give a clear and just Idea of Perspective. To dwell longer on it, in this place, is unnecessary; seeing that, Perspective itself, in Theory, is nothing more than a continuation of the same Science. But, before I enter upon the subject of Perspective, I shall, in this Section, give my objections to the received Opinion of the cause of Vision.

It is not an easy matter to remove prejudices, which are deep rooted; especially, if they are strengthened with the sanction of Men of known abilities. I am at a loss to conceive why it is, that the generality of mankind so willingly impose on their own judgments, or are so ready to be imposed upon by others; eager of being amused with deep and profound Discourses, pretending to penetrate into the mysteries of Nature; which, neither themselves nor their Authors know any thing of. In fact, we see and know nothing further than the surfaces of Bodies; the formation of them, or their constituent parts, we are, and ever shall be, totally ignorant of. We daily see the effects of one Body acting on another, and, by repeated Experiments, we find their full force; that they are regularly and uniformly the same; but, the true cause of those effects are to us unknown; we cannot form an adequate Idea, how, and in what manner, they act and produce such wonderous effects, although so simple in their Nature.

We have not the least Idea how a Solid can become a Fluid and a Fluid become again a Solid, by different Degrees of Heat; what Fire is we know not, nor how it acts on Bodies, so as to dissolve some, to change the nature of others, and, in a great measure to annihilate them. We know as little of the Element of the Air we breathe in; or, by what means it is often so violently agitated in one place (as in high Winds) whilst it remains at rest, or has a contrary motion in another. We know not how any Solid is dissolved by a Fluid, and the Solid become perfectly liquidated; how two different Fluids mix and incorporate, so as to be beyond the power or art of Man to separate again, if they have an equal specific Gravity; nay two Solids, as Metals, &c. may be so incorporated as to be quite another kind of Body; and the constituent parts, of each, beyond the power of Art to dissever, or to distinguish them apart. Some again may be separated, if they have not the same specific gravity, as Gold or Silver, from any other baser Metal. Some Fluids can never incorporate; as Oil or Spirits with Water, &c.

How a Fluid is composed, or what form its minutest particles are of, we are totally ignorant. Is it not ridiculous to suppose Particles of any Figure? It is the common Opinion that they are globular; because, the least drop of Water or other Fluid assumes that form, than which nothing is more absurd; because, perfect Globes may be piled into a Pyramid, as firm and permanent, almost, as Cubes, with very little lateral pressure; consequently, a Fluid might be suspended in the same Figure, if its constituent Particles were solid Globes. For, on the supposition of Particles, they must necessarily be solid, or each Particle must, again, be composed of Particles.

To compare a Fluid with fine Sand is truely ridiculous. Sand will support itself in a pyramidal form; and, if the grains were perfectly globular, it would always assume one, of the figure of its Base, terminating in a single grain at the Vertex.

It

It is not so with Fluids; a Fluid cannot rest till it recovers an equilibrium in every part; presently acquiring a perfectly level surface. Besides, if its Particles were globular, there must be interstices between them filled with Air, which cannot possibly be incorporated with Water, as, in that case, it would be, to all sense. Also, if Metals, &c. when in a fluid state, by means of Fire, were composed of globular Particles, it is reasonable to suppose they would remain so when fixed; consequently, they would be extremely porous and admit Air to pass freely through, which is repugnant to common experience. No, say those acute reasoners, it is by reason of the Particles changing their Figure that they become fixed. Amazing truely! that the least particles of Matter, which I should suppose (on their Hypothesis) impenetrable by any means, can be varied in their Shape or Figure. It is also amazing, that, by being globular, they are so easily separated, when in a Fluid state, and, when fixed, are so extremely difficult to dissever.

If the Particles of solid Bodies are kept together only by cohesion, why does not the same power act on them in their fluid state? or why, when the particles of a solid Body are, by any means, separated, do they not cohere together again? or, what reason can be given, why they should cohere more strongly one way than another; as in Wood of all kinds? It is downright Sophistry, all; a language, or rather jargon, without meaning. That Bodies may be separated into Parts or Particles is undoubted; but, that they are composed of Particles, is what I cannot give them credit for. Every Body, whether Solid or Fluid, before any of its parts are separated, is one entire Mass of Matter.

Again, if Fluids are composed of Particles, and allowing them to be transparent in their nature, their globular form would render them, in some degree, opake; so that, a Fluid would not admit of distinct Vision through it. Now, I am clearly of opinion that all the parts of a Fluid lie perfectly close to each other, without any Cavities interspersed; it is impossible there can be any; consequently, there is not the least particle of Air contained in Fluids. If any dissoluble Body be immersed, as Sugar, &c. in Water; does not the Air, which is contained in its cavities, immediately ascend to the Surface, as soon as it is at liberty, by the dissolution of the immersed Body?

The cause of Transparency, either in Fluids, or, more particularly, in solid Bodies, is, to us, an impenetrable mystery. It is astonishing that there should be distinct and perfect Vision through Glass, or pellucid Stones; and, with all the knowledge and penetration human wisdom can pretend to, we cannot form an Idea tending to comprehend it. Yet, this aspiring, presumptuous, all-sufficient Man, would pretend to account for the cause of Vision, itself, and for all the other Senses; nay (if we will take their words for it) for every operation in Nature. But I shall make free to tell them, plainly, that they cannot account for one. First, let them account for their own existence; can they tell where the first spring of self-motion begins? no, not so much as for the motion of a Finger.

As for the generality of Mankind, they neither do nor are capable of judging for themselves, in many cases; and only give their assent, or dissent as they are influenced, or depending on the judgment of others, whom they either know or believe to have Judgment. That this is the case, in general, is certain; and is the reason why the opinions of some great Men are, though ever so arbitrary, adopted as orthodox; because, there are but few who dare venture to contradict them, and think or judge for themselves. And, because those Persons have advanced some received and established truths, these therefore conclude, that all which they presume to advance must necessarily be so; than which, nothing is more uncertain.

Those profound Gentlemen knowing their own importance, and the deference which the World pays to their Opinions, are but too apt to be intoxicated with it,

and often take too much Consequence to themselves; imagining, that they are really endowed with a penetration beyond the rest of Mankind. Or, from the knowledge they have of Mankind, the great deference they pay to their own superiority over them, and the implicit assent which is given to all their productions, make them forget, that they, as well as the rest, are fallible.

Respecting Vision. It is said in Smith's Optics, Page the first, and that from the Authority of Sir Isaac Newton, "that we shall find it difficult to conceive, "Light to be any thing else but very small and distinct particles of Matter, incessantly thrown out from shining substances; and every way dispersed by reflection from all others." I freely own, that I am at a loss to conceive how it can be Matter, and that for several reasons which may be given.

First; it is an indisputable Maxim, universally allowed by all Men of common understandings, that two Bodies cannot occupy or fill the same Space at the same Time; even Air cannot be where there is already any other Body. Steam, or Water expanded, almost, if not totally, excludes Air; consequently, no other Body can occupy the same Space where there is Air. Now, since the whole of Space, within our Atmosphere, at least, is filled with Air, how can it be also filled with Light, if Light be, also, a Body composed of Matter? for, since it is manifest, to occular conviction, that, in full Day Light, there is no part of the Space around us but what is wholly illumined; because, if any part was unillumined it could not be seen; consequently, being wholly and in every part filled with Light, if it be a Body, Air must be totally excluded; agreeable to the general Maxim, that two Bodies cannot fill the same Space. But, I presume, we do not find that to be the Case; for, the Space is filled with Air, and cannot possibly be excluded by Light.

I expect it will be alledged, that Air, being a Fluid, is composed of globular Particles; and, that Light fills up the interstices between them. Of what shape and magnitude, then, are the particles of Light, compared with the particles of Air? If Light be an existing Medium it must also be a Fluid; consequently, its Particles are also globular. And, if it be supposed to proceed from the Luminary, progressively, in Right Lines, in all directions, I own I am at a loss to conceive how that can possibly be effected, through innumerable globular particles of Air; to say nothing of the inconceivable velocity of its motion, being supposed to proceed from the Sun to the Earth in seven minutes and a half. For which extremely nice calculation we are indepted to Romer; who first observed, that the Eclipses of the Satellites of Jupiter happen, when in Opposition, that is, when the Earth is in a Right Line between the Sun and Jupiter, fifteen minutes sooner than when in Conjunction, or the Sun in a Right Line between the Earth and Jupiter.* The difference of the distance of Jupiter from the Earth, at these two periods, is the whole Diameter of the Earth's Orbit; computed at more than 160 millions of Miles; half of which is the Distance of the Sun from the Earth. From which curious observation it is conjectured, that we receive fresh Light from the Sun every seven or eight minutes; that is, the Sun being supposed to be continually emiting Light, it fills a concave Sphere, equal in Diameter to the Earth's Orbit, in that Time. Quere; how is the Sun supplied? and, being Matter, what becomes of it?

* It very rarely, if ever, happens, that they are in a Right Line, in a strict sense; because, the Plane of the Orbit of Jupiter is inclined to the Ecliptic, that is, to the Plane of the Earth's Orbit, in an Angle of one Degree and 20 Minutes. But, the Line of the Nodes, that is, the Line of intersection of the Planes of the two Orbits, is supposed to have a revolution (as the Moon's Orbit in 19 Years) but in what time is not, I believe, as yet determined. Consequently, they can never be perfectly in a Right Line, but only when the Line of the Nodes passes through the Centers of all the three; viz. the Sun, the Earth, and Jupiter. Wherefore, at a Conjunction, Jupiter will appear either above or below the Sun, according as he is in the upper or lower part of his Orbit; and, at an Opposition, the Earth will be either above or below a Right Line passing through the Centers of the Sun and Jupiter.

Now,

Now, admiting this extraordinary observation to be truely calculated, it undoubtedly proves that Vision is not instantaneous, at any distance; which seems reasonable; but I cannot see that it proves the progression of Matter from Jupiter to us. Besides, the Light which proceeds from Jupiter, or his Satellites, is but reflected, at second Hand; perhaps, Light, coming immediately from the Sun, may be much swifter in its motion. I am almost of opinion, that if it were possible to screen the Sun from our sight, by the interposition of an opake Body, close to its Surface, being removed in an instant, that we should see the Luminary instantaneously; and, the whole Space between be instantly illumined.

The Law of reflected Bodies, that is, the velocity with which an elastic Body rebounds, according to the force by which it is first impelled, I am not acquainted with. But, of this I am well convinced, that the velocity of any impelled Body is not uniformly regular, but is continually varying, and the Body, in motion, tending to rest; consequently, it cannot acquire an additional force by striking against another Body, at rest, so as to rebound with the same velocity by which it was at first impelled; and, if Light, as a Body, acts on the same principles, it must come, immediately from the Sun to us, with a greater velocity than when reflected from any other Body.

The Distance of Jupiter from the Sun is computed at 424 millions of Miles; and the Earth but 81 millions; which, is not one fifth part of the Distance of Jupiter. Consequently, the velocity with which Light is first impelled must be greatly diminished before it reaches Jupiter, and more so in being reflected back from Jupiter to the Earth. I shall endeavour to illustrate what I advance.

Let S represent the Sun, in the center of our System, of Planets, &c. and CDEF the Earth's Orbit, whose semidiameter SE is 81,000,000, of Miles. Plate II. Fig. 9.

Let AB be a portion of the Orbit of Jupiter, and J the place of Jupiter therein; whose Distance, SJ, is above five times SE or SC. Let a b be the Orbit of one of Jupiter's Satellites, in which the Satellite is supposed to be, at a, emerging from behind Jupiter, after an Eclipse.

2. It is affirmed that we see Objects, only by means of material Rays of Light, reflected from them to the Eye; which impress on that Organ the perception of Vision.

Suppose the Satellite, at c, to be in a direct Line behind Jupiter; it is plain that it could not be seen at C or E, because it would be hid by the Body of Jupiter; but, as soon as it emerges, that is, as soon as it can appear, as at a, in the Right Line Ca or Ea, it becomes visible; not, merely, because there is no other Body between, but because it becomes illumined from the Sun, in the Right Line Sa; which, whilst behind Jupiter, from a to b, was deprived of Light.

Let S a represent a Ray of Light from the Sun to the Satellite at a; the Earth, at C, being directly between. The same individual particles of Light, which pass from the Sun to Jupiter, pass by the Earth to Jupiter, and back again, before either Jupiter or the Satellite can be seen; according to their Hypothesis.

It is reasonable to suppose, that the velocity with which those particles of Matter first set off, from the Sun, is greater than when they arrive at the Earth; how much, then, must it be decreased before they reach Jupiter? Which Opinion, Rohault (Chap. 27, Art. 34,) endeavours, though in a very lame manner, to confirm, by a comparison with a conical Vessel filled with Water; being raised, at its Surface, by ejecting in more Water at the Vertex. The comparison has indeed some affinity, though a very gross one, because Light is supposed to expand as it spreads; by which means, its direct motion may be equible.

Suppose

Plate II. Fig. 9. Suppose Light loses one fourth of its velocity before it reaches Jupiter, it can have lost but one twentieth part at the Earth; and it is reasonable to suppose that the force with which it returns from Jupiter, cannot be equal to what it came with. Suppose then it has lost a fourth part more of its force, before it reaches the Earth again, at C; which, considering how languid its Light is, compared with the Light of the Sun, is very moderate. According then to this supposition, Light comes from Jupiter to the Earth, with little more than half the velocity with which it comes from the Sun, directly.

C represents the place of the Earth, at an Opposition and E at the Conjunction; the difference in the Distance of those two situations, from Jupiter to the Earth, is CE, the Diameter of the Earth's Orbit.

Now the motion of a Particle of Light (for a Ray, or continued Right Line of Light, I have no conception of) in its passage from the Sun to the Earth, from S to C, is, I presume, at its first and greatest velocity, always the same; but, at J, it has lost one fourth of its force; and, from J to C again, it has lost a fourth more; and consequently, in its passage from C to E, which is nearly half JC, it must necessarily have lost one eighth of the velocity it had, at its return to C; at which time the calculation is made.

Wherefore, the motion of Light, from C to E, has, at most, but half the velocity it has from S, to C or E, directly; and, it passes, according to Romer's calculation, from C to E in 15 Minutes; consequently, it passes from S to E (with double velocity) in a fourth part of the time, that is in 3 minutes and 3 quarters.

But, if the velocity, to and again, be regular and uniform (which I believe is contrary to the laws of motion, of Bodies impelled and reflected again) being 15 minutes in passing from C to E, half of which, SE, must consequently take 7 minutes and a half. For, a Satellite, emerging at a, when the Earth is at C, appears 7 minutes and a half sooner, and at E 7 minutes and a half later, than at D or F, where the distance of Jupiter from the Sun and the Earth is equal, according to the calculations made by the ablest Men. But since it is not possible, or, at least, probable that the velocity of Light can be uniform and equal, at all Distances from the Sun (if it has any motion at all) the greatest velocity must be at its first emission from the Luminary; and consequently, it takes less time in passing from the Sun to the Earth than half CE, in its return from Jupiter.

3. It is certain that no opake Body, that is, such as are not luminous, can be seen at a great Distance; unless its magnitude be sufficient to obstruct a great quantity of the Light of the Heavens; or when in a direct Line with a luminous Body, so as to obstruct its Light; or, when it is so situated, in respect of a luminous Body, that the Light, it receives from the Luminary, is reflected again;* of which, the various appearances of the Moon is sufficient proof. Consequently, we could not discern either Jupiter or his Satellites (whose immense Distance is more than seventeen hundred times the Moon's Distance) if they were not illumined by the Sun.

Jupiter being a superior Planet, that is, he moves in an Orbit beyond, yet concentric with the Earth's, he is, consequently, always illumined on that Face towards the Earth; and consequently, may always be seen, when above the Horizon, by the naked Eye; except when in near vicinity with the Sun. But, the Satellites are secondary Planets or Moons (of which Jupiter has four) accompanying the primary ones, as the Moon does the Earth; and are so very small, in comparison of their primaries, that they cannot be seen at that distance without a good Telescope.

* By Light being reflected, I would not be understood to mean, that there is any kind of real Matter projected from the reflecting Body; but only, by being illumined, itself, it becomes, in some degree, luminous, so as to shine with its borrowed lustre, and illumine others.

Now,

Now, if those Satellites cannot be seen, after they emerge from behind Jupiter, untill the Light, which is always ready at hand to illumine them, is reflected, back again, from the Satellite to us; and, suppose this Light to be a material Body, I am firmly persuaded that they would never be seen at all, by us; such an immense Space, from a to E, above 500 millions of Miles, for Matter to be projected in about 45 minutes, is beyond the power of my reasoning faculties to find credit for (with God nothing is impossible.) Besides, they not only reflect the Light directly, but also obliquely in all Directions, filling a Hemisphere; which is too gross an improbability (being Matter) to pass with a thinking rational Being.

The whole of this Phænomenon I suppose is this; (for I have never seen the Experiment; as it must require extraordinary Telescopes, which magnify to a great Degree, to discern the emersion distinctly) either it is impossible to perfect Telescopes sufficient to discern the emersion, till after the Satellite is considerably advanced from behind Jupiter; and also, from the near vicinity of the Sun, when in Conjunction with the Earth, at E, whose superior splendor may hinder the Satellite from being seen, for some time; together with the so much greater Distance, than when at C; all which, may concur to render it invisible for 15 minutes, after it might be visible from C. For I make not the least doubt, that their motions and revolutions are as regular and equal as the motion of the Earth itself; which, to us, is the only standard measure of Time.

4. If we see Objects, only by means of material Rays of Light passing from the Object to the Eye; by what means are opake Objects, which are immersed in Shade, seen at all? as they do not receive Light, immediately, from any luminous Body; nor, perhaps, from any other, opposed to them, by reflection. But, with such sophistical reasoners, who can give what properties they please to Matter, there is no arguing; seeing that, they can make Light reflect and rebound from one Object to another, at pleasure. But, are not those ideal properties of Light of their own creating, entirely? That Light is reflected from one opake Object to another is beyond a doubt; but, that real Matter is reflected, to and again, in every direction, I cannot acquiesce in.

Let AB represent, what is called, a Ray of Light, from some luminous Object, falling on any Surface, as CD. Fig. 10.

I shall suppose this Ray of Light to be a physical Right Line, the least in its dimensions, of breadth and thickness, that can be conceived; consequently, this physical Ray of Light can strike any Surface, on which it falls, in a physical Point only, the smallest that can be conceived.

Now, by the laws of reflection, according to all the writers on Optics, whenever a Ray of Light strikes or impinges on any Body, it is reflected again from that Body, making an equal Angle with a Right Line (BF) perpendicular to the Surface, at the Point in which it strikes the Surface.* Or, if it be a Plane Surface, it makes equal Angles with the Plane (ABI = EBK) both the Original Ray, AB, which is called the incident Ray, and the reflected Ray, BE, being in a Plane, which is vertical to the other Plane.

H It

* It is advanced by Sir Isaac Newton, (Prop. 8, Part 3. of the Second Book of his Optics) that Light is reflected from Bodies before it impinges on their Surfaces. But I must own, that in all the seven reasons he gives for that Opinion, I cannot find one of weight, nor all of them together, sufficient to prove it. Which, Smith, from that authority, and from Sir Isaac's first and fourth Quere, endeavours to prove, is not reflected immediately in an Angle, but makes a regular parabolic kind of curve, at a very small distance from the Body, which he calls the space of activity; and which space, he saies, is so extremely small, that, consequently, in physical Experiments, the incurvation of a Ray of Light may he considered as performed in a physical Point. Emerson saies (in a Scholium to Prop. 1. of his Catoptrics) that the Curve, described by a Ray of Light, is so extremely small, that it may be looked upon as a single Point.

It is, to me, astonishing, how any Person dare presume to advance such Opinions! and, for what purpose they do advance them! except it be to make the World have a high veneration for their extraordinary sagacity

Plate II. Fig. 10. It is reasonable to conclude, if this Ray of Light, AB, be material, and it is by means of material Rays striking the Organs of Sight, that Vision is performed, or the sense of Seeing is effected, that, the Point B could only be seen by an Eye placed any where in the direction of BE, as at E; BE being considered as the same individual Ray, AB, reflected or broken at the Surface, in the Point B. It is certain, that if the surface CD be Water, or a polished Mirrour, the Image of any Object, at A, will be seen at B, only, by the Eye at E, or any where in the direction BE.

But, experience convinces us, that the Point B, or any other Point in the Surface CD, may be seen as perfectly in a thousand other Directions; as at F, G, H, &c. Consequently, if Eyes were placed all around, within the compass of a Hemisphere, of which the Plane CD may be considered as the Base, or being elevated but a few Degrees above the Plane CD, the Point B may be seen by them all, at the same moment of Time.

Now, since the Point B can receive but one Ray of Light from the luminous Body, how can that identical Ray be reflected in ten thousand Directions, or in all Directions above the Plane CD, and those Rays to be material? at the same Time, the whole Space is filled with Air in every part. But, the same Eyes can also see every other physical Point in the whole Surface CD, as C, D, &c. consequently, every Point in the Surface must also reflect Rays in all Directions, crossing and cutting the former in every Point; which, unless, not only two, but, an infinite number of Bodies may occupy the same Space, and at the same Time, cannot possibly be, on the supposition that Light is a material Body.

5. Again. Since, as I have observed before, it is asserted that Vision is performed by means of those material Rays of Light, which enter the Aperture of the Eye, and, impinging on the Retina, affects the Optic Nerves with the perception of the Objects from which they flow, how shall we account for Vision through Glass, or much harder diaphanous Substances, as Crystal, or Adamant? which is impenetrable by any tool made of the hardest Steel.

Let any Object, as AB, be seen, by an Eye at E, by means of the Visual Rays AE, BE, &c. and, let any transparent uncoloured Stone, as Crystal, &c. be interposed directly between the Eye and the Object, as CD, of any thickness; having its opposite Surfaces parallel Planes, and well polished. The Object, AB, will be seen thro' it instantaneously, as represented at a b, with every circumstance of Colour, &c.

Fig. 11.

Now, I should be glad to know, how or by what means those Rays of Light, transmitted or reflected from the Object, AB, to the Eye, at E, pass in an instant through this solid Body of Crystal or Adamant, if they are material? and first, I should be glad to know, what is the cause of Transparency? or, in what does the difference consist, between transparent and opake Bodies?

The reasons given by Sir Isaac Newton, in the third part of the second Book, and which, Smith, in his Optics (Ch. 8, P. 95) has very wisely copied, word for word, concerning the cause of Transparency, Opacity, and Colours, in Bodies, is nothing at all to the purpose; it does not convey, to me, the least Idea of the cause of Transparency, and how Vision is conveyed through transparent Mediums, in every direction, instantaneously.

sagacity and penetration. Can they, either by experiment or otherwise, prove the truth of their Hypothesis? or do they suppose mankind so credulous as to give them credit for it, from the ridiculous experiment of a Hair, or of the Knives, and fringes of Colours produced by them? And yet it is certain, that some (not to be behind them in penetration) either do or pretend to do; for I have heard the same thing advanced, verbally, by a Person who has a tolerable share of mathematical knowledge.

See Fig. 10. No. 2. AB is supposed to be an incident Ray, and CD the reflected Ray; making a Curve, BEC; before it touches the Surface GH; which Curve is performed in so small a Space, that the Points, B and C, are supposed to coincide. Quere; how is the Curve to be ascertained, and determined? or, how shall we have conviction that it does not touch the Surface GH?

The

The learned and reverend Dr. Harris, in his Lexicon, tells us (under the article Transparency) that, a diaphanous or transparent Body is one which, probably, has its Pores all Right or Direct, and nearly perpendicular to the Plane of its Surface.

By the Pores being all right or direct is meant, I presume, that they go directly through, in Right Lines, without any obstruction; that they are perpendicular, or nearly so, to the Surface, I must needs say, he was extremely sagacious who first found out that extraordinary quality. But how shall we interpret the Word, probably? is it not indeed a candid acknowledgement, that it is all nothing more than Conjecture? I cannot say that it is even a probable Conjecture.

I am afraid that the Doctor was not very happy in his memory; for, in another place he tells us (under the article Diaphaneity) that the Pores of a diaphanous Body are so ranged and disposed, that the Beams of Light can pass freely through them, every way; which plainly contradicts the former assertion, that they are nearly perpendicular to its Surface; and which, in reality, is saying nothing, because, a Surface may be made at pleasure; and we cannot suppose that the Pores can change their Position, as a Surface may be altered at discretion; or how a Ray of Light, in passing through a triangular Prism, can be nearly perpendicular to both Planes, I am at a loss to devise.

For my part, I look on it as ridiculous and presuming, to tell us that Glass and all transparent Bodies are full of Pores and minute Interstices, through which the Rays of Light have a free passage; because, these Pores must be in every part of it, and in all Directions; neither can there be any solid part, between one Pore and the contiguous or adjoining Pores; for, if there be, it must necessarily impede, or entirely stop the progress of some Rays of Light in their passage to the Eye, and, consequently, will prevent the Vision of the Object from being perfect. Wherefore, since these Pores are so very numerous, have no solid part between them, and that, in all Directions; for it is plain, that they are not confined to one position, but must lie in all directions, which is evident; for, turn the Glass or Crystal, CD, as you please, the perception of the Object is as distinct as before; from which it is clear, that the Pores must be the same in all Directions, and then, I would ask, what becomes of the solid Body? for, in reality, it must be all Pores.

It is an insult on our understandings, on our reason, and on common sense, to suppose, or for any person to attempt to persuade us, that Glass, or Adamant, the most compact of Stones, is more porous, or that the Pores are more direct than in soft Wood, or in any Wood; which every one, who has considered it, knows to be full of Pores (like Veins in the animal Body) through which the Juices pass for its nutriment; and, in many kinds of Wood, they are perceivable to the naked Eye, and lie in direct Lines. Can any Person persuade himself, that the Pores are more numerous, or more direct, or that they are more capacious, so as to admit a freer passage, through Adamant, for the most subtle Fluid whatever (which I will suppose Light to be) than through a thin piece of Wood; in which the Pores are obvious, and clearly visible? Air, which is much denser than Light, or Water, still denser than Air, has a free passage through; but not through Glass, I presume. Yet, if you oppose it to a Candle (being cut very thin across the Pores) you may perceive, indeed, a few scattered Rays of Light pass through, but very far from having distinct Vision of the Candle, or even the out Line of the Flame, only. The reason is very obvious; because, the parts between the visible Pores, being more condensed and compact, receive the Light which falls on them, and either absorb or reflect it; which, therefore, does not pass through to the Eye; consequently, the parts of the Object, from which it flows in a direct line to the Eye, cannot be seen; therefore, the Vision of the Object is imperfect.

As this is evidently the case when it is opposed to a luminous Body, which is seen by its own Light; what will be the consequence when it is opposed to an opake

Plate II. opake Body, which (as we are told) is ſeen only by means of reflected Light? Why, that there is not the leaſt appearance of it to be ſeen at all; not even through fine oiled Paper, which, I muſt needs ſuppoſe, is infinitely more porous than any Glaſs or Stone whatever.

Whereas, through perfectly pellucid ſubſtances, every viſible Point, in any Object whether luminous or opake, is diſtinctly ſeen, without the leaſt impediment; conſequently, if Light is material, and a Ray is tranſmitted from every Point, in a viſible Surface, to the Eye (for every phyſical point may be ſeen) and if thoſe Rays are conveyed through tranſparent ſolid Bodies, they muſt neceſſarily paſs through Pores, direct in all poſitions.

Now I am fully convinced, that the Pores in Glaſs &c. are of thoſe ingenious Gentlemen's own creating; who, when they are at a loſs for proof of certain Hypotheſes (for want of better) they imagine Bodies to poſſeſs ſuch and ſuch qualities as may beſt anſwer their purpoſe. But, are thoſe Chimeras, of their own fertile imaginations, to paſs on the World for real exiſtencies? are the Concluſions drawn from ſuch Premiſes candid? by no means, they are very diſingenuous, inſomuch, that I deny it to be in the power of any Man, to give occular or other demonſtrative proof that there are Pores in Glaſs or tranſparent Stones; and, I do believe that the moſt pellucid ſubſtances are the freeſt from Pores; for all porous Bodies are compreſſible into leſs compaſs, which neither Glaſs nor Stones can poſſibly be; nor Water, which is perfectly tranſparent.

Pores are, in my opinion, rather the cauſe of Opacity than of Tranſparency in Bodies, ſeeing that they abſorb the Light in their receſſes. Yet, I do not ſuppoſe that all Bodies, as Wood or Metals, which are the freeſt of the kind from Pores, have any degree of tranſparency, but when they are exceeding thin; as leaf Gold, &c. but, they approach nearer to tranſparency than the more groſs and porous kinds. The real cauſe of Tranſparency, and how Viſion is conveyed through tranſparent Bodies, are (I am firmly perſuaded) among the hidden myſteries of Nature, which is not given Man to explore.

6. It is, I believe, a Paradox, not eaſily accounted for, that, if two triangular Priſms, of Glaſs or other diaphanous Subſtance, having equal Angles, are ſo placed together (as ABC, BCD) that the outſide Faces (AB and CD) are parallel, we have direct Viſion through them both (from the Eye, at E, to an Object at F); whereas, if either of them (as BCD) be taken away, the Object is loſt to ſight from that Point of View, by reaſon of the ſuppoſed Refraction of the Rays of Light, in paſſing through the Priſm ABC.

Fig. 12.

Now, if the Pores, through which the Rays E a, E b, &c. paſs, go directly through both Priſms (from a to c, and from b to d) how can they be varied by the removal of one of them? the taking away of one Priſm can certainly have no effect on the Pores of the other, ſo as to alter their Direction; yet (whatever be the Cauſe) it is certain, if either Priſm, as BCD, be removed, that the Viſion, of the Object F, is turned aſide out of a Right Line, and totally loſt to ſight, from that Station; for, it will be ſeen by an Eye at *E*, and not at E, and appear to be in the direction *EF*. Or, the Object, at F, being removed to G, will appear to be at F, in the direction of EF; and F will be its apparent place.

Again; if the Pores go direct through Glaſs, &c. the Rays of Light do not ſtop at the Surface, and conſequently, they cannot ſuffer either Reflection or Refraction by it. For, I preſume, the true definition of a Pore is a ſmall Cavity or Interſtice, which admits a free paſſage for Fluids. If, therefore, they do enter, and paſs freely through, how can the ſurface affect them? or how can the Rays of Light, if Light is a Body, be turned aſide, within the Glaſs, in any other direction than that of the Pores?

It

It is affirmed and allowed, that the Angle of Refraction is always to that of Incidence in a certain Ratio or Proportion; and since the Angle of Incidence may be any Angle at pleasure, it necessarily follows, that Light passes freely through in all Directions; which (according to the established Hypothesis, that it is corporeal) implies Pores in all Directions, according to the Doctor's Definition; and if it were possible for Pores to go right or direct through, in all Directions (which is repugnant to reason and absurd to suppose) the whole Prism must be all Pores; which omniporous quality, being attributed to any kind of bodily Substance, I am persuaded, no Man, in his senses, will acquiesce in.

7. I shall just give one more objection to the materiality of Light, and conclude this Section, and Subject.

Suppose the Eye, at E, viewing an Object, AB. There is supposed to be Rays of Light, AE, BE, &c. transmitted from every point to the Eye, forming an Image of the Object on the Retina, which is generally allowed to be inverted; although a late writer has given some reasons to the contrary. Be that as it may, it is certain, that if these Rays of Light enter the Eye, they must pass through the Aperture or Pupil, and converge to a Point, E, within, before they can diverge again to form an Image of the Object; at a c b. Fig. 13.

Now if the Eye be so situated (in respect of Distance) to the Object, that the extreme Rays, AE and BE, incline to each other in an Angle, AEB, not exceeding 60 Degrees; it is certain, that the Eye is capable of taking in the whole of that Object at one View; although, every part cannot be distinctly seen at once. Every physical Point, in the Surface of the Object, as C, D, F, &c. is supposed to transmit a Ray of Light to the Eye, as well as in all other Directions, at the same moment of Time. Consequently, the whole System of Rays generate a Cone or Pyramid of Rays, close wedged together in every part; which, all enter the Eye at the same instant, passing through a Point of the same dimension as one single Ray, at E, the Vertex of the Pyramid or Cone; which Circumstance is so egregiously absurd, that it is sufficient, in my opinion, to refute all that can be alledged concerning the Rays of Light being material.

For, can a great quantity of Air (the rarest Fluid that we are acquainted with) pass through a Pin-hole in an instant? Will it not (like Water) require more or less time, according to the dimensions of the Hole, to pass through? can any force drive the same quantity of Air through a Hole of half an inch in Diameter, in the same Time as through one of an inch? No; the ratio of the Time required, with the same force, will ever be in proportion to the Area of the Aperture; that is, the Ratio of different Apertures, to each other, is equal to the Ratio of the Time required, with equal force.

And yet, a Pyramid of Light, whose Base is equal to St. Paul's Cathedral, nay, millions of times greater (for the Eye can take in, not only the Sun, but, a great part of the Hemisphere, at once) and its Altitude of any Dimension, as far as the fixed Stars; yet, I say, this prodigious Pyramid of Light, a material Body, a Fluid, can pass (without any known impulse) through ten thousand imaginary Pores in the Cornea in their passage to an imaginary Point within; through which, the whole is conveyed in an instant, to the seat of Vision; and what becomes of it after? the Eye must be very capacious to contain it all. Nay more; the very same Base, or Object, can send forth millions of Pyramids of Light, to other Eyes all around, at the same Time, and in all Directions. Impossible! being Matter. Can any Person form an Idea, to comprehend how the same Body, or Surface, simply, can emit, or reflect what it receives, this instant, from any other Body, in innumerable directions at the same Instant? Let those, who can, find belief and give credit for it.

Much more might be said in support of this argument; but, as it is not directly to the purpose of Perspective, I shall not trespass any longer on the Readers time and patience, in this Digression from the Subject. I shall only beg leave to draw from it this conclusion; that Light is not an existing Medium composed of Particles; which, being reflected from Objects in all Directions, and striking on the Organs of Sight, conveys the Vision of them to the Mind, and occasions the sense of Seeing; intimating, by means of the different qualities of its heterogeneous Rays, not merely the existence of the surrounding Objects, but, they are also supposed to excite, in the Mind, the idea of Colours on their Surfaces, which otherwise have no existence: strange Doctrine!

But, with submission, I think, that the perpetual existence of such a Medium is repugnant to the notion of luminous Bodies emiting Light, incessantly; which, proceeding from them progressively, in Right Lines, excites Vision; not instantaneously, but propagated in Time: which, Opinion, is more consistent with the notion of its being reflected from other Bodies. For, how a stagnating Medium, a Fluid, can be so actuated, as to be reflected, from Objects, in all Directions within itself, and consequently, in direct Opposition to its first, or incident motion, and with such amazing velocity, is beyond the reach of human reason to conceive; much less to comprehend and explain.

It is the distinguishing Property of lucid or luminous Bodies to dispense Light all around them; but how, or in what manner, it is not my intention to enquire into; being well assured, that the attempt would be as fruitless as presumtuous. Such Phœnomena are, and ever will be, to Man, impenetrable and inscrutable; mysteries not to be unfolded but by infinite wisdom, itself. To us, there is a large field of knowledge, open and in view, whereon to exercise our reasoning faculties, and which lies within our reach; let us not, then, step aside into intricate Mazes and Labrinths, out of which it is impossible to extricate ourselves; in which, the farther we wander the more we are bewildered; till, wearied with the vain pursuit, we are, at last, obliged to own, that all our boasted knowledge is but to know how little can be known.

SECTION V.

Of Refracted Vision.

THE last Section touched, though very slightly, on that part of Optics called Catoptrics, or reflected Vision. In this I shall briefly touch on Dioptrics or refracted Vision; both which, I look on, in many respects, as Deceptions in Vision. For it is evident, that, in the case of Reflection, on the surface of Water, &c. or polished Mirrours, we do not see the Object, but only its Image or Appearance. So likewise, in the case of Refraction through transparent Mediums, solid or fluid, we do not see the real Object, but its Image or Representation only; which is likewise a Deception; seeing that, the Object appears, in some cases, larger, in others, smaller than it really is, and always appears to be where it really is not; although, we imagine, that we are looking directly at the Object.

Perspective Representations, are also manifest Deceptions in Vision. The business of Perspective is to represent Objects on a Plane Surface, by the rules of Optics; which,

which, in the true Point of View, will give the Idea of a real solid Object (being properly and judiciously shaded) having the appearance of projectures and recedeings, of one part before or behind another; and having also the same hue or teint of Colour, the Deception becomes stronger; insomuch, that it is possible so to deceive the Eye, as to imagine the Representation to be real and substantial. Plate II.

In looking at Objects through a Telescope, either reflecting or refracting; although we level it exactly to the Object, in a Right Line, and imagine we look directly at the Object, through the Tube; yet, it is plain that we do not see the Object. For, in a Reflector, we look directly at a concave Mirrour, which is placed direct between the Object and the Eye; consequently, in that case, it is impossible to see the Object, but only its reflected Image on the Mirrour, which is first received on another Mirrour, at the other end through which we look, and reflected again to the former, opposite to the Eye, (a most curious and ingenious invention; which was first constructed by Gregory, and thence called the gregorian Telescope.) The Image, of the Object received on the Mirrour, is magnified, by means of convex glasses in the small Tube; so that, we do not see even the reflected Image on the Speculum, but the Image of that, only, magnified to a great degree. In respect of the refracting Telescope, the Object itself is magnified, in the same manner as the Image on the Mirrour, in the Reflector; so that, we do not, in either case, see the real Object, but only its magnified Image, between the Object Glasses and the Eye. I shall illustrate it by a single magnifying Lens.

1. Suppose an object, as AB, and a convex Lens, CD, placed between the Object and the Eye, at E. It is manifest, since Vision is conveyed in Right Lines from the Object to the Eye, that, if we saw the real Object, by the Visual Rays AE and BE &c. it would appear, in proportion to the Lens CD, of the magnitude *a b*, only; but we find, that it appears larger than its real magnitude, in proportion to the Lens. Fig. 14.

For, if the Rays A a and B b fall perpendicularly on the convex Surface, CHD, towards the Object, they will pass directly to the other Surface, which is concave towards the Object, cutting it at a and b, and are thence refracted, to the Eye at E, as aE, bE; then if A *A* and B*B* be drawn parallel to the Axe of the Lens, EI, and Ea, Eb is produced, cuting A*A* and B*B* in *A* and *B*, then, is *AB* the apparent place of AB, or its Image; which, being seen under a larger Angle *A*E*B* then the real Object AB, if the Lens was removed, or Plane, it will consequently appear larger; by Theorem 1st.

Hence it is manifest, that the real Object, AB, is not seen; for it is not possible that the real Point, A, can be seen under the refracted Lines Aa, aE; and if Ea, Eb be produced, it is evident that the Object A B, in that place, must be larger, to appear equal to ab, on the Lens.

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Eye

Plate II. Fig. 14. Eye being removed to *E*; for *ab*, the Image of AB, appears, to the Eye at *E*, under a greater Angle, *aEb*, than the Lens CD, under the Angle CED; and therefore, AB, or its Image *ab*, appears, to the Eye at *E*, larger than the Lens C D, which it never could do, if its apparent place was on the other side of the Lens.

In this Case, the Object AB appears erect; but, if another Object, FG, be placed beyond the Focus of the Lens, and Right Lines F*f*, G*g*, be drawn, through the Center, I, of the opposite Surface, consequently perpendicular to it; and if F*G*, G*F* be drawn parallel to the Axe cuting F*f* and G *g* in *G* and *F*; then is *FG* the Image, or apparent place of FG, which, in this Case, is inverted; and consequently, the real Object FG is not seen; for the extreme F is seen at f, and G at g, on the surface of the Lens, in a contrary direction to their true places.

The contrary effect is produced through a concave Lens, in which, the Object always appears less than its real magnitude, according to its Distance; which, being but the reverse of the other, 'tis needless to illustrate it.

It is evident, that Vision is conveyed through any transparent Medium whatever, obliquely situated, in refracted or broken Lines; consequently, in such Case, the Object never is where it appears to be. For, the Point A, which is supposed to be seen by the Eye at E, appears to be at *A* or a, and not at A where it really is; and the Point S, which is seen directly through the Center of the Lens, although it be seen in the Right Line E S (for there is no refraction perpendicularly thro' parallel Surfaces of any kind) yet, its apparent place is at *S*; and, if either the Eye or the Lens be removed, the apparent place of the Object is varied.

2. Lenses of all kinds, excepting such as are plane on both Sides, or such Meniscus, (hollow on one Side and convex on the other) as being portions of concentric Spheres, consequently parallel Surfaces, partake of the nature and properties of Prisms, whose surfaces are inclined to each other. Wherefore, the Rays, in passing through them, are more or less refracted, according as the Surfaces are more or less inclined; that is, the less the radius of the Sphere, to which the Lens is formed, the greater is the magnifying power; because, the refracting Angle is greater; and, consequently, projects the Rays to a greater distance from the Center.

Hence it becomes a Paradox, how a large Object (I mean one, which, according to its Distance, occupies, or appears equal to the whole surface of the Lens) can be regularly magnified; for since it is manifest, that through parallel Surfaces there is no sensible refraction (except the Surfaces are considerably distant from each other and seen through obliquely as *E* b c e, Fig. 12.) and the surfaces of every Lens are parallel at the Vertex, and nearly so at a small Distance from it; and, the greater the angle of Inclination, the refraction is greater; wherefore (the surfaces of a convex Lens, being least inclined at its extremes, i. e. the refracting Angle is the greatest; and, seeing it is continually varying, from the Vertex to the Extremes) it seems reasonable, to conclude, that the Object would be more distorted about the edges of the Lens than near the middle; which I do not find to be the Case. Quere; by what means is the Object magnified equally.

3. There is another circumstance, which I do not remember to have seen noticed, by those who treat of the properties of Prisms, in respect of Refraction; and which, I do not see a sufficient reason for. Right Lines, seen through a triangular Prism, or any two inclined plane Surfaces, being parallel to the intersecting Line of the two Planes, that is, to the refracting Angle of the Prism, do not appear Right Lines, but, curved; and which is more or less curved, as the Angle of refraction is greater or less; the Curve being concave towards the Angle of Refraction.

Now

Now it is reasonable to suppose, that the System of Rays from any Right Line to the surface of the Prism, which is a Plane, would also form a Plane; which, by its intersection with the surface of the Prism, must necessarily generate a Right Line, in any position of the Prism; which, would be the Representation of the original Line; and, when the angle of Refraction is parallel to the original Line, they will also cut the other Surface, towards the Eye, in a Right Line, parallel to the opposite; how is it then that it appears curved? The vertex of the Curve, whatever it be, is where it would be cut by a Plane, passing through the Eye, perpendicular to the Original Line and the Planes of the Prism.

4. I have already described a Prism, in the last Section, and the nature of Refraction by it, as much as is my design, in respect of the Deception of Vision through a Prism. Those who would be more acquainted with the properties of Prisms and Lenses, in general, I refer to Sir Isaac Newton's, or, where they are treated more at large, to Smith's Optics; where, if he has patience to go through it, he will find enough to exercise his patience on. I do not mean to disparage the Work, for I believe it to be the best of the kind, in many respects; yet, I think it deficient, in some Cases; but, in respect, of Colour, &c. it is of a piece with the rest, and, being dwelt too long on, becomes tedious, if not trifling.

5. As to the Phænomenon of Colours seen through a Prism it is really suprising; but, it is only the Edges of Objects that are tinged or produce the Colours, on which there is any bright reflection of Light, not the whole Surface; and those edges or apparent Lines are most coloured when they are parallel to the refracting Angle; for, when they are perpendicular to it, and seen direct through, they have no Colour at all, but the natural Colour of the Body, whatever it be. Also, the Colours seen through the Prism, do not always follow in the same order as when a Beam of the Sun's Rays passes through; for the Red and Blue are often joined; an intense Red always follows after any Line or edge of an Object which is opposed to, or which obstructs the Light in one position of the Prism, the brighter Colours succeeding; which being reversed, is a deep Purple and Blue following, the other Colours are but very faint. The Colours are more intence and vivid the brighter the Object, or the greater the opposition or obstruction of the Light by an opake Object; which, being uniformly coloured, and having no sudden reflections of Light thereon, is not at all varied in looking through a Prism; but only at the edges, where there is an immediate opposition with a darker Body. And, how it is possible to deduce or to draw a conclusion, that all Objects, which we perceive, are by Nature of the same, or have no Colour at all, save what is effected by the Light reflecting from their Surfaces is, I must freely own, to me, as much a Paradox as ever. (See the second Paragraph, Sect. 1. Page 2.)

6. As, in Nature, there is necessarily Refraction at the common Surface of any two different Mediums, whether solid or fluid, except when we look perpendicularly through the Surface or Surfaces at the Object, although the true reason of it be to us unknown; so it would be superfluous and foreign to my Design to multiply Cases wherein it happens. I shall, therefore, only take notice of one common Case, which is so very common, though but little attended to, as to be obvious to every Person who is blessed with the faculty of Seeing.

Let any Person put a streight Stick into Water, slanting in any direction, and it immediately appears to be broken, at the surface of the Water; the part within, taking a contrary direction to that which is out of the Water; and the more it deviates from a Perpendicular, the more it is refracted or broken, till the Stick makes an Angle, with the surface of the Water, of about 45 Degrees; and then, the Refraction is continually less, which is evident; for, when the inclination of the Stick, to the Surface, is such, that the Angle it makes with the Surface is very small, consequently, the Refraction cannot be great; and being immersed perpendicularly,

Plate II. dicularly, there is no Refraction; which is the same thing as looking directly into the Medium, at an Object in the Water, or through a Body of Glass, very thick; in which Case, the Object appears to be considerably nearer, but, it is in the same Direction as we see it. Also when we look at the Heavens of Stars, &c. those which are in the Zenith are seen in a Right Line; and the farther they are from that Point, the more the line of direction is refracted or broken, so that, they are not in that place in the Heavens where they appear to be; which Refraction, is owing to the Atmosphere of Air and Vapours, which surrounds the Earth, being denser than the upper Regions.

7. It is a common, though curious and entertaining, Experiment, to immerse a piece of Money or other Object in a Bason full of Water; which will appear to be raised considerably higher than when the Vessel is empty.

Fig. 15. Let ABCD represent a Vessel, suppose of Glass; and suppose E an Eye looking at an Object, at F, at the bottom of the Vessel, being empty. Whilst the Eye remains fixed, at E, so that the Object, at F, can just be seen over the edge, at c, let there be Water poured gently into the Vessel, and the Object will appear to rise, gradually, to G, so as to be seen quite clear of the edge of the Vessel, in the direction EG.

Now it is certain, that the Object is at F, which appears to be at G; consequently, if the real Object be seen at all, it is seen in the refracted Lines FI, IE, where the Right Line EG, to its apparent place, cuts the surface of the Water; but, more probably, the Image of the Object, only, is seen at G. Consequently, a streight Stick or Wire being put into the Water, from E to F, will appear broken at the Surface (at C) and appear to go in the Direction CG.

Again. Let the Eye be removed to *E*, the Water remaining in the Vessel, so that the Object is apparently seen over the edge, at C; and, whilst the Eye remains fixed, let the Water be drawn gently off, by means of a Cock, or otherwise, at the bottom; the Object, apparently at G, will gradually sink lower in the Vessel, and totally disappear from that Station.

Hence it is manifest, that when the Water is in the Vessel, the real Object, at F, is not seen by an Eye at *E*; for, if it was possible, it must be seen through the Side of the Vessel, at H, which it is plain is not the Case; and, hence it is plain, that the Object being seen, apparently at G, is a manifest Deception in Vision. The Case is the very same, in Objects seen oblique through Glass, or any other pellucid Substance; excepting some small variations in the degree of Refraction.

8. Refraction in Water, according to all writers on Optics, is subject to one invariable Law, without any sensible error, but it is not to be demonstrated mathematically. I have made the Experiment myself, as accurately as it will admit of, and find it to be nearly as follows.

Fig. 16. If AB be supposed the Surface of Water, and CE, DE or FE an incident Ray of Light falling on it, at E; on which Point, if a Circle be described, and a Perpendicular, EG, be drawn, let CK, DL, &c. be drawn, perpendicular to EG. Then, if CE be produced to c, and a c, equal CK, be drawn; take a b equal to three fourths of a c, and draw b d perpendicular to a c, cuting the Circumference in d; the Ray CE is supposed to be refracted, at E, and go in the direction E d, into the Water. By the same Rule, DE goes in the direction Ee, and FE in E g.

But, the same Rays CE, DE, &c. will also be reflected in equal Angles GEH equal DEG, &c. Can the same Ray be both reflected and refracted? impossible; yet, an Object, o, may be seen at H, by the reflected Ray EH, as well as at e, by the refracted

refracted Ray E e. It is however certain, that if a Circle be described on a Plane, as AGBg, and the Lines are described thereon, as in this Figure; being immersed perpendicularly in Water, to the Diameter AB; CEd, DEe, and FEg will appear Right Lines; which is very surprising; but I do not see which way it proves, that the Rays of Light go in those Directions into the Water; they rather come in those directions, out of the Water.

9. Before I conclude this Section and Subject, I cannot help taking notice of an extraordinary passage; which is in the Conclusion drawn from Prop. 8, Part 3, of the second Book of Newton's Optics, Page 69; concerning the extraordinary porosity of Water; which is 19 times lighter, and consequently, he saies, 19 times rarer than Gold; and Gold is so rare, that Water may be forced through its Pores. For, as he was informed, by an Eye witness, a Globe of Gold being filled with Water and sodered up (but of what thickness he does not tell us; I have heard say above a quarter of an Inch) and being pressed with great force, the Water squeezed through its Pores; and stood, all over its Surface, in multitudes of small drops like Dew. From which, he concludes that Gold has more Pores than solid Parts; but how such a conclusion can possibly be drawn I cannot conceive. I should suppose, that, in such Case, the Water would spout out in streams, rather than stand on the Surface like Dew. Yet Gold will not admit either Air or Light through its Pores, though much rather Fluids than Water; which, he also concludes, from the same Experiment, has above forty times more Pores than Parts; and consequently, Gold (the compactest Metal we know of) according to that Ratio, contains above twice as much empty Space as Matter.

It is no wonder that Water is pellucid, being so extremely porous, and admits Light so freely through its Pores; but, it is somewhat surprizing that it is not compressible, which all porous substances must be. And I think it also surprizing, that a Man of Sir Isaac's sagacity should advance so much on the credit of any Experiment he never tried himself; which he certainly had opportunity enough for, being Master of the Mint near thirty years.

If Water be so full of Pores, as Sir Isaac imagines, it might be compressed into less Space; which, the same Experiment absolutely proves that it cannot. If it be composed of globular Particles, they may certainly be squeezed so close together, that (supposing every Globe inscribed in a Cube) the difference between the quantity of Matter and of Space, is no more than the difference between a Globe and a Cube, whose Diameter and Side are equal, which is nearly double (I mean, that the quantity of Matter contained in Cubes, which admit of no Space between them, is nearly double of that which is contained in the inscribed Globes; and consequently, the Space between Globes, lying in that position, is nearly equal to the Globes) for the same Ratio must continue *ad infinitum.*

But it is manifest, that Globes will lie considerably closer together; as Circles leave only triangular spaces between them; so Globes of equal Diameters, will lie so together as to leave a kind of triangular pyramidal or prismatical Space between, joining each other. The difference between the Globes and the Space may be very nearly ascertained, by puting a quantity of equal Globes or Bullets into a hollow triangular Pyramid, which may easily be made, the true figure of a pyramid of Globes (for it is a regular Tetrahedron) and, having placed them regularly, pour in Water just to cover them, filling up the Interstices between; then take out the Globes and fill the Box equally with Water. I myself have tried the Experiment, and find the difference between the Globes and Space to be, as ·2 to 1, as near as can be; in the former Case they are nearly equal. Quere; where is the remaining 40 times to be found? are the least particles of Matter, which cannot be conceived of any dimensions, to contain, in each, 40 times more Space than Matter? for, as I have observed before, the ratio, of Globes to the Space between them, will continue to infinity; and what other regular Figure can there be, to contain more Space between them than Globes? The

Plate II. The Question then is, whether we are to give credit to such Assertions, or are we to credit the evidence of our own Senses and Reason? I shall trust the latter, before the bare assertion of the greatest Man that ever lived.

10. As I have had neither time nor inclination to go through the whole of Smith's Optics; I shall just take notice of one passage in it, in which his geometrical reasoning is very erroneous; but, whether it be of Consequence, in the following part of the Work, I have not enquired; as the Subject is not to my present purpose. I shall, literally, quote his own words, as follows.

In Vol. 1. Page 59. Art. 157, he saies. "The apparent magnitude of a given "line, AB, seen very obliquely at a given distance, OA, increases and decreases "in proportion to the increase and decrease of OP, the perpendicular distance of "the Eye from the line AB produced; provided the distance AO be very large in "comparison to AB. For let the Ray BO cut a line AC perpendicular to AB "in C; and while the Eye is raised or depressed in the perpendicular OP, the "line AC will increase and decrease as OP does; and so will the angle AOC, "subtended by AC, and this Angle measures the apparent magnitude of AB."

Fig. 17. First, he saies, that the apparent magnitude of AB increases and decreases as the Eye is raised or depressed, in the perpendicular OP; and the Angle, it subtends, measures the apparent magnitude of AB.

Let PO be produced to T; in which, take several Divisions, each equal to PO, at Q, R, S, and T; and draw QA, QB and RA, RB, &c.

Now, because PQ is double PO, and PR is triple, the Angle BQA, (according to him) will be double BOA, and BRA triple; for, AF being drawn perpendicular to AB, it is certain, that the Divisions AC, CD, &c. are equal, AF being parallel to PO;* consequently, AC increases as OP does; and so, he saies, does the Angle AOC, subtended by AC; wherefore, the Angle at Q is double, and, at R triple, of the Angle AOB. But, the Perpendiculars AC, AD, and AE are not the measures of those Angles; for they are truly measured by an Ark of a Circle, of the same or an equal Radius, only.

* Cor 6. 6. El.

Acccording, then, to his Words, the Angle AOB increases as PO increases; which I deny, and will prove to the contrary,
It is obvious, that the Angle AQB is considerably larger than AOB, but it is not double, although PQ is double PO; but, ARB is very little larger than AQB; and notwithstanding, PR is triple PO, yet the Angle ARB is not double AOB; as the Arks *a b*, *c d* and *e f* evinces.

Now by the same reason, the Angles ASB, ATB are continually greater than ARB; but, they are continually less. For, if PR be a mean Proportional between PA and PB, the Angle ARB is the greatest that can be made, by Right Lines drawn from the Extremes of AB, and touching the Perpendicular PT.

DEM. Describe a Circle through the three Points R, A and B; by Prob. 40 Geo.
Then, because PR is a mean Proportional between PA and PB (by Hyp.)
the square of PR is equal to a Rectangle under PA and PB; } Cor. 9. 6. El.
i. e. $PB \times PA = PR\square$ — — — — — }
Consequently, PR will touch, or be a Tangent to that Circle, drawn through the Points R, A, B — — — — — P. 16. 3. El.
But, the Angle ARB is at the Circumference of that Circle; conf. ASB and ATB, which are beyond the Circumference, are less than ARB. — 20. 6.
Therefore, ARB is the greatest Angle, touching the Tangent PT, subtended by AB; and consequently, the Angle ASB is not encreased, as the Eye is raised in the Perpendicular OP.

Again.

Plate II.

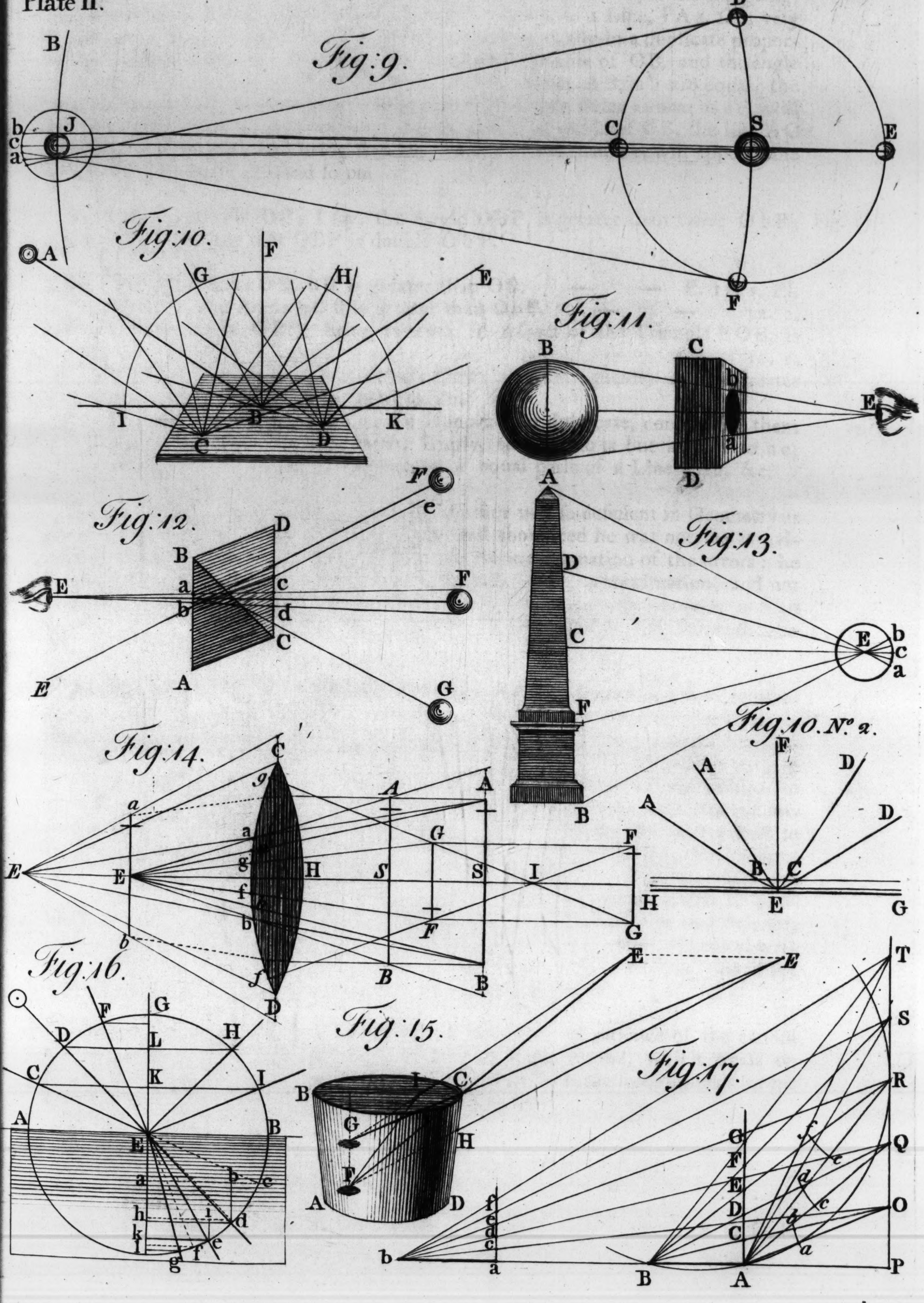


Fig:9.
Fig:10.
Fig:11.
Fig:12.
Fig:13.
Fig:14.
Fig:10. No. 2.
Fig:15.
Fig:16.
Fig:17.

Again. He saies, in the following Paragraph, as deduced from the foregoing, that " the apparent magnitudes of equal parts, AB, ab, of a Line, PAa, seen very " obliquely, at great distances from the Eye, are reciprocally in a duplicate propor- " tion of those distances. For example; let Ob be double of OB, and the angle " OBP will be double of ObP; and accordingly, since AB, ab are equal, the " perpendicular AC will be double of ac, and being seen twice as near as ac, will " appear four times bigger than ac. Again, if Ob be treble of OB, the line AC " will be treble of ac, and being seen three times nearer than ac, will appear nine " times bigger than ac; and so on.

Let Ob be double OB; I say, the Angle OBP is greater than twice ObP; and he expresly saies that OBP is double ObP. Fig. 17.

DEM. Because Ob=2OB, bB is greater than OB. — — P. 13. 1. El.
wherefore, the Angle bOB is greater than ObP. — — 12. 1.
But, the Angle OBP, being external, in respect of the Triangle bOB, is equal to bOB+ObB or P — — — — — 10. 1.
Wherefore, it is greater than double ObP; and consequently AC is greater than double ac; AB being equal to ab.

Therefore, the ratio of AC to ac is more than duplicate, considering them as similar Surfaces; but, as Lines, simply, their Ratio is but as AC to ac; and he saies, the apparent Magnitudes of equal parts of a Line PAa, &c.

Now I am far from supposing, that this Author was so deficient in Geometry as these examples seem to indicate; nay, I am well convinced he was not, as is evident in the next Book; and his Provisos are some extenuation of the errors: he ought, then, to have told us that they are nearly so, by approximation, and not that they are so, in express Terms. But, in any Case whatever, there is not an equal ratio, or nearly, between the increase and decrease of the Perpendicular AC, and the Angle AOC.

As this and the preceding Article are the only passages which any way tend to advance the Theory of Perspective (having been quoted for that purpose) I have, therefore, been more particular in my remarks on them. And, as what I have advanced in the two last Sections, and part of the first, is not directly to the purpose of Perspective, it will, I know, by some, be deemed impertinent and foreign to the Design of this Treatise; let them, if they please, pass it over and proceed immediately to the Subject, which is not vitiated by it. It was not my intent to swell either the bulk or price of the Book by such means (having other Matter enough) for I have run it to a much greater length than I at first intended, and that, after the price was fixed, in my Proposals. But, as I do not intend to publish a Treatise on Optics, and as Perspective has a near affinity to that Science, what exceptions I have always had, to sundry passages in the works of optical writers, I thought proper to give here, where I was treating on an essential part of the Theory of Perspective, and a branch of the Science of Optics.

I hope I have not trespassed, too much, on the time and patience of the candid and impartial Reader. I shall now proceed to Book the second, which treats on the Theory of Perspective; where, I shall endeavour to make some amends for his time spent, I hope not lost, in perusing this Digression.

B O O K II.

Of the Theory of Perſpective, rectilinear and curvilinear.

S E C T I O N I.

Containing a general INTRODUCTION to PERSPECTIVE.

TO define the Terms of Art peculiar to any Science with brevity and perſpicuity, I have always looked on as a particular excellence in the Work; but I have frequently found, that an affectation of brevity has left the Term, intended to be explained, rather obſcure, at leaſt doubtful; whilſt others, endeavouring to render it clear by a multiplicity of words, have, at laſt, involved it in perplexity.

Although my deſign is to be as brief as the nature of the Subject will admit of, yet, I am afraid I ſhall rather be thought prolix, than otherwiſe, in ſome of the following Definitions or Explanations; but certain I am, that, the Time ſpent in acquiring a perfect knowledge of all the Terms I have defined, will not be loſt, as they contain many uſeful hints and leſſons to a young Student. I have endeavoured to explain every Term in the moſt eaſy and familiar manner; not ſaying more than was neceſſary, yet, where it is needful, enough to be clearly underſtood. To be too brief is worſe than prolixity, ſo it be not tedious and trifling; the one leaves us doubtful, the other, probably, makes it clear at laſt.

I may, perhaps, be particular and ſingular in my opinions; but, I think it better to define each Term ſeparately, than to include ſeveral in one Definition, as is very frequent; and I always chooſe to name the Term, I mean to define, firſt, rather than end the Definition with it, or name it promiſcuouſly. My reaſon for which is, that 'tis much eaſier to find when refer'd to; or, when there is occaſion to look for any particular Definition, without reference, the Number not being known.

The Order in which Definitions are treated is, with me, a material circumſtance; beginning at the Foundation, and going gradually on in a regular ſucceſſion; never uſing any Term, if poſſible to avoid it, in the explanation of another, which has not already been defined. In general, I find them promiſcuouſly jumbled together, without Order; going, as it were from one end of the Science to the other, to and again; too haſtily introducing, perhaps, ſome new or favourite Term, before others which ſeem neceſſary to be firſt known, in order to prepare the way, by removing ſome impediment. How I have ſucceeded, muſt be left to the deciſion of the candid and impartial Reader.

I am no favourer or admirer of new and uncommon Terms; nor, indeed, do I think that every writer, on any Science, has a right to impoſe new Terms; unleſs he has found out ſome new Principles, on which it was not poſſible for him to expatiate, by the Terms already known and in uſe; as Dr. Brook Taylor has done in Perſpective;

ſpective; on which, I have ſpoke more largely in another place. It was impoſſible for him to convey the Ideas he intended to inculcate by the old Terms, and therefore, he was under the neceſſity of inventing and enforcing new ones; which are, moſt certainly, extremely expreſſive of the thing meant. But, if every writer on that or any other Subject (who, becauſe he knows ſomething of it, imagines that he knows more than any who have writ before him) was to take the liberty to impoſe new and unmeaning Terms, of his own, ſuited only to his own trifling Ideas of the Subject, the Science would, by that means, become perplexed and intricate; each Perſon, who happened to receive his knowledge of it from different Books, would, conſequently, underſtand and call the ſame Thing by different Names; than which, eſpecially when they are abſurdly or falſly named, nothing tends more to perplex, and involve the Science in obſcurity.

I have already, in the Preface, given my reaſons for omiting, in this Treatiſe, geometrical Definitions and Problems; becauſe, I ſuppoſe the reader already tolerably verſed in Geometry; if not, I adviſe him firſt to ſtudy it, at leaſt Practical Geometry; without which, it is uſeleſs to attempt, and impoſſible to ſucceed in the Study of Perſpective: the better he is acquainted with Geometry, the greater progreſs will he make in Perſpective; of which, Geometry is the foundation. To aſſiſt him in it, I have compiled and compoſed a Volume, which may be called an abridgment; it, nevertheleſs, contains all that is eſſential. In it, I have, for particular reaſons, collected all the uſeful Problems in Plane Geometry into one Book (which is the firſt) before I begin with the Elements, entitled Practical Geometry; in which, I have, in a great meaſure, omitted, and refer to the Elements for further Demonſtration.

My chief aim, in that work, was to make it uſeful, the Study of it pleaſant, and attainable to any tolerable Capacity, and applicable to various uſes in Life; particularly ſubſervient to this Treatiſe of Perſpective, as I always refer to it for Demonſtration: in which caſe, it may be deemed a part of this Work, and ought always go together. But, although I have, there, fully defined a Plane (Def. 6th) yet, as it is ſo very eſſential in Perſpective, it was by no means proper to omit it here; ſeeing that, on it the whole Theory of Perſpective is built.

A PLANE is a perfectly even, ſtreight, and regular Surface, which is neither convex nor concave in any part; but agrees, in every part, with a Right Line or ſtreight Ruler, applied, any how, to the Surface.

2. By the motion of a Right Line, a Plane may be conceived to be generated; either by a direct, lateral motion, on two parallel Right Lines, or, by ſuppoſing it whirled around (ſo as not to generate a Cone) on any Point in it.

3. It is eaſy to conceive, that, if the Eye of a Perſon be in a Plane, or in a continuation of it, the neareſt extremes or limits, towards the Eye, hide all the reſt of the Plane; for, the whole Plane vaniſhes, and is loſt to an Eye in the Plane; which appears but a Right Line, extended in length to the apparent dimenſions of the Plane, conſidered as having only length and breadth: thickneſs or ſubſtance is the property of Solids, a Plane has none.

N. B. The Picture (in Perſpective) is always underſtood to be a Plane. Therefore, the Board, Canvas or Paper, on which we draw, is the Plane of the Picture. See Def. 2d.

N. B. 2. A Plane may be of any Shape or Figure; and, in Perſpective, it is frequently conſidered as being infinitely extended, without regard to Figure, or to its limits.

If

Plate III. If a Solid be composed of, or bounded on all sides by Planes, its Surfaces, only, are the Planes; of which, no Solid can be formed having less than four; and that, must necessarily be a Pyramid.

§ Fig. 1. A Cube is composed of six Planes, which are all Squares; as A, B, X§. Every other Parallelopipid has also six Planes, which are all Parallelograms; either right angled, as Fig. 2, or acute angled, as Fig. 3.

* Fig. 4. † Fig. 1, 2, and 3. ‡ Fig. 5. A Prism is a Solid whose Base, A and Top, B, are either similar Triangles, * Quadrangles, † or Poligons ‡ of any Number of Sides. The other Planes or Surfaces (X,X) are always Parallelograms.

Fig. 6. Pyramids have their Bases (A) of any Number of Sides. The Planes of the Sides are always Triangles. As Y,Y.

Of PLANES and their POSITIONS in GENERAL.

1. Horizontal, is the first and most natural position of Planes. Such, are all Planes which are parallel to the Horizon; consequently, all horizontal Planes are parallel amongst themselves; as Z,H,H. Fig. 7.

2. Vertical, are all such Planes as are perpendicular to, or which cut the Horizon at right Angles; as V. Fig. 7, 8, and 9.

Fig. 8. Vertical Planes may be in all positions, in respect of each other, viz. parallel, perpendicular or inclined; as may be conceived, by revolving a vertical Plane, on a Right Line, AB, perpendicular to the Plane of the Horizon; consequently, they will, if produced, all pass through the Zenith and Nadir * of our Horizon.

3. Inclined. All Planes whatever, which are neither parallel nor perpendicular to the Horizon, are Inclined Planes.

For, if a Plane cut the Horizon, or would if produced, in an acute Angle, it is not Vertical or Perpendicular; consequently, it inclines to the Horizon on one side more than the other; which, Inclination, is always measured on that side making the acute Angle. e. g.

Fig. 9. The Plane X inclines to the Horizon, in the Angle BCA; and it is also said to incline to a Vertical Plane, in the Angle BCE; which Angle, if they have the same Intersection, CD, or parallel Intersections with the Horizon, is the Complement of its inclination to the Horizon. See N. B. Def. 13th. Geo.

4. It may not be improper, here, to observe, (for it is necessary to know and understand well) that the Angle of Inclination, of one Plane to another, can be measured, only in a Plane to which both the other are perpendicular; or, which is the same thing, if a Line be drawn in each Plane, from the same Point in their common Intersection, and perpendicular to it, an acute Angle made by these Lines is the Inclination of the Planes; for it is evident, that a Plane drawn through those two Lines will be perpendicular to the common Intersection † and consequently to both Planes. To illustrate it.

† 2. 7. El.

Suppose the horizontal Plane Z raised up into the Position of X, inclined to the Horizon; the line CD, on which it was supposed to turn, may be considered as the common Intersection of the two Planes, X and Z.

It

* Imaginary Points in the Heavens, diametrically opposite to each other; the one perpendicular over our Heads, the other under our Feet, in the lower Hemisphere.

It is evident, that the point A will, in that motion, have described the Ark AB; and, if the Angle ACD be a Right one (as it is supposed) BCD is still a Right Angle; wherefore AC and CB are both perpendicular to CD; § and the Plane ABC, described by the motion of CB, is also perpendicular to CD,† and consequently, to the two Planes X and Z. ‡ Therefore, the Angle ACB, in the Plane ABC, is the Angle of Inclination of those two Planes. § Def. 11. Geom. † 2. 7. El. ‡ 9. 7.

Through A, draw AD, at pleasure, cuting the common Intersection, CD, in D; and, from the same point D, draw DB, in the inclined Plane X.

DEM. Now, if ACD be a Right Angle, ADC is acute — Cor. 3. 10. 1. El.
wherefore, AD is longer than AC; and also, BD than BC — P. 12. 1. El.
But, the Chord, or Subtense, AB subtends both the Angles ACB and ADB; consequently, the angle ADB is less than ACB. — — Cor. to 14. 1. El.

Or, suppose BF perpendicular to the Plane Z; and let FD be drawn.

Then, a Plane DBF, passing through BF, is perpendicular to the Plane Z, but not to X. And, because FD is longer than FC, and BD than BC, it is manifest, that the Angle BCF is greater than BDF.

But, the Plane CBF is perpendicular to both the Planes X and Z (as before) and, consequently, to their common Intersection CD.

From which it is clear, that the Angle made by a Plane cuting two other Planes, perpendicular to their common Intersection, is the Angle of Inclination of those two Planes; seeing that, the Angle made by any other Plane, passing through AB or BF, will necessarily be less, the greater the Inclination of AD, or FD, to CD.

To set this matter in the clearest Light possible, it being so very essential in Perspective, as well as in other Sciences and Arts, I have added the following Figure.

Let ABC and CBD be two rectangular Planes, cuting each other in BC, their common Intersection. Let EF and FGH be two Right Lines, one in each Plane, perpendicular to their Intersection BC, at the same Point, F. Fig. 10.

Wherefore, the Plane EIFG, passing through those Lines, is perpendicular to both Planes, AC and CD; § and, the Angle EFG, made by that Section, is the largest that can possibly be made by a Plane which is perpendicular to either of them. § 2. 7. El.

For, suppose the Line EH perpendicular to the Plane ACB, only; and, a Plane EIKL to pass through that Line, it will be perpendicular to the Plane AC; † and because the Plane EFG is perpendicular to both the Planes AC and CD, and passes through the same Point E, it will, also necessarily, pass thro' the Line EH; wherefore, EH is the common Intersection of those two Planes,* EIFG and EIKL produced. I say, that the Angle EFH, made by the Plane EIFG, is greater than EKH. † 9. 7. El.

DEM. Now, EH, the common Intersection of the two Planes IFG and IKL, together with the Intersections EK and KH, of the Plane IKL with the two Planes AC and CD, form a Triangle; and so does the same line EH with the two Intersections EF and FH, made by the Plane IFG, with the same Planes AC and CD.

But HFK is a Triangle, and, the Angle HFK is presumed to be a Right one; wherefore, HK is longer than HF; ‡ and, for the same reason, EK is longer EF; and consequently, the Triangle EFH, having one Side, (EH) common with the other Triangle EKH, and, having the other two Sides, EF and FH, less than the two Sides EK and EH, respectively, they, therefore, contain a larger Angle, viz. EFH than EKH. — — Cor. to 14. 1. El. ‡ 12. 1. El.

* EI is the true Intersection, perpendicular to the Plane AC; and consequently, EH would be IE produced. But, as it would have run into the Figure below, I thought it best to dispense with it, as the Demonstration would be the same.

M But,

Plate III. But, the Plane IFG is perpendicular to both the Planes, AC and CD; and the Plane IKL is perpendicular to one of them (AC) only.

Fig. 10. Therefore, the Angle EFG, made by the section of a Plane which is perpendicular to both the other, is larger than any Angle made by any other Section, of a Plane perpendicular to one of them only; and consequently, the Section EFG measures the true Angle of Inclination of the Planes ABC and CBD.

N. B. The Angle made by the section of a Plane inclined to both Planes, AC and CD, may be either greater or less than EFG.

5. Although it is not possible to conceive an Idea of a Plane abstracted from one or another of the three Positions I have explained, yet, in the application of Planes in Perspective, as in Geometry, no particular regard is had to them; for one Plane is said to be perpendicular to another, if it makes Right Angles with the other Plane, as H to V: each of which is said to be perpendicular to the other, notwithstanding one of them (H) is really horizontal.

Fig. 11.

The Planes X and Y are also said to be perpendicular to each other, altho' both are inclined to the Horizon; and, whatever their Inclination to the Horizon may be, it matters not, if they make Right Angles with each other, as at C. For, if the Plane Y was turned up, on AB, its intersection with the Horizon, into the vertical Position W, and, along with it, the Plane X, into the horizontal Position Z; their Position, in respect of each other, is not altered, if the Angle, at D, be still a Right one, as before, at C.

6. So likewise, one Plane is said to incline to another, if they do not intersect at right Angles, as H and X, or would not if produced, as X and V.

The Plane X being inclined to both H and V, (Art. 3. of Planes) they are, for the same reason, both inclined to X; yet, one is horizontal and the other vertical; for, the inclination of two Planes is mutual. So that, when it is said that one Plane is perpendicular or inclined to another, it means nothing more, than, that they are at right Angles, or otherwise with each other; no regard being had to the horizontal or vertical Position of either; except the Position, of one, is previously known or determined, to which the other is said to be perpendicular or inclined.

7. In Perspective, it is also frequently said, that Lines are perpendicular to certain Planes; whereas, if the Plane be vertical, it is easy to conceive, from what has been said, that all Lines, which are perpendicular to a vertical Plane, are horizontal, and parallel amongst themselves; as AB, EF, and CD, to the Plane GIK. Yet, the Planes, in which these Lines are, may be either horizontal, as ABEF; vertical, as EFCD; or inclined to the Horizon, as ABCD.

Fig. 12.

If the Plane be horizontal, the Lines perpendicular to it are really perpendicular, i. e. to the Horizon; as AG, ED and FC. But, if the Plane be inclined to the Horizon, then, the Lines, which are perpendicular to it, are also inclined to the Horizon, yet parallel amongst themselves.

Suppose the Plane ILMN vertical, and perpendicular to the inclined Plane GH; the lines LI and MN, which are at right Angles with its Intersection, IN, are perpendicular to the Plane GH. But the line IP, which is perpendicular to the Horizon, is inclined to the Plane GH, in the Angle LIO. And OI, at right Angles with IP, is horizontal; but, it is also inclined to the Plane GH, in the Angle OIN equal LIP.

DEM. Let NI be produced, towards H. LI is perpendicular to NH.

Then, because LIN is a Right Angle, LIH is, also, a Right one - C. 2. 1. El. consequently, LIO, equal LIN−OIN, is equal to PIH, or LIH−LIP.

Every

Every Right Line, therefore, which is neither parallel nor perpendicular to a Plane, is, consequently, inclined to that Plane; and its Inclination may be known, by drawing a Perpendicular from the extreme, or any other Point, as MN, in the Line IM, to the Plane GH; the Complement of the Angle IMN, i. e. IML equal MIN, is the Inclination of IM to the Plane GH.

Or, if a Plane (ILMN) be drawn, through the Line IM, perpendicular to the Plane GH; the Angle MIN, which the line IM makes with IN, the intersection of the perpendicular Plane with GH, is the Angle of its inclination to the Plane GH.

N. B. IN is the Seat of the Line IM or IO, and also of ML, or, of the Plane NL, on GH; produced by the Intersection of a perpendicular Plane passing thro' the Line, as above. Therefore, the Angle which any Line makes with its Seat, on a Plane, is the Angle of its inclination to that Plane.

8. PQRS is a horizontal Plane, cuting the inclined Plane KPQ in the Line PQ. But, PQ is not the Seat of the Plane PR, on KQ, it being inclined to KPQ, in the Angle SPT; for, if ST and RU be drawn, perpendicular to the Plane KPQ, the Lines PT and QU (joining the Points, T and U, where the perpendiculars cut the Plane, with P and Q) are the Seats of the Lines PS and QR; TU is the Seat of the Line RS, or of the Plane TURS; and consequently, PQUT is the Seat of the Plane PQRS, on KPQ. Also, turs is the Seat of that part which is over the Ground Plane.

I would advise the young Student, who is not well versed in these things, to make them familiar to him; for which, the reading over a second time, with due attention to the Figures, will be sufficient.

I have been more particular on this Subject, because I have frequently known Pupils to be mislead, by calling Lines and Planes perpendicular, imagining them to be really so, i. e. to the Horizon; from the common acceptation of the Term, Perpendicular, (to hang down as a plumb Line) not considering the Position of that Plane or Line, to which the other Planes or Lines are said to be perpendicular.

9. Suppose the Object AIKC to be, in the lower part, a right angled Parallelopiped (the most general form for Buildings, or the several Parts of a Building) the Planes BEDC, and AIB, of the Front and End, and their opposites, are Vertical; for they are perpendicular to the Horizon, or Ground (considered as a Plane) on which it stands. Fig. 13.

Now, the Lines AB and FE, in the Plane AIB, are perpendicular to the Plane BEDC; and the Lines BC, ED, and IK, are all perpendicular to the Plane AIB (i. e. the originals of those Lines are so, in the real Object) yet, they are all parallel to the Horizon, and the three last, parallel between themselves, though in different Planes; § for, a Plane may pass thro' any two Lines that are parallel. (Ax. 5.) § 4. 7. El.

10. In the Practice of Perspective, it is often necessary to suppose the Object, we are delineating, transparent, as if the whole Object was Glass; and, the Planes or parts of the Building, which are adjacent, are supposed to be seen through the hither Planes; as ABCH, the ground Plane, and FEDG parallel to it; AFGH parallel (or otherwise) to BEDC, and HGDC opposite to AFEB.

These six Planes, forming a right angled Parallelopiped, compose the Body of the Building; every Angle of which, A, B, E, D, &c. is a solid Right Angle; each being composed of three plane Right Angles, as ABC, ABE, and EBC, of the Angle B; consequently, AB, BC, and BE are each perpendicular to the other.

By means of this supposed transparency, the connection of the several parts of an Object are distinctly seen, and delineated with greater accuracy; which, in some Cases, could not, without that expedient, be so certainly ascertained.

11. The

Plate III. Fig. 13. 11. The Roof, FIKDGE, is a triangular Prism, in its construction. The Planes, of which, are inclined to the Horizon; EIKD on this side, and FIKG on the other side, seen through; yet, these Planes may be perpendicular to each other, if the Angles, FIE, GKD, are Right. (See Art. 5.)

They are also inclined to the Front and its opposite, which are vertical Planes; but observe, that, their Inclination is not the internal Angle IEB or IFA, for those Angles are obtuse; but, if either of the Planes be produced, as BEDC to LM, there is made an acute Angle (IEL) with that Plane, equal to the Complement of IEB to two Right Angles; which, is the Angle of Inclination of those Planes.

N. B. The inclined Planes of the Roof, as well as the horizontal and vertical Planes AC, EC, &c. are perpendicular to the Plane AIB, and to HKC, its opposite.

This, I hope, is sufficient to make all I have said, relative to Planes and Lines, clearly understood; if so, it is a very material point gained towards understanding Perspective clearly; and will, also, greatly facilitate the Practice.

SECTION II.

Of the several kinds of PROJECTION, and other introductory Matters.

Plate IV. PROJECTION, is the description or delineation of Objects in Plano, or on a Plane, according to a certain Law; by means of Right Lines, called Rays, supposed to be drawn from every Angle of the Object to some Plane.

Fig. 14. As A a, B b, C c, &c. which, taken altogether, are called the SYSTEM OF RAYS.

When those Rays are all united in a Point (as AO, BO, &c.) it is called a CONE OR PYRAMID OF RAYS; and that Point (O) being supposed an Eye, and the Right Lines OA, OB, &c. Visual Rays, the System of Rays is then called, the OPTIC CONE. (See the 4th Definition, in Optics; Book I. Page 9.)

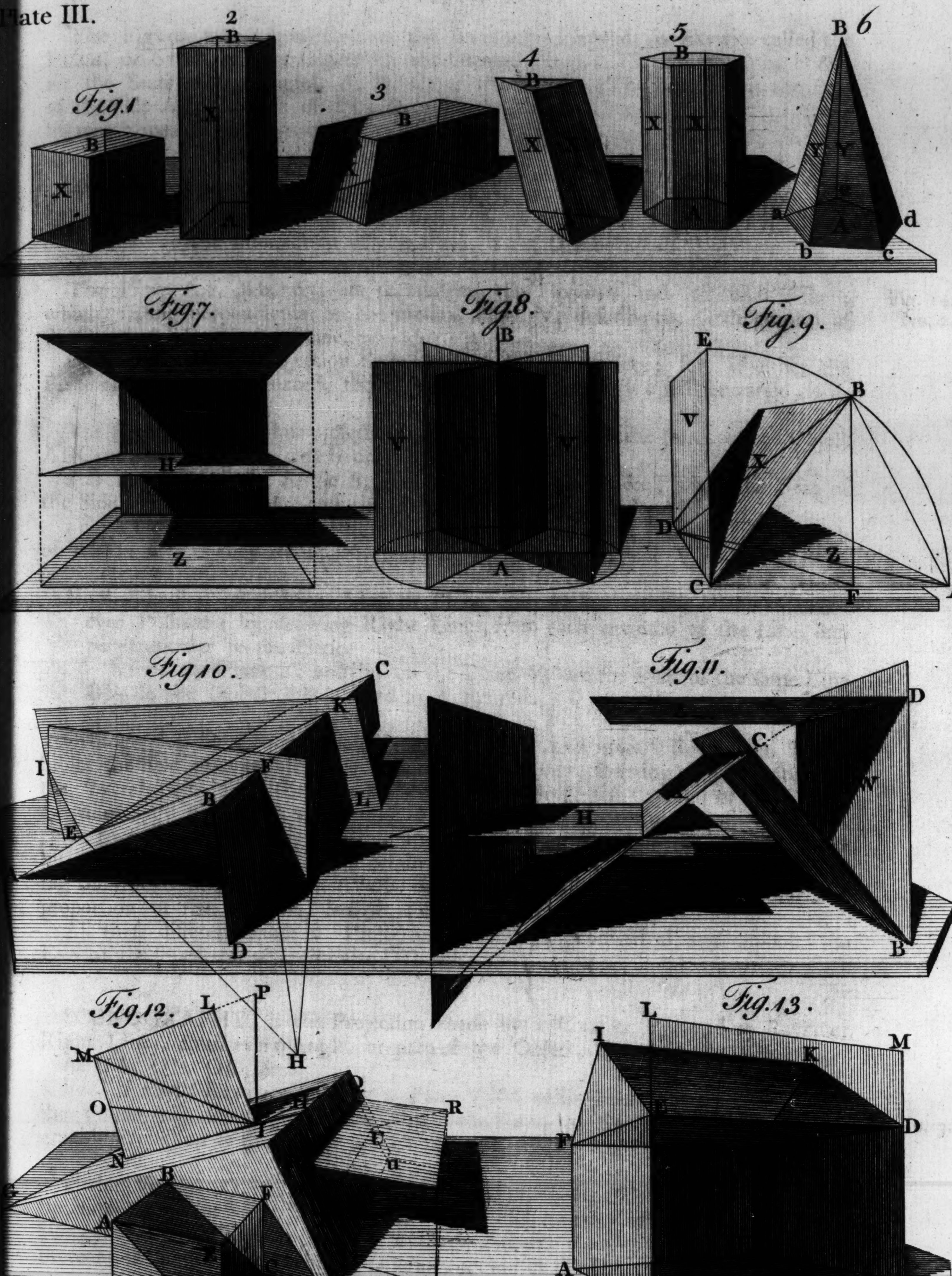
The Figure described, as a e f b, on the Plane V, a b c e, on the Plane Z, or, a b d e, on the Plane X, is the Projection of the Object ABC.

N. B. When the Object to be delineated is circular or globular, the System of Rays is then properly called a Cone, for it is really so; but when the Object is right lined, or mixed, it is more properly called a Pyramid, as in Fig. 6. Plate 3. the Base, A, only, being considered as the Object, and aB, b B, &c. as Visual Rays to the Eye, at B, in the Vertex of the Pyramid, a B d, formed by the Rays.

ICHNOGRAPHY, or Ichnographic Projection, is that which is described by Right Lines, parallel amongst themselves and perpendicular to the Horizon, from every Angle of the Object, on a Plane parallel to the Horizon.

Fig. 14. No. 1. As A a, B b, C c, &c. from the Angles A, B, C, &c. of the Object, to the Plane Z, which is horizontal. The points, a, b, c, d, &c. where the perpendicular Lines

Plate III.



Lines, or Rays, cut that Plane, being joined by Right Lines a b, b c, c d, &c. and diagonal wise, as a c, b d, &c, is the Ichnographic Projection of the Object, ACB.

The Figure, a e c b, projected on the horizontal Plane Z, is likewise called the PLAN, or SEAT of that Object on the Ground Plane. The Points a, b, c &c. are the Seats of the Angles, A, B, C, of the Object. The Line a b is the Seat of the Side AB, and b c of BC, &c. also the Diagonal a c is the Seat of AC; by which means, the Plan of the whole Figure is formed.

ORTHOGRAPHY. If the projecting Rays are parallel to the Horizon, and fall perpendicularly on a Plane (consequently Vertical) the Figure described, on that Plane, by the Intersection of the Rays, is the Orthographic Projection.

The Lines A a, B b, &c. are parallel to the Horizon and to each other; which, falling perpendicular on the vertical Plane V, describe the Orthography of the Object ABC, on that Plane. Fig. 14. No. 2.

Of these kinds of Projection there may be infinite variety; for, if either the Plane or the Object be turned, though ever so little, the Figure will be varied on it.

The Figure, a e f b, thus projected, may likewise be called the SEAT of the Object ABC on that Plane; as a e c b on the Ground Plane.

a is the Seat of the Angle A, and b of the Angle B, &c; a b is the Seat of the Side AB, a e of AE, and the Diagonal a f of the Side AF.

a e may also be considered as the Projection, or Seat, of the Plane AEC, and a b of ABC; for, if they were produced, they would cut the Plane V in those Lines.

N. B. The Seat of a Point, Line, or Plane, may be had on any Plane in whatever Position; by drawing Right Lines from each extreme of the Line, &c. perpendicular to the Plane.

b f on the Plane Z, and b f on the Plane V, are the Seats of the same Line BF, in the Object ABC; and so of the rest.

Orthographic Projection is usually called the ELEVATION. But, when it exhibits the End of a Building, or part of the End only, shewing the Projecture of the several parts, from the main Body of the Building, the Contour of the Curves of Moulding, &c, it is called a PROFILE. And, when the Building is supposed to be cut by a vertical Plane, in any direction through the Building, the hither part being supposed to be removed, and the inside exposed to view, shewing the thickness, &c. of the Walls and Floors, the structure of the Roof and proportion of the Timbers, &c. it is called, a SECTION.

All these different kinds of Projection are entirely geometrical, and supposes the Eye of the Spectator at an infinite Distance.

SCENOGRAPHY, is the Projection made by a Cone or Pyramid of Rays, or Right Lines, from every Angle, or part of the Object, converging to a Point.

As OA, OB, OC, &c.

For, if those Rays are cut by a Plane (X) passing between the Object and the Vertex (O) the Figure projected, by the Rays, on that Plane, is the Scenographic Projection of the Object ABC. Fig. 14. No. 3.

If the Vertex, O, of this Pyramid of Rays, be considered as the Eye of a Spectator, and the Right Lines OA, OB, &c. as visual Rays, the optic Pyramid being cut by the Plane X, considered as a Picture, the Figure, a b d e, projected thereon, is the PERSPECTIVE REPRESENTATION of the Object ABC.

N STEREO-

Plate V. STEREOGRAPHY comprehends the whole Art of repreſenting Solids * on a Plane; which differs from geometrical Projection, in what was explained in the preceding Definition; that the projecting Rays, ſuppoſed to proceed from every part of the Object, terminate in a point, at any diſtance from the Object; which Point, is always conſidered as the Eye of a Spectator, and the interſecting Plane as the Picture. The Section of the Rays is the Repreſentation.

By this kind of Projection, the ſeveral dimenſions of Bodies, viz. length, breadth and thickneſs, are all repreſented at one View, in the greateſt degree of perfection that Art is capable of; inſomuch, that, with the aſſiſtance of Colour and Shade, judiciouſly diſpoſed, it is poſſible to deceive the Eye and Judgment; in ſuppoſing the Repreſentation, on a Plane, to be a real Object.

As the Stereographic Projection (of Objects) is various, I ſhall explain it, more fully, under the three ſeveral Heads, Perſpective, Projected Perſpective or Projection, and Tranſprojection. And firſt, of PERSPECTIVE, which is more imediately the Subject of the following Treatiſe; that part of Stereography being more particularly adapted to the purpoſe of delineating all kinds of regular Objects, than either of the other; which are, therefore, but ſeldom practiſed; and indeed, becauſe they are but little underſtood, or the difference between them known; yet, they have their ſeveral uſes in delineating.

They are particularly uſeful in projecting the Shadows of Objects, delineated, in ſome poſitions of the Luminary; which, produce the beſt effect in the Picture.

I ſhall now proceed to a full explanation of the Term PERSPECTIVE, and endeavour to place it in the cleareſt point of view that I am capable of.

N. B. Let the Reader take particular notice, that all the References to Fig. 15. refers, likewiſe, to the Apparatus.

PERSPECTIVE, is a Science founded on Geometry; which teaches how to delineate, or draw, on a plane Superficies (or, ſimply, on a Plane) the true repreſentations of Objects, according to their Diſtance and Situation, and Bearings of the Objects to each other; from any Station, at pleaſure, real or imaginary.

To give a clear and perfect Idea of the Principles on which the Science of Perſpective is built, it is neceſſary that the Objects, to be repreſented, are conſidered as being beyond, or on the other Side of the Picture; that is, having fixed on the Station, from which you intend to delineate an Object, imagine a tranſparent Plane interpoſed between the Eye and the Object, or Objects; through which you have diſtinct Viſion, of the Objects on the other Side.

It is evident, to all who have conſidered it, that if, with a ſteady hand, you trace every Line, of Objects, accurately, as they appear on the tranſparent Plane (which may be ſuppoſed a Window, or a perfectly regular Plate of Glaſs) keeping the Eye fixed in a Point, there will be a true, linear Perſpective, Repreſentation of all the Objects on that Plane; which, is conſidered as the Picture. Now, the performance of this, by geometrical Rules, is what is properly called PERSPECTIVE.

Fig. 15. Let BFIKL be an Object having the three Dimenſions, length, breadth, and thickneſs; which may be ſuppoſed a Building, or what you pleaſe, to be repreſented on the Plane, or Picture MNOP, ſtanding perpendicular on the Ground Plane, SRZ; whoſe Interſection with it, MP, is at right Angles with the line of Station, SL. The Picture, MNOP, is, therefore, direct, between the Object and the Spectator, ES. EA, EB, EF, &c. may be conſidered as Viſual Rays, in which the viſion of the Object, BFIL, is conveyed to the Eye, at E.

2. It

* Every Object, having length, breadth and thickneſs or depth, comes under the Denomination, geometrically, of a Solid; the concavity, if any, not being conſidered; but only the external Form, conſiſting of Planes or other Surfaces, variouſly diſpoſed.

2. It is obvious, that if this Plane, or Picture, was transparent, the Representation, a i f c, on that Plane, of the Object, BFIL, on the other Side, would, by means of the Right Lines EA, EI, EF, &c, from each Angle of the Object to the Eye, exactly coincide, and agree in every part, with the original Object.

For, as it is not possible for Vision to be conveyed but in Right Lines to the Eye (except through dense, refracting Mediums) the Angle A, of the Object, must necessarily appear at a, on the Picture, B will appear at b, and F at f, &c. where the Right Lines, from the Angles of the Object to the Eye, pass through the Picture; and is the supposed reason for the genesis of a Point on the Picture, in Theory. The Right Line a b, or b g, on the Picture, joining the representations of the Angles, or Points, A and B, or B and G, in the Object, will also coincide with, and hide the Original Line, AB, or BG, from the Eye, at E; and is, therefore, its Representation; and so of all the other Lines on the Picture.

This, I think, needs no other Demonstration, for it is occular, and evident, that the Angle, made by the Pyramid of Rays forming a solid Angle (AEF, FEC) at E, is, not only equal, but, the same, under which, both the Object and its Representation are seen.

3. So likewise, the Representation, f i or f d, on the Picture, of any Line, FI or FD, in the Object, is seen under the same Plane Angle, IEF or FED. Consequently, f i coincides with FI, and f d with FD; the representations, a b g h of the Plane ABGH, f g h i of FGHI, and b f c of the Plane BFC, coincide with each other, respectively; and consequently, the whole Representation, a i f c, or Projection of the Object, AIFC, perfectly coincides with the Original, in the Point of View E, in that Position and Situation of the Picture and Object.

Wherefore, since the Eye is affected, in the same manner, by the Lines and Angles on the Picture, as by the corresponding Lines and Angles of the Object, it is evident, that, if the Picture had the same degree of Light and Shade, and also the true teint of Colour, as the Object, it would be impossible for the Eye, at E, to distinguish whether it was a Picture, delineated on the Plane MNOP, or the real Object, on the other Side, that was perceived.

4. Hence it is manifest, that there is, and may be, great deception in Vision; and also, that there may be various Representations, of the same Object, from the same Station or Point of View; which, notwithstanding their difference in figure and dimensions, will have the same Appearance in the true Point of view.

For, if any other Plane, as *MNOP*, be placed between the Eye and the Object, not parallel to MNOP, the Representation, of the same Object, projected on that Plane, by its intersection with the Visual Rays, will not only be smaller, but, also, very different in Figure and Proportion. For, having drawn the Right Lines SA, SB, SC, from the Foot of the Spectator, at S, to each angle of the Object, towards the Picture, on the Ground Plane, they will determine the extreme width of the representation of that Object on each Picture; and also the proportion, of the representation of the Plane AG to that of GC, which is, as k l to l m, on the Picture MNOP, where the lines SA, SB, and SC cut the bottom edge of the Picture; and, on *MNOP*, the Proportion is as n o to o p; but, *MP* is not parallel to MP; consequently, the proportion, of n o to o p, is not as k l to l m.

If the Picture was placed on MQ, it is obvious that the Representation would be still less, and also different in its Figure and Proportion; as the parts Q r, r q, intercepted between the lines SA, SB, and SC, sufficiently evinces.

Thus, may there be as many different Representations, of the same Object, as you please, and from the same Point of View; from the different Position and Distance of the Picture; all which, will affect the Eye alike, at the Point E, in the Vertex of the Optic Cone or Pyramid of Rays.

PRO-

Plate IV. ## PROJECTED PERSPECTIVE, or PROJECTION.

Fig. 15. No. 4. If the Viſual Rays EB, EC, ED, &c. are ſuppoſed to be produced, or projected beyond the Object ABCD (a quadrangular Pyramid) and there fall on, or are cut by a Plane (V) the Repreſentation (a d b c) of the Object, projected on that Plane, is the Projection of the Object ABC.

Projected Perſpective is the very ſame, except in the operation, as common Perſpective, *i. e.* when the Rays are cut by a Plane paſſing between the Object and the Eye, with only this difference, that, in common Perſpective, the Repreſentation is always leſs than the Object; becauſe the ſection of the Rays, by the Plane X, is on this Side, towards the Eye; in projected Perſpective, the Repreſentation muſt neceſſarily be larger than the Object, becauſe the Plane of the Section, V, is beyond the Object; but, if the Planes are parallel between themſelves, whether the Rays are cut on this or on the other Side of the Object, or both, the Repreſentations will be perfectly ſimilar.

No. 5. No. 4.

a, b, c, and d are the projective Repreſentations of the ſeveral Angles of the Pyramid, A, B, C, D; which, joined by Right Lines, is the Projection of that Object on the Plane V.

GHIF is the Plan or Seat of the Object ABCD, on the Ground Plane, to which the Baſe, ACdD, is parallel.

By means of the Seat and the Station Point, S, the Repreſentation, abcd, on the Plane V, may be projected.

The difference between Perſpective and Projection is very obvious.

In Perſpective, the Object, to be repreſented, is always ſuppoſed beyond the Picture, X; in Projection, the Picture V, is beyond the Object; which is projected, or ſuppoſed to be thrown forward to the Picture, and which is full as rational to ſuppoſe. Nor is there occaſion, in this Caſe, to ſuppoſe the Picture tranſparent, as in the former, when the Picture is ſuppoſed to be on this Side of the Object.

The difference in the Operation is very little, and is illuſtrated by frequent uſe in the practical part of this Work.

Fig. 15. No. 6. TRANSPROJECTION. If the Viſual Rays, from the Object (ABCD) are ſuppoſed to paſs through the Eye (at E) forming an oppoſite Pyramid of Rays (a E b) and there fall on, or are cut by a Plane (Y) the Figure (a c b d) projected on that plane, by its interſection with the Rays, is the Tranſprojection of the Object (ABCD).

It is evident, that, in this kind of Projection, the Repreſentation may be either larger or ſmaller, or equal to the Object; as the Plane, Y, is removed further or nearer to the point E; or, as the point E is removed nearer to, or further from the Object. For, if the point E be in the middle, between the Object and the Plane of the Section, the Repreſentation will be equal to the Object; and, whether it be nearer to, or further from the Object, the Repreſentation will have that Proportion, to the Original, as their Diſtances from the point E.

It is alſo evident, that Projections of this kind muſt neceſſarily be inverted; as the Rays all paſs through one common Point (a is the tranſprojected place of the Angle A, on the Plane Y, c of C, and b of the Vertex, B, of the Pyramid) in the ſame manner as Optical Philoſophers endeavour to account for Viſion; by ſuppoſing an Image, of every Object perceived, formed in the back part of the Eye, on the Retina, in the ſame inverted Poſition. The Eye being in this reſpect like a *Camera Obſcura*, which is a kind of artificial Eye; in which, the Picture is always inverted. (See Page 10, 1ſt. and 2nd. Par. Alſo, see Fig. 13, Plate II.)

Notwith-

Notwithstanding, if the Plane Y, of the transprojective Picture, be parallel to either the perspective, X, or the projective, V, the Representation thereon will be similar to the other, or to both, if they are all parallel amongst themselves.

An ORIGINAL OBJECT is any Object whatever, which is the Subject of the Picture we are delineating. Fig. 15.

BFIL is an Object, of which, the Projection, aifc, on the Plane MNOP, or *aifc*, on the Plane M*NOP*, are Representations.

Also, parts of an entire Object, as a Column, a Chimney, &c. are the Originals of their separate Representations.

By ORIGINAL PLANE is meant, not only, according to Dr. Brook Taylor, the Ground, or other Planes upon which Objects are seated, but also, the Planes of which original Objects are composed.

ABGH and BFC, &c. in the Original Object BFIL, are Original Planes, as well as the Ground Plane Z on which it stands.

It may be necessary to make a distinction between Figures and Objects, although every Plane Figure may be called an Object; but, I think, that Term is, more properly, applicable to Solids than to Plane Figures.

By ORIGINAL FIGURE, I shall therefore mean, only, Figures in Original Planes, as Doors, Windows, &c. in Original Objects; or the Figure of the Original Plane itself. As FGHI, or BGFDC, &c.

Any geometrical Plane Figure whatever, in Original Objects, is, therefore, an ORIGINAL FIGURE.

ORIGINAL LINE is any Line in an Original Object, whether Right Line or curved; as the Bounds or Limits of all Original Planes, or Figures, are Lines.

AB, BG, FG, FD, &c. are Original Lines, in the Original Planes ABGH and BFC, each, of which, is likewise in another Plane; for, each is the common Intersection of two Planes; consequently, each Line is in two Planes.

By Prop. 1. 7. El. the common Intersection of two Planes is a Right Line. 3. 11. Euc.

AB is the common Intersection of the Ground Plane and the Plane ABGH, and is, therefore, in both Planes; and, BG is, for the same reason, in both the Planes ABGH and BFC; and so of all the rest.

2. The Intersection of two Lines is a Point; and the extremes of Lines are also Points; wherefore, the Angles F, G, H, &c. of the Object BFIL, are Points; for they are the intersections, and also the extremes of Lines; and are called ORIGINAL POINTS.

All Original Objects, whatever, that are formed by Art, as Buildings of all kinds, &c. (which are the fitest subject for Perspective) are composed either of Planes or of curved Surfaces. Every Building and parts of Buildings come under the Denomination of some one or other geometrical Solid; as Parallelopiped, Prism, Pyramid, Cylinder, Cone, or Sphere; or they are compounded of several, together. For, the Body of the Building is either one Parallelopiped, or it is composed of several, variously disposed, at the discretion of the Architect. The Roofs are, generally, either triangular Prisms, or Pyramids. The Planes, of which Roofs are chiefly composed, are either Triangles, Parallelograms, or Trapezia. The Planes which compose the Fronts and Ends, &c. of a Building are, for the most part, Parallelograms; sometimes Pentagons (as BGFDC) or other Poligons;

which contain other geometrical Figures, variously disposed, as Doors, Windows, &c. which are generally Parallelograms, sometimes Circles, Ellipses, or mixed Figures.

Temples, in Gardens, Cupolas, &c. are either cylindrical, or poligonal Prisms; the Roofs, of which, are either pyramidal, spherical, or mix'd curved Surfaces.

Columns approach nearly to Cylinders; in the lower part they are perfectly so.

Thus, may every part of a Building, or other regular Object, be reduced to some geometrical Figure or other. Solids are composed of Planes or other Surfaces, all which, may again be reduced, to their first Principle Lines; for, as the boundaries of Solids are Planes, or other Surfaces, so the bounds of Planes, &c. are Lines.† Also the Figures in Planes, whether Doors, Windows, &c. or other Figures, are composed of Lines; all which, are ORIGINAL LINES.

† Def. 55, Geo.

So likewise, Mouldings, Steps, &c. are represented by Lines; which, in the Originals, are generated either by Planes, only, as streight Steps, or by Planes and curved Surfaces, as in Mouldings and circular Steps. Each Moulding, if it be right lined, is composed of Planes and cylindrical Surfaces; which, by their parallel intersections, generate Right Lines. Circular Mouldings, in Cornices, &c. are composed of Planes, with cylindrical and other curved Surfaces; which, by their regular intersections, generate circular Lines.

The Edges of Columns and other cylindrical Objects, which are represented by Lines, on the Picture, have no real existence in the Originals, but are only apparent; for there is no real Line; as it is but one continued Surface which returns again into itself, without cuting or intersecting, by which Lines are generated. The Lines, which form the representations of Bases, &c. of Columns, are, in the Originals, some of them real and some only apparent; as the contour of the Curve of the Torus, &c. and have various Forms, on the Picture, according to the situation of the Eye, or Picture.

Having thus reduced compound Original Objects into their first Principles or Elements, viz. Planes and Lines; the next thing, to be considered, is the Position and Situation of those Planes and Lines, in respect of the Picture and of each other; which being premised and well considered, the whole mystery of linear Perspective will be found comprised in a small Compass, both in Theory and Practice. The Principles on which the Theory is built are few, but they are general, and applicable in all positions of the Picture and situation of the Object, or of the Eye. Wherefore, the Distance and Situation of the Object being determined; that is, the Station being fixed, from which an Object is to be delineated, and the Position of the Picture determined, the Representation is also determinable; which Representation, is always in proportion to the Distance of the Picture.

Thus far I have proceeded, merely by way of Introduction. I have called it an Introduction to Perspective, because, it cannot be called a part or branch of that Science; seeing, all which it contains may be, and is, known to several, who are not acquainted with one Theorem, or any Rules for the practice of Perspective. Nevertheless, I am well convinced, that, the knowledge inculcated by this Introduction is by no means to be dispensed with, being as essential to be previously known, as the Definitions of the Terms used in Perspective; which, might also be contained in the Introduction, but not with propriety, being elementary. It is certainly possible, by Rules laid down, for a Person to practise Perspective without knowing what Perspective means; but that is not compatible with my Design, having entitled this Work a Compleat Treatise, which could not possibly be, without accounting for the Rules given; which I shall do, as briefly as is consistent with the Subject, not dwelling on any unnecessary part of the Science.

I shall now proceed to define the more elementary Terms, on which the whole Theory of Perspective is built; the Rules for Practice are deduced from that Theory.

SECTION

SECTION III.

Containing the ELEMENTS OF PERSPECTIVE.

IN order to investigate the Theory of Perspective, with clearness and precision, it is necessary to have recourse to certain imaginary Planes, which may be conceived to pass through the Eye of a Spectator, or Point of View, from which an Object is supposed to be delineated, in all Positions as occasion requires.

The Picture, as it has already been observed, (under the Article Perspective) is supposed to be between the Eye and the Object.

Three of those imaginary Planes, together with the Picture and any Original Plane whatever, are supposed to be constructed as they are represented in Plate IV, Fig. 16, 17, 18, 19, 20 and 21. The Planes, thus constructed, I shall first define; afterwards, the Lines generated by their Intersections; and lastly, the Points produced by the intersections of the Lines; all which, are so essentially necessary, that, in short, without the assistance of these five fundamental Planes, and the Lines and Points generated by their Intersections with each other, Perspective would be a very intricate and perplexed Study; and, in Theory, a most imperfect Science.

I would, before I proceed further, particularly advertise young Students, not to pay the least regard to the general positions of the Planes I am about to define, but only their position in respect of each other; for which reason I have given them in various Positions, and advise the Reader not to give particular attention to the first. For, in the Theory of Perspective, the position they have to the Horizon is not considered, at all; as the Theorems are general, and applicable in all positions and situations of the Picture, whatever; since (as Dr. Brook Taylor, in the Preface to his second Treatise, justly observes) all Planes, simply as Planes, are alike in Geometry, and have the same properties however situated.

DEFINITIONS.

Plate IV.

I. Let ABGH be considered as a part, or a continuation of an ORIGINAL PLANE; being supposed to be produced (if necessary) from any Original Object to the Picture; and, til' it cuts a Plane passing through the Eye of a Spectator, parallel to the Picture. Fig. 16, 17, 18, 19, 20 and 21.

DEFINITION II.

The PLANE of the PICTURE (ABLM, or the Plane X) is considered as the Board, Paper, or Canvas, on which is to be delineated the representation of some Original Object, or Plane Figure.

The PICTURE is, generally and for the most part, vertical, and direct between the Eye and the Object to be delineated; as in Fig. 16, 18 and 19; but may be in any Position whatever; as in Fig. 17, 20 and 21.

DEFINITION III.

VANISHING PLANE. If a Plane be imagined to pass through the Eye parallel to any Original Plane, it is called the VANISHING PLANE of that Original Plane; or, simply, the PARALLEL of the Original Plane.

As V, or IKLM, parallel to Z or ABGH, Fig. 16, 17, &c.

And, a Perpendicular, EC, from the Eye to its Intersection with the Picture, is the RADIAL of the Vanishing Plane.

DEFI-

Plate IV.

DEFINITION IV.

The DIRECTING PLANE is an imaginary Plane, ſuppoſed to paſs through the Eye of a Spectator, parallel to the Picture.

GHIK, or the Plane Y, parallel to the Picture, X, is the Directing Plane; the Eye of a Spectator being ſuppoſed at E, in the Directing Plane.

N. B. The Diſtance of the Directing Plane, from the Picture, is always equal to the Diſtance of the Eye (as EC) and being parallel to the Picture, it makes equal Angles with the Original Plane, as the Picture. (IHA equal MAN).

DEFINITION V.

The VERTICAL PLANE (ECDF) is ſuppoſed to paſs through the Eye, perpendicular to the Original Plane and to the Picture. Wherefore, it cuts all the four preceding Planes at right Angles, in all poſitions of the Picture whatever.

DEFINITION VI.

RADIAL PLANE is an imaginary Plane, paſſing through the Eye and any Original Line, whatever.

Fig. 20. As EV*ID*, or E*CPD*; which, being produced, would paſs through the original Line ON, or QP.

In Fig. 16. the Original Plane (AG) is ſuppoſed horizontal, and the Picture (AL) vertical; conſequently, the Varniſhing Plane (MK) and Directing Plane (KH) have the ſame Poſitions, and cut each other at right Angles; and alſo the Vertical Plane (ECDF).

In Fig. 17. ſuppoſe the Picture (ABLM) and the Directing Plane (HIKG) in the ſame vertical Poſition; and ſuppoſe the four Planes, viz. the Original Plane, the Picture, the Vaniſhing and the Directing Planes, ſo fixed together, as to be moveable on their Interſections, AB, GH, IK and LM, as on hinges; and then, let us ſuppoſe, the Picture and Directing Plane (and along with them the Vaniſhing Plane) puſhed into an inclined poſition, on either Side of the original, vertical Poſition (A*M*B, H*IK*G; or AmlB, HikG) the Interſections AB and GH remaining, as they were, unmoved.

It is evident, that the paralleliſm of the Planes is not deſtroyed by this motion; for, the Vaniſhing Plane (IKLM) is ſtill in the ſame horizontal poſition as before, though removed lower, to iklm; and, the Directing Plane, HikG or H*IK*G, is ſtill parallel to the Picture, AmlB or A*MB*, both being inclined to the Original Plane and its Vaniſhing Plane, in equal Angles.

In Fig. 18. the Picture (X) and the Directing Plane (Y) are ſtill Vertical, but the Original Plane (Z) and its Vaniſhing Plane (V) are inclined to them, and to the Horizon; making equal Angles with each other, as in the former Caſe.

Nor, is there underſtood to be any difference in the Poſition, between this and the preceding Figure; for, either Plane (AL or HK) may be ſuppoſed the Picture, and the other the Directing Plane; by which means, it has both the Poſitions of the former; only, ſuppoſing the Original Plane horizontal, the Picture and Directing Plane are conſequently inclined.

In

In Fig. 19. both the Original Plane and Picture (NBH, and ABLM) are supposed Vertical; therefore, the Vanishing and Directing Planes are, also, Vertical; and the Vertical Plane (CDFE) in this Case, is consequently Horizontal.

In Fig. 20. the Planes are all inclined to the Horizon, and also to each other; excepting the Vertical Plane; which always cuts the other four at right Angles, and is, therefore, perpendicular to them all.

This Figure I recommend, more particularly than any of the other four, to be contemplated by the young Student (although it is the same in all) in order to divest him entirely, of partiality to any particular Position, respecting the Horizon.

In Fig. 21. The four primary Planes are all moveable, and may be put into all the Positions of the former; either at right Angles, as Fig. 16; or inclined, on either Side, as in Fig. 17, in any Angle at pleasure.

As these Planes are all marked with the same Characters, as in the five preceding Figures, it would be superfluous to particularize them here; and if a Plane be supposed to pass through the four Points, E, C, D and F, it will be vertical or perpendicular to them all, in every Position.

N. B. Any one of the four Planes may be the Original Plane, the opposite one is, consequently, its Vanishing Plane; either of the other may be the Picture, and the opposite to it is the Directing Plane; ECDF is still the Vertical Plane.

Of LINES, generated by the Intersections of the five elementary Planes.

DEFINITION VII.

INTERSECTION of the PICTURE, with an Original Plane, is the Line in which any Original Plane cuts the Picture; or, in which an Original Plane, being produced, would cut the Picture.

AB; is the Intersection of the Picture ABLM, with the Original Plane ABGH. Fig. 16, 17, 18, 19, 20 and 21.

DEFINITION VIII.

VANISHING LINE is a Line produced by the Intersection of an imaginary Plane, passing through the Eye parallel to any original Plane, with the Picture.

LM, the Intersection of the Vanishing Plane, IKLM or V, with the Picture ABLM or X, is the Vanishing Line of the Original Plane, ABGH or Z.

DEFINITION IX.

PARALLEL of the EYE is the Line IK, in which the Vanishing Plane (V) and the Directing Plane (Y) intersect each other.

As both these Planes are imagined to pass through the Eye (at E) consequently, their Intersection (IK) passes through the Eye; and, because the Directing Plane is parallel to the Picture, the Parallel of the Eye is parallel to the Picture.

DEFINITION X.

DIRECTING LINE is the Line GH, in which, an Original Plane (Z) cuts, or would, if produced, cut the Directing Plane (Y).

Plate IV.

DEFINITION XI.

Fig. 16, &c. The VERTICAL LINE is the Line CD, in which, the Vertical Plane (ECDF) cuts the Picture; at right Angles with the Vanishing Line and Intersection of the Original Plane.

DEFINITION XII.

The DIRECTOR of an Original Line. If an Original Line be produced till it cuts the Directing Plane, a Right Line passing through the Eye and that Point is the Director of the Original Line.

E*D* (Fig. 20 and 21) is the Director of the Line NO, produced to *D*; where it cuts the Directing Plane.

DEFINITION XIII.

VISUAL RAY. With optical writers, this Term signifies an imaginary Ray of Light; by which, Vision is supposed to be conveyed from the Object to the Eye; therefore, in Perspective, it is a Right Line drawn from any Point, in an Object, to the Eye.

EA, EI, EF, &c. (Fig. 15.) and EN, EO, EP, &c. (Fig. 20 and 21) are Visual Rays.

DEFINITION XIV.

RADIAL LINE is the parallel of any Original Line, producing its Vanishing Point. (See Def. XXII.) As EV, Fig. 15, 20, and 21.

DEFINITION XV.

DIRECT RADIAL is a Right Line, from the Eye or Point of View, perpendicular to the Picture. As EC.

N. B. If the Original Plane be at right Angles with the Picture (as in Fig. 16.) the Direct Radial (EC) is the common Intersection of the Vanishing Plane, (IKLM) and the Vertical Plane (ECDF); and is, always, in the Vertical Plane.

Of POINTS, and their Distance from the Eye.

DEFINITION XVI.

The POINT of SIGHT is that Point where the Eye, of a Spectator, ought to be placed to look at a Picture; for, in that Point, only, can a perspective Picture be seen perfectly.

E is the place of the Eye, or Point of View, to look on the Picture ABLM. It is the Point where the three imaginary Planes, viz. the Vanishing, the Directing and the Vertical Planes, intersect; consequently, the Eye is in all the three. Or, it is the Point of Intersection between the Parallel of the Eye, IK, and the PRIME DIRECTOR, EF.

Fig. 15. N. B. It is the Vertex of the Optic Pyramid of Rays (EA, EB, EF, &c.) the only Point in which the Images, or Representations (aifc,) on the Planes or Pictures, MNOP or *OP*, can exhibit a true Appearance of the Original Object (BFIL) on the other Side.

DEFI-

DEFINITION XVII.

The CENTER of the PICTURE is the Point C, in which the Picture is cut by a perpendicular Line from the Eye, or Point of Sight.

The Direct Radial (EC) being perpendicular to the Picture, is, therefore, in the Vertical Plane (ECDF) consequently, the Center of the Picture, produced by the Perpendicular EC, is in the Vertical Line (CD) and, when the Original Plane is perpendicular to the Picture, it is the Intersection of the Vertical Line (CD) and its Vanishing Line (LM) Fig. 16.

DEFINITION XVIII.

DISTANCE of the PICTURE, or principal Distance, is the Direct Radial, or perpendicular Line, EC, from the Eye to the Picture, or to its Center.

N. B. The Center and Distance of the Picture are very essential, and ought to be well understood; for they govern all the rest.

DEFINITION XIX.

CENTER of a VANISHING LINE, is the Point where it is cut by a perpendicular Line from the Eye or Point of Sight.

EC (Fig. 15 and 16) or E*C* (Fig. 18, 19 and 20) being a Perpendicular from the Eye (E) to the Vanishing Line (LM) C or *C* is, therefore, its Center.

DEFINITION XX.

DISTANCE of a VANISHING LINE, is the Perpendicular, EC or E*C*, from the Eye to its Center.

For, it is the shortest Line that can be drawn from the Eye to the Vanishing Line.

DEFINITION XXI.

POINT of INTERSECTION, is that Point in which any Original Line (being produced, if needful) cuts the Picture; or the Plane of the Picture produced, if it be necessary

I is the Intersecting Point of the Line NO, produced to the Picture; *B* and *F* are the Intersecting Points, of the Lines AB and FI, on the Pictures MNOP. Fig. 20. 21. Fig. 15.

DEFINITION XXII.

VANISHING POINT. If a Line be drawn from the Eye, parallel to any Original Right Line, the Point, where it cuts the Picture, is the Vanishing Point of that Original Line.

NO is an Original Line, in the Original Plane HNBG; EV is a Line from the Eye, (E) parallel to NO, cuting the Picture in V, which is, therefore, the Vanishing Point of NO; and EV is its RADIAL, or simply, the PARALLEL of the Original Line; and it is, consequently, in the Vanishing Plane, or parallel of that Plane the Original Line is in. Fig. 20. and 21.

EC or EV being parallel to the Original Lines AB, GH, EI, &c. C or V, where it cuts the Picture, is, therefore, the Vanishing Point of those Lines. Fig. 15.

N. B. The RADIAL, EC or EV, is the Distance of the Vanishing Point, C or V.

DEFI-

Plate IV.

DEFINITION XXIII.

DIRECTING POINT is that Point in which any Original Line, being produced, would cut the Directing Plane.

Fig. 20 and 21. *D* is the Directing Point of the Original Line NO, or PQ being produced to the Directing Plane (GHIK).

It is the Intersection of the Original Line and the Directing Line, of the Plane the Original Line is in.

In Fig. 23. O, P, and Q are the Directing Points of the Lines CB, AB, and AD, where those Lines cut the Directing Plane, *GIKH*, produced.

DEFINITION XXIV.

STATION POINT is the Point in which a perpendicular Line, from the Eye or Point of Sight, cuts the Ground, considered as a horizontal Plane; or any other horizontal Plane, on which an Object, to be delineated, is seated.

Fig. 15. S, is the real Point of Station, at the foot of a Spectator (ES.)

In Theory, it is that Point, in which a Right Line, from the Eye, cuts the Directing Line perpendicularly.

Fig. 16, 17, &c. As F, in the Directing Line GH; EF being, supposed, perpendicular to GH.

AXIOMS.

I. One part of a Right Line cannot be in any Plane, and another Part of the Line out of that Plane.

II. Two Planes which are parallel cannot intersect.

III. The common Intersection of two Planes is a Right Line.

IV. The Intersection of two Lines is a Point.

V. Two Right Lines, that are parallel, are in the same Plane; i. e. a Plane may pass through both Lines.

VI. Two Right Lines, meeting in a Point, or which, if produced, would intersect, may be in the same Plane.

VII. Two Right Lines, being parallel, and both cut by another Right Line, are all in the same Plane.

VIII. Three Right Lines, touching or intersecting each other, are all in the same Plane; consequently, every right lined Triangle is in a Plane.

Several of these Axioms are demonstrable Propositions, in the eleventh Book of Euclid's Elements; but, when a true Idea of a Plane is inculcated, they are so very evident, that it is but trifling to little purpose to attempt to prove them. Nevertheless, as the Student is supposed to understand Geometry, they may, without scruple, pass for Axioms here.

SECTION

Plate IV.

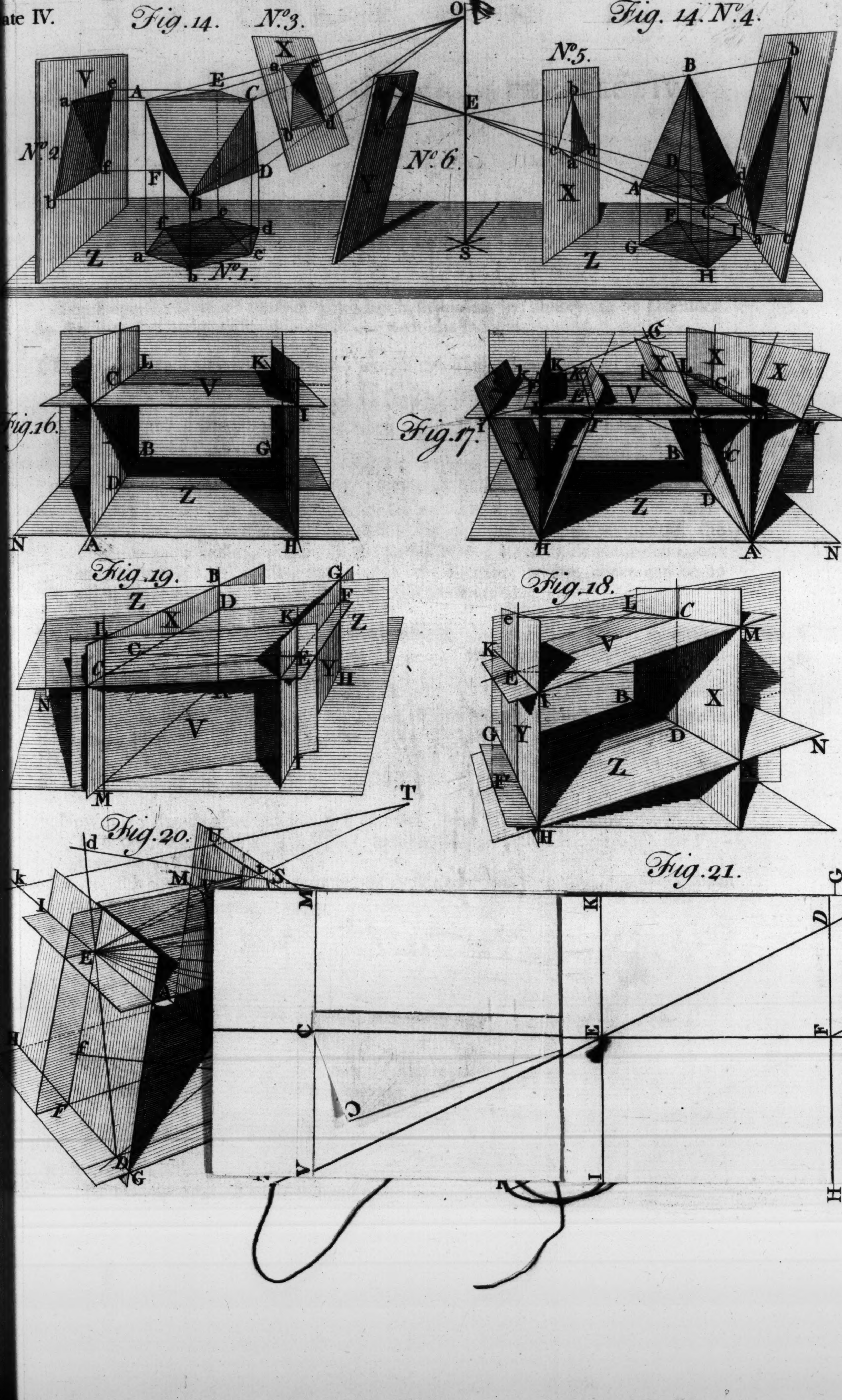


Fig. 14.
N.o 1.
N.o 2
N.o 3.
Fig. 14. N.o 4.
N.o 5.
N.o 6.
Fig. 16.
Fig. 17.
Fig. 19.
Fig. 18.
Fig. 20.
Fig. 21.

late IV.

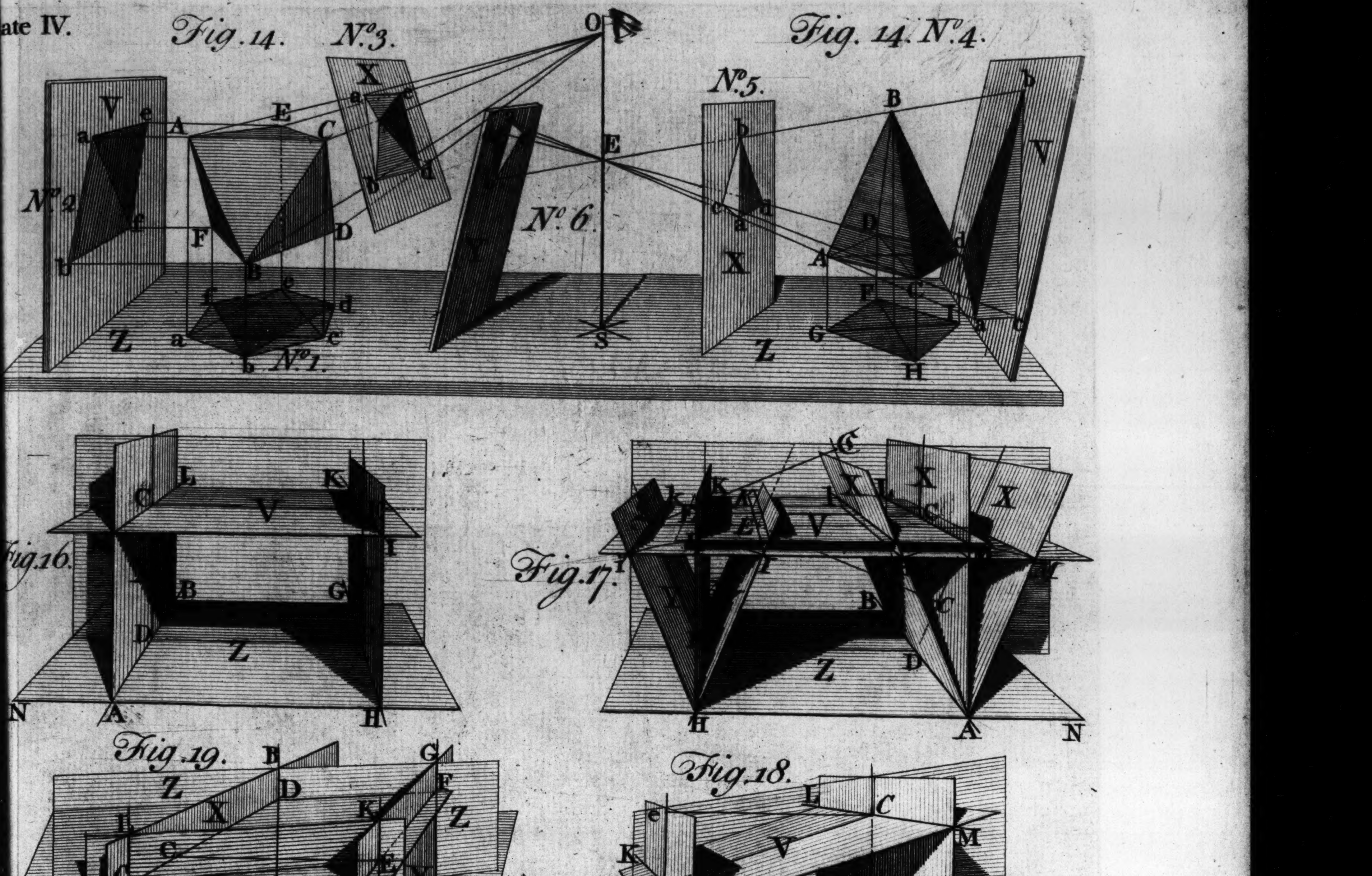


Fig. 14.
N°3.
Fig. 14. N°4.
N°5.
N°2.
N°6.
N°1.
Fig.16.
Fig.17.
Fig.19.
Fig.18.
Fig.20.
Fig.21.

SECTION IV.

Containing the THEORY of Rectilinear PERSPECTIVE.

THEOREM I.

THE Perspective Projection of every original Right Line is a Right Line, in every Position or Situation whatever.

The Representation of every Right Line is supposed, in Theory, to be produced by the Intersection of an imaginary Plane with the Picture.

DEM. Suppose a radial Plane to pass through the Eye and any original Right Line.

If the Line be situated on the other Side of the Picture, the imaginary Plane must, necessarily, cut the Picture; and, if it lie on this Side, the Plane will cut the Picture if produced; provided, the original Line be not in the Directing Plane.

But, the Intersection of two Planes is a Right Line - - P. 1. 7. El.

And, since the Eye is in this imaginary Plane, the whole of that Plane appears but a Line. (See Definition of a Plane, Page 43.)

But, the original Line is supposed to be in that Plane; consequently, the Original Line and every Line in the radial Plane, have their Representations in the Intersection of that Plane with the Picture; seeing, there can be no other Line common to both Planes. To illustrate it.

EXAMPLE. Let, BFIL be a rectilinear Object, composed of Planes in various positions. Let MNOP be the Picture, placed direct between the Spectator, at ES, and the Object, BFIL. Plate V. Fig. 15.

FI, FG, or FD, is an Original Line (or, if you please, take any other Line in the Original Object BFIL. E is the Point of Sight, or place of the Eye; and EI, EF, EG, &c. are Visual Rays, in which the extreme Points, I, F, G, &c. seen, are forming the Optic Angles FEI, FEG, &c.

Now since the Original Line FI, FG, &c. is on the other Side of the Picture, MNOP, the Visual Rays EF, EI, &c. must necessarily, pass through the Picture, to the Eye.

But, FEI is a Triangle; for, it is three Right Lines touching or cuting each other, and is, therefore, in a Plane. - - - - Axiom 8.

The Ray EF cuts and passes through the Picture in the Point f, and EI in the Point i, &c. the Points f, i, g, &c. are, therefore, the Representations of the Original Points F, I, and G. For, they coincide with their respective Originals.

Wherefore, the Line fi, or fg, joining the Points f and i, or g, is the Intersection of the Plane Triangle FEI, or FEG, with the Picture.

But, the common Intersection of two Planes is a Right Line. - - Ax. 3.

And fi is the Representation, or perspective Projection of the original Line FI, or fg of FG. (See Projection, Scenography, Page 49.)

For, to the Eye, at E, the Point f coincides with the Original Point, F, i with I, and g with G; consequently, the Line fi coincides with its Original FI, and fg with FG.

N. B. It is the same, in respect of any other Right Line in the Original Object, BFIL; or applied to any Right Line whatever.

Plate V.

THEOREM II.

An Original Plane, which is parallel to the Picture, has no Intersection with the Picture, nor Vanishing Line.

DEM. The Original Plane being parallel to the Picture cannot intersect it. Ax. 1. And, a Plane passing through the Eye parallel to the Original Plane, which should produce the Vanishing Line, by its Intersection with the Picture, is, in this Case, also parallel to the Picture, and the same with the Directing Plane; consequently, it can never cut the Picture, and therefore, cannot produce a Vanishing Line. Q. E. D.

Fig. 15. EX. Let MNOP be the Plane of the Picture, parallel to the Plane BFC, of the Original Object BFIL.

Then, because the Plane BFC is parallel to the Picture, it cannot cut the Picture (though produced infinitely) therefore can have no Intersection with it. Ax. 2. And a Plane RSTU passing through the Eye, at E, parallel to the Original Plane, BFC, is also parallel to the Picture; therefore, it cannot cut the Picture and produce a Vanishing Line; (agreeable to Def. 8.) for, it is the Directing Plane of that Picture; which is parallel to the Picture. - - Def. 4.

COR. Original Lines, which are parallel to the Picture, have neither Vanishing Point, Intersecting, nor Directing Points.

For, an Original Line, parallel to the Picture, is, or may be, in a Plane that is parallel to the Picture; and, a Line passing through the Eye parallel to the Original Line, which should produce its Vanishing Point ‡, is, in this Case, also parallel to the Picture; and is, therefore, in the Directing Plane, which is parallel to the Picture. Wherefore, it can never cut the Picture and produce a Vanishing Point; or, it may be supposed at an infinite Distance.

‡ Def. 22.

Neither can the Original Line, for the same reason, cut the Picture or Directing Plane; therefore, it has neither Intersecting nor Directing Point.

EX. Because the Plane BFC is parallel to the Picture, every Line in that Plane is also parallel to the Picture; therefore, BG, GD, &c.

Consequently, since one part of a Right Line cannot be in any Plane, and another Part of it out of the Plane†, the Line BG, or GD, &c. can never cut the Picture and Directing Plane, and produce an Intersecting and Directing Point; agreeable to Definitions 21 and 23.

† Ax. 1.

For the same reason a Line passing through the Eye parallel to the Original Line BG, GF, or GD, &c. must necessarily be in the Directing Plane RSTU; consequently, it can never cut the Picture and produce a Vanishing Point; agreeable to Definition 22.

THEOREM III.

The Vanishing Line, the Directing Line, and the Parallel of the Eye of any Original Plane, not parallel to the Picture, are parallel to the Intersection of that Plane with the Picture.

The Construction of the five elementary Planes with their uses and relation to each other, (as explained or defined), 'tis presumed, is well understood.

Sup-

Suppose, then, the first four, viz. the Original Plane, Z, its Parallel, or Vanishing Plane, V, the Picture, X, and Directing Plane, Y, forming a right angled Parallelopiped; or that part of them, only, which lies between their Intersections AB, ML, KI, and GH. Plate IV. Fig. 16.

DEM. Now, the Picture and the Directing Plane are parallel, and they are cut by an Original Plane and its Vanishing Plane, which are also parallel.

But, if two Planes, being parallel, are cut by another Plane, their common Sections are parallel. - - - - - - - - P. 8. 7. El. Consequently, if two parallel Planes are cut by two parallel Planes, their Intersections are all parallel amongst themselves.

But, AB, the Section of the Original Plane, Z, and the Picture, X, is the Intersection of the Picture. - - - - - - - Def. 7.

ML, the Section of the Picture, X, and the Vanishing Plane, V, is the Vanishing Line of the Original Plane, Z. - - - - - Def. 8.

KI, the Section of the Vanishing Plane, V, and the Directing Plane, Y, is the Parallel of the Eye. - - - - - - - - Def. 9.

And, GH, the Section of the Directing Plane, Y, and the Original Plane, Z, is the Directing Line - - - - - - - Def. 10.

Therefore, the Vanishing Line, ML, the Directing Line, GH, and the Parallel of the Eye, KI, are all parallel amongst themselves, and to the Intersection, AB, of the Original Plane with the Picture. Q. E. D.--4. 7. El.

N. B. Whether the Original Plane be at right Angles, or otherwise, with the Picture, the Demonstration holds good, in all positions whatever; since the Directing Plane is always parallel to the Picture, and the Vanishing Plane to the Original Plane; consequently, their common Intersections are still parallel amongst themselves.

To assist the Imagination, I have given a Figure with the four principal Planes, moveable; which may be raised up, to a Right Angle or any other, at pleasure. Fig. 21.

In every Position, it is evident, that the parallelism of the Planes, and consequently of their Intersections, remains; and the distance of the Intersections from each two, which are in the same Plane, is invariable.

The Vertical Plane may be supposed to pass through the middle, and perpendicular to them all; cuting the Vanishing and Directing Plane, in EC and EF; the Picture and Original Plane, in CD and DF.

COR. 1. If any one of the four Lines be given or determined, and one Point, in any of the other, be also given or found, the whole Line is determined.

For, it is a Right Line, drawn through that Point, parallel to the given Line; seeing, they are all parallel amongst themselves, by the Theorem.

COR. 2. If the Original Plane passes through the Eye, its Intersection and Vanishing Line is the same; and, the Parallel of the Eye is the same as the Directing Line.

Plate VI. Fig. 22.

For, seeing that the Original Plane (U, or cde, being produced) passes through the Eye (E) there can be no other Plane pass through the Eye parallel to it; but, in this Case, the Original Plane and its Parallel coincide; consequently, its Intersection (*LM*) with the Picture (AMLB) is also its Vanishing Line.

And, for the same reason, the Parallel of the Eye (IK) coincides with the Directing Line; being produced by the same Original Plane.

In

Plate IV. In which case, no Figure in the Original Plane can be represented.

For, as the Original Plane passes through the Eye, the whole Representation of that Plane, and every Figure in it, is its Intersection with the Picture; and is the reason, why, Figures, in Horizontal Planes, that are on a level with the Eye, cannot be represented; for they are all in the Vanishing Line of Horizontal Planes, and, therefore, cannot appear in the Picture.

THEOREM IV.

A Line drawn from the Center of the Picture, to the Center of a Vanishing Line, is perpendicular to that Vanishing Line.

DEM. For, because the Direct Radial, producing the Center of the Picture, is perpendicular to the Picture§; and the Center of every Vanishing Line is produced by a Right Line from the Eye perpendicular to the Vanishing Line‡; consequently, the Radial of the Vanishing Line, producing its Center, and the Direct Radial, together with the Line which joins the two Centers, form a Triangle, and are all in the same Plane† which Plane is perpendicular to the Picture. - - - - - - - - - - - P. 9. 7. El.

§ Def. 15.
‡ Def. 19.
† Ax. 8.

For, it passes through the Direct Radial, which is perpendicular to the Picture.

And, it is also perpendicular to the Vanishing Line; because, it is perpendicular to the Picture, and also to the Plane which produced the Vanishing Line. - - - - - - - - - - Cor. 1. 9. 7. El.

Wherefore, since the Radial of the Vanishing Line, and the Right Line joining the Center of the Picture and the Center of the Vanishing Line, are both drawn to the same Point in a Plane, at which Point the Vanishing Line is perpendicular to that Plane; consequently, the Line, joining the Center of the Picture and the Center of the Vanishing Line, is perpendicular to the Vanishing Line. Q. E. D. - - - - - - Cor. 2. 2. 7. El.

Fig. 18. and 19. EX. LM is the Vanishing Line of the Original Plane Z; *C* is its Center, and C the Center of the Picture. Draw C*C*.

I say, that the Line C*C* is perpendicular to the Vanishing Line LM.

By Def. 17. EC producing the Center of the Picture is perpendicular to the Picture; wherefore, it is in the Vertical Plane (E*C*DF.)

For, it is perpendicular to the Picture, X, and also to the Vanishing Plane, V.† Consequently, LM, the common Section of the two Planes, V and X, is perpendicular to the Vertical Plane, E*C*DF. - - - - - 9. 7. El.

Wherefore, E*C*, the Intersection of that Plane and the Vanishing Plane, V, is perpendicular to LM. - - - - - - - - - - - 2. 7. El.

But, E*C* produced the Center (*C*) of the Vanishing Line LM§; and consequently, C*C*, the Intersection of the Vertical Plane with the Picture, is, also, perpendicular to LM; for, E*C* and C*C* are two Lines, cuting each other, in the same Plane (E*C*DF) to which the Vanishing Line, LM, is perpendicular, at the Point, C, of their common Section.

† Def. 5.
§ Def. 19.

COR. 1. When the Original Plane is perpendicular to the Picture, the Center of the Picture is the Center of the Vanishing Line.

Fig. 16. For, a perpendicular (EC) from the Eye to the Vanishing Line (ML) producing its Center is the Direct Radial, which is perpendicular to the Picture; and, consequently, produces the Center of the Picture; which is the Center of every Vanishing Line, of Planes perpendicular to the Picture. See Th. 6.

COR.

COR. 2. The Distance of every Vanishing Line, which does not pass through the Center of the Picture, is the Hypothenuse of a Right angled Triangle, whose Catheti, or Legs, are the Distance of the Picture, and the Distance between the Center of the Picture and the Center of the Vanishing Line.

For, EC is the Distance of the Picture, *C*C is the Distance between the two Centers; and, E*C*, the Hypothenuse of the Right angled Triangle EC*C*, is the Distance of the Vanishing Line LM. Fig. 18. and 19.

To set this Matter in the clearest light possible I have illustrated it by moveable Planes, in Fig. 21.; which, let be raised up, in any Angle, at pleasure.

Underneath the Vanishing Plane (IKLM) is a triangular Plane (EC*C*) which, being turned down, perpendicular to the Vanishing Plane, will represent a Part of the Vertical Plane, and determines the Center of the Picture, in a certain angle of inclination, inclined from the Eye, in the Angle E*C*C.

The Triangle EC*C* is right angled, at C; consequently, EC is perpendicular to the Picture, and C is therefore its Center.† † Def. 17.

It is evident that *C*E and *C*C are both perpendicular to LM, or VM, the Vanishing Line of an Original Plane, NGH.

But, *C* is the Center of the Vanishing Line, and C is the Center of the Picture; therefore, a Right Line joining the Center of the Picture and the Center of a Vanishing Line is perpendicular to that Vanishing Line.

Above the Vanishing Plane is another Right angled Triangle; which being erected, perpendicular to the Vanishing Plane and the Picture, determines its Center, when the Picture is inclined towards the Eye, in the Angle E*C*C.

In which Case, the Center of the Picture necessarily falls above the Vanishing Line; as in the former Case it falls below.

Both these Positions are represented in the 17th Figure, and also in the 18th, considering either Plane, X or Y, as the Picture, and E or C as the place of the Eye.

But when the Picture and its Directing Plane are perpendicular to the Original Plane, as in the 16th, and the 17th Figure also shews the same, the Center of the Vanishing Line is the Center of the Picture.*

SCHOL. In this Scheme, and some of the following Diagrams, the original Plane, NGH, must not be considered as necessarily being horizontal; for, the whole Theory, here given, is quite general, and applicable to all positions whatever, either of the Original Plane or of the Picture.

* It may perhaps, to some, seem unnecessary to dwell so long on this Proposition, which, at first, may not appear of much Consequence; but, trifling as it may seem, I know it to be of the greatest Consequence, in the most difficult part of Perspective, viz. in the practice on inclined Planes; the Theory, of which, being well digested and understood, the Practice will be found almost as easy as in the most familiar Cases, being founded on the same invariable Principles, as will be exemplified in Practice; and therefore, the pains I have been at to enforce it, I am confident, is by no means to be dispensed with.

The Reader may be well assured, that every Part of the Theory, given in this Work, is of utility, and necessary to be known; and that, the better it is understood, the progress in practical Perspective will be greatly facilitated.

Plate VI.

THEOREM V.

All Original Planes, which are parallel amongst themselves, but not to the Picture, have the same Vanishing Line.

DEM. Vanishing Lines, in general, are produced, in Theory, by imaginary Planes, passing through the Eye or Point of View, parallel to the Original Planes. - - - - - - - - - - - - - Def. 8.

Wherefore, a Plane passing through the Eye parallel to any Original Plane, producing its Vanishing Line, is also parallel to every Plane which is parallel to the Original Plane; and since this parallel Plane can produce but one Line on the Picture, by its Intersection with it, that Line is, consequently, the Vanishing Line of all the Planes to which it is parallel.

This Theorem needs no further Demonstration.

EX. V, U and W are three Original Planes, parallel amongst themselves;
Fig. 22. ABLM is the Picture, and E the Eye, in the Directing Plane, IKGH.

Now, the Vanishing Line of any Original Plane is produced by a parallel Plane passing through the Eye and cuting the Picture. - - - - Def. 8.

IK*LM* is parallel to the Plane U; or, rather, they coincide, for it is a continuation of that Plane; consequently, it is parallel to the Planes V and W; and, seeing that no other Plane can be drawn through the Eye parallel to them, therefore *LM* is the Vanishing Line of the three parallel Planes V, U and W.

This may be further illustrated by Fig. 15. and by the Apparatus.

Plate V.
Fig. 15. Suppose a Plane (RSTU) to pass through the Eye (E) parallel to the Planes BFC and AIL in the Original Object BFIL, cuting the Picture, MNOP, produced, in RU; which is, therefore, the Vanishing Line of those parallel Planes; and, consequently, of all other Planes parallel to them.

For the same reason, a Plane, TOPS, passing through the Eye, parallel to the Planes ABGH and CDKL, cuting the Picture MN*OP* in *OP*; and the Picture RUOP in OP; *OP*, or OP, is the Vanishing Line of those Planes, on each Picture, respectively.

But, the Plane RSTU, passing through the Eye, being parallel to the Planes BFC and AIL, is parallel to the Picture MN*OP*, and therefore, can never cut it; consequently, those Planes have no Vanishing Line on that Picture; but, the Representation, *bfc*, is parallel, and similar to its Original, BFC.

SCHOL. All horizontal Planes have horizontal Vanishing Lines; except, the Picture be also horizontal; in which Case they have no Vanishing Line, the Picture being parallel to the Original Plane.

It is also evident, that all vertical Planes have, when the Picture is also Vertical, perpendicular or vertical Vanishing Lines; i. e. the Vanishing Lines are, in such Case, perpendicular Lines on the Picture, and at right Angles with such as are horizontal.

For, 1st. The Intersection of a horizontal Plane, with any other Plane, is necessarily a horizontal Line, seeing it is produced by a horizontal Plane, in which the intersecting Line must always be.

2nd. The common Intersection of two Vertical Planes, is a Right Line perpendidular to the Horizon; seeing that, the Line of their common section, is, necessarily, in both Planes.

COR.

COR. 1. All Original Lines that are parallel amongst themselves, and not to the Picture, have the same Vanishing Point.

For, by the same reasoning as in the Theorem, it is evident, that, since the Vanishing Points of Original Lines are produced, in Theory, by imaginary Lines passing through the Eye parallel to the Original Lines †; it necessarily follows, that a Line passing through the Eye, producing the Vanishing Point of any Original Line, by its Intersection with the Picture, being parallel to the Original Line, is also parallel to all Lines which are parallel to the Original Line; and, since it canproduce but one Point on the Picture, it is, consequently, the Vanishing Point of them all. (See Theor. 2. Book 1st.) † Def. 8.

In the same Figure, the Original Lines AB, GH and FI are parallel amongst themselves; ECV is a Right Line from the Eye, parallel to them all, cuting the Picture RUOP in V, and MN*OP*, in C, its Center; the Points C and V, on each Picture, respectively, are, therefore, the Vanishing Points of the Original Lines AB, GH, and FI. Fig. 15.

The same may be said in respect of the parallel Lines GF and HI, whose Vanishing Point, on the Picture RUOP, produced, is *V*.

Also, of GD and BC, whose Vanishing Point is Y, on the Picture RUOP; but, being parallel to MN*OP*, they have, consequently, no Vanishing Point on that Picture, but are parallel to their Representations.

COR. 2. Hence, it is demonstrable, that all Lines which are perpendicular to the Picture vanish in the Center, of the Picture.

For, AB, GH and FI are Original Lines, perpendicular to the Picture MN*OP*, whose Vanishing Point is C; produced by the Line EC from the Eye, E, parallel to the Original Lines. - - - - - - Def. 22.

Now, the Original Lines are perpendicular to the Picture, therefore, EC, producing their Vanishing Point is perpendicular to the Picture.

But, a Right Line from, or passing through the Eye, perpendicular to the Picture, produces the Center of the Picture. - - - - - - Def. 17.

Therefore, the Center of the Picture is the Vanishing Point of all Lines that are perpendicular to the Picture.

SCHOL. It may be necessary to observe here, that, in this last and in most of the Schemes or Diagrams made use of in this Theory, (being drawn perspectively) whenever any two or more Lines are said to be parallel, if they are not really so, they tend to the same Point; agreeable to Coroll. 1. For, I think it unpardonable, in a Treatise on Perspective, to give an Appearance of Objects, or Schemes, not truly and perspectively drawn; merely, for the sake of preserving the parallelism of certain Lines; as several Authors have done. Unluckily, for them, they cannot also preserve the true Angles, which are worse represented by that means.

Therefore, whenever Angles are said to be Right, or otherwise; and such or such Lines, to be parallel or perpendicular; if they are not really so, they must be understood to be so; and, I am persuaded, that they will always appear to be so, when truly delineated, perspectively.

THEO-

Plate V.

THEOREM VI.

The Vanishing Lines of all Planes, which are perpendicular to the Picture, pass through the Center of the Picture.

DEM. For, since the Vanishing Line of a Plane is produced by an imaginary Plane passing through the Eye parallel to the Original Plane†, and the Center of the Picture is determined by a Perpendicular, from the Eye to the Picture §, the Original Plane being, in this Case, perpendicular to the Picture, it follows, that its Parallel or Vanishing Plane (and the Parallel of every Plane that is perpendicular to the Picture) seeing it passes through the Eye, must, necessarily, pass through the Direct Radial, which is perpendicular to the Picture ‡, and consequently, through the Center of the Picture.

Therefore, the Vanishing Line, produced by the Intersection of every such Plane with the Picture, must pass through the Center of the Picture.

† Def. 8.

§ Def. 17.

‡ Def. 15.

Fig. 15.

EX. 1. All horizontal Planes are perpendicular to a evrtical Picture; wherefore, the Plane GHKD, and also the Ground Plane BL, or Z, are perpendicular to both Pictures, MNOP or *OP*; their Vanishing Plane, therefore, passes through the Direct Radial, EC, of each Picture, and consequently, through C, their Centers.

CX and XV (on each Picture, respectively) parallel to those Planes, and drawn through the Centers, C, of both Pictures, are the Vanishing Lines of those Planes.

For, by Theo. 3d. the Vanishing Line of every Plane is parallel to its Intersection; and consequently to the Original Plane.

EX. 2. The Planes AHGB and CDKL are perpendicular to MN*OP*, only; their Vanishing Plane, ST*OP*, therefore, passes through its Radial, EC, and their Vanishing Line, *OP*, through the Center, C, of that Picture.

As these two Examples are particular Cases; viz. when the Original Plane is either Horizontal or Vertical, I shall give another general Case, which will illustrate the Theorem universally.

Fig. 15. Nº 2.

The Original Plane *IKLM* is moveable on the Line TU, its common Section with the Ground or other horizontal Plane. Let AONB, the Plane of the Picture, be raised into a vertical Position; and, let TU, the common Section of the Original Plane, be at right Angles with AB, the Intersection of the Picture with the Plane Z.

Now, it is evident, that, if the Original Plane be turned over (on TU) to the other Side, the point *L* will describe a Semicircle; in which motion, the Plane *IKLM* is always perpendicular to the Plane or Picture, AONB; for, in every position of the Plane in that semi-revolution, the Line TU is still perpendicular to the Picture.

The Plane GHIK, being raised up with the Picture, and parallel to it, is the Directing Plane.

Let E be supposed the Eye of a Spectator, and EC the Direct Radial, producing the Center of the Picture, C.

Now, if a Plane IKLM be supposed to pass through the Eye of the Spectator, at E, and make also a semi-revolution, being all the while parallel to the Original Plane, it must revolve on EC, the Direct Radial, which is parallel to TU; and, in every Position, its Intersection with the Picture must necessarily pass through C, the Center of the Picture; produced by the Perpendicular EC.

For,

For, if the Plane IKLM, passing through the Eye, makes the same Angle with the Horizon, viz. PQR; i. e. if it be parallel to the Plane *IKLM*, it will cut the Picture in *O*N, the Vanishing Line of that Plane, and pass through C, the Center of the Picture; making the Angle, NCM, with LM, the Vanishing Line of Horizontal Planes, equal to the Angle PQR, of the Original Plane with the Ground Plane.

After the same manner, the Vanishing Lines of all Planes, which are perpendicular to the Picture, may be ascertained and drawn on the Picture; knowing the Angle which the Original Plane makes with any horizontal Plane; or with a vertical Plane, which is perpendicular to the Picture.

N. B. The Center of the Picture is the Center of all Vanishing Lines, of Planes perpendicular to the Picture; and they have all the same Distance or Radial, (EC) by Def. 19. and 20.

COR. Hence, also, it is evident, that the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture.

For, EC, a perpendicular Line from the Eye, producing the Center of the Picture, is the Common Radial of all Vanishing Planes, that are perpendicular to the Picture; and consequently, parallel to all Lines that are perpendicular to the Picture (for, they are all parallel amongst themselves, and to the Direct Radial, EC) as AB, GH, FI, &c. Fig. 15.

Therefore, since EC is parallel to all such Lines, the Point C, which it produces on the Picture, is their Vanishing Point; by Def. 22.

But, C is the Center of the Picture; consequently, the Center of the Picture is the Vanishing Point of all Lines that are perpendicular to the Picture.

This Corollary is also deduced from the last Theorem.

In this single and particular Vanishing Point, many Artists seem to rest all their knowledge of Perspective (commonly called the Point of Sight) and, whenever a number of Lines tend to the same Point, it is, by them, called a Point of Sight; not considering, or perhaps knowing, that the Center of the Picture is a Vanishing Point, in common with all other, only by virtue of the general Definition (Def. 22.) And, because the knowledge of it is more general, and, as they imagine, the practice much easier, we find it more used than any other; as most regular Objects, particularly Buildings, are right-angled; which, having one Side or Front parallel to the Picture, all the Horizontal Lines in the adjoining Sides are, consequently, perpendicular to the Picture, and therefore vanish in its Center; which, from the certainty of the Position of such Lines, is determined and fixed at once, though often very injudiciously.

THEOREM VII.

Original Planes, whose common Intersection is parallel to the Picture, have parallel Intersections with the Picture, and parallel Vanishing Lines.

DEM. The common Intersection of the Original Planes being parallel to the Picture, a Plane may be supposed to pass through that Intersection which is also parallel to the Picture.

Now, the Original Planes being produced, through their common Section, to the Picture, each Plane must necessarily cut the Picture in a Line parallel to their common Section† (which is in a Plane parallel to the Picture, by Supposition) wherefore, since the Intersection of each Original Plane, with the Picture, is parallel to the common Section of both Planes, they are, consequently, parallel between themselves. — Q. E. D. - - - - - 4. 7. El. † 8. 7. El.

S EX. W

Plate VI. Fig. 22. EX. W and X are two Original Planes, whoſe common Section, NO, is parallel to the Picture, ABLM. g h i is a Plane, ſuppoſed to paſs through their common Section, NO, parallel to the Picture.

Now, the Plane W being produced to the Picture, cuts it in *AB*; and the Plane X, being produced, cuts it in AB; wherefore, ſince the Picture and the Plane gh i are parallel, and are both cut by the Planes *AB*NO and ABNO, the Interſections, *AB* and AB, are, each, parallel to the common Section, NO, and therefore they are parallel between themſelves.

DEM. 2. The ſecond Part, viz. that their Vaniſhing Lines are alſo parallel, is already demonſtrated, in the 4th Theorem; which proves, that the Vaniſhing Line of every Original Plane is always parallel to its Interſection.

Wherefore, having proved that the Interſections are parallel, and, by the 4th, the Vaniſhing Line of each, reſpectively, is parallel to its Interſection, conſequently, the Vaniſhing Lines, of the two Planes, are parallel between themſelves. Q. E. D. - - - - - - - 5. I. El.

EX. LM is the Vaniſhing Line of the Plane X, and *LM* of the Planes V, U, & W; which are, reſpectively, parallel to the Interſections AB, JF, &c. (by Theo. 4.) and therefore they are parallel, alſo, between themſelves.

3. After what has been ſaid, it is manifeſt that the Directing Lines of Planes, in ſuch Caſe, are alſo parallel.

For, ſince their Interſections with the Picture are parallel, conſequently, if they were produced till they cut the Directing Plane, their Interſections with it would likewiſe be parallel; the Directing Plane being parallel to the Picture.

COR. 1. The common Interſection of a Plane, inclined to the Horizon, with any horizontal Plane whatever, being parallel to the Picture, the Vaniſhing Line, of that inclined Plane, is a Line parallel to the Horizon.

For, becauſe the common Interſection is parallel to the Picture, the Vaniſhing Lines of both Planes are parallel to each other; by Theorem.

But, one of thoſe Planes is horizontal, therefore its Vaniſhing Line is parallel to the Horizon; by Def. 8.

Conſequently the other Vaniſhing Line is, alſo, parallel to the Horizon.

COR. 2. All Original Planes, that have parallel Vaniſhing Lines, have the ſame Vertical Plane; and, conſequently, the ſame Vertical Line; and, alſo, the ſame Parallel of the Eye.

Z is an Original, horizontal Plane; and X, is a Plane inclined to the Horizon; their common Interſection, a b, is parallel to the Picture, AMLB.

*A*IK*B* is the parallel of Z, cuting the Picture in *AB*, the Vaniſhing Line of Z, and all horizontal Planes; and KLMI is the Parallel, or Vaniſhing Plane of the Original Plane X, paſſing through the Eye, at E, and cuting the Picture in LM, the Vaniſhing Line of that Plane; whoſe common Section, a b, with Z, being parallel to the Picture, its Vaniſhing Line, LM, is, conſequently, parallel to *AB*, by the Theorem.

But, QTQ is the Vertical Plane of both X and Z (alſo of W and Y) and of all Planes parallel to them; i. e. it cuts them both at right Angles, being perpendicular to their common Section a b; and it cuts the Picture in CD the Vertical Line of them both (or of them all) for, the ſame Plane cannot cut the Picture in two Lines.

IK is alſo the Parallel of the Eye of all thoſe Planes, for, it is the common Section of all their Vaniſhing Planes, IKLM, IK*LM*, and IK*BA*, with the Directing Plane IKGH.

This

This Theorem, and the last Corollary, may also be illustrated by Figure 15.

1. For, BG, the common Intersection of the two Planes ABGH and BFC, is parallel to both Pictures, MNOP or *OP*; their Intersections *BE* and *BK*, on MNOP, are therefore parallel; also their Vanishing Lines, RU and OP, on that Picture. Fig. 15.

The other Picture MN*OP*, being parallel to the Plane BFC, has but one Vanishing Line, *OP*; which is parallel to the Intersections *BG* and MN.

2. The two Planes, ABGH and BFC, being Vertical, a Plane vertical to them, is, consequently, Horizontal; for, no other Plane can cut both, or either of them and the Picture, perpendicularly. (See Fig. 19. and Par. 1. p. 57.)

Wherefore, they have the same Vertical Plane; which, in reality, is a horizontal Plane, passing through the Eye; and, consequently, they have also the same Vertical Line; which, in this Case, is the Vanishing Line of horizontal Planes; viz. CX or VX, on either Picture, respectively.

N. B. It is the same in all Positions, whatever, of the Original Planes.

THEOREM VIII.

The Planes, which produce the Vanishing Lines of two Original Planes, are inclined to each other in the same Angle as the Originals; and, have their common Intersection, passing through the Eye, parallel to the common Intersection of the Original Planes.

DEM. For, first, since the Planes, producing the Vanishing Lines of any two Original Planes, are, respectively, parallel to the Originals, they have consequently the same Inclination to each other, as the Originals.

For, if all the Planes are produced (viz. the Original Planes and their Parallels) they will cut each other in parallel Lines, and form a Parallelopiped, between their Intersections. - - - - - - - 8. 7. El.

Therefore, they have the same Inclination to each other, respectively. - 13. 7.

EX. W, X, and Z are Original Planes, and K*M*, KM and K*A* their respective Vanishing Planes. Fig. 22.

The Angle, which the Original Plane, X, or *A*NOP, makes with Z, is Oak (aO and ak being both perpendicular to their common Section ab) and it is equal to the Angle MI*A* or S, which their Vanishing Planes KM, and K*A*, make with each other.

Because IM is parallel to aO, and I*A* to ak, the Angle MI*A* is equal to Oak ‡. ‡ 6. 1. El.
For, let I*A* be produced till it cuts aO in S, the Angle MI*S* is equal to ISa †; † 4. 1. El.
And, because I*A* is parallel to ak, the Angle ISa is equal to Sak; by the same. consequently, MI*A* is equal to Oak; by the 3d Axiom. 1st El.

Therefore, the Planes, extended through those Lines, being respectively parallel, (KM to X, and K*A* to Z), are inclined to each other in the same Angle; i. e. KM has the same inclination to K*A* as X to Z.

Also, the Plane KM being parallel to X, and K*M* to V, U, and W; they are, therefore, inclined to each other in equal Angles, viz. MI*M* equal *A*Oa.

Secondly. Because, in this Case, the common Sections (ab and NO) of the Original Planes are parallel to the Picture, the Intersection, IK, of the Vanishing Planes, passing through the Eye, E, is also parallel to the Picture; and consequently, to the common Sections, ab and NO, of the Original Planes. Q. E. D. - - - - - - - - 4. 7. El.

For it is in the Directing Plane, which is parallel to the Picture; by Def. 4.

To

Plate V. To illustrate this Theorem more familiarly, by moveable Planes.

Fig. 15. No. 2. 1. Raise up the Plane of the Picture AONB, perpendicular, or otherwise, to the Horizon. The Directing Plane, GEH will be paralel to it; and IKLM to the Plane of the Horizon.

No. 3. Then, if the Plane *ADFB* (any Original Plane) whose Intersection, *AB*, with the Horizon, or any horizontal Plane, is parallel to the Picture, be raised up, on *AB*, making any Angle (PQR) with the Horizon, and, a Plane, *IK*ON, pass through the Eye, at E, parallel to that Original Plane; it is, occularly, manifest, that this Vanishing Plane, *IK*ON, makes the same Angle with *IKLM*, as *ADFB* with the Horizon; and *IK* (the parallel of the Eye) the Intersection of the Vanishing Planes, is parallel to *AB*, the common Intersection of the Original Planes.

2. The Picture, &c. remaining in the same Position, perpendicular to the Ground Plane, and to the common Intersection TU of the Original Plane *IKLM*.

No. 2. Then, the Vanishing Plane IKLM being placed parallel to the Original Plane *IKLM*, it is evident, that it will make the same Angle with the Horizon, or with a Plane supposed to pass through the Eye, parallel to the Horizon, and cuting the Picture in LM (the Vanishing Line of horizontal Planes) as the Original Plane *IKLM* with the Horizon.

Also, EC (the Direct Radial) the Intersection of those Vanishing Planes is parallel to TU, the common Intersection of the Original Planes.

For, they are both perpendicular to the Picture.

To say more on this is unnecessary, for it is evident, that let the Picture be situated, as you please, to the Original Planes; since the Planes producing their vanishing Lines are, respectively, parallel to them, they are inclined to each other in equal Angles, respectively; their common Sections are also parallel, and make equal Angles with the Picture, if they are not parallel to it; as in Case 1st.

In Fig. 23. this Theorem is quite general, i. e. the Position the Original Planes have to the Horizon, or to the Picture, is not regarded; I mean, as to parallel or perpendicular.

Plate VI. Fig. 23. X and Y are two Original Planes, both inclined to the Horizon, and to the Picture JTU*N*; which is also inclined to the Horizon, towards the Eye, at E. AB is the common Intersection of those two Planes.

Then, if a Plane, IKLM, be supposed to pass through the Eye (E) parallel to the Plane, X; and another Plane, *IKLM*, parallel to ABD, or Y, cuting the Picture in the Vanishing Lines, LM and *ML*, of those Planes; they will, necessarily, be inclined to each other in equal Angles, respectively; i. e. the Vanishing Planes, as the Original Planes; and the common Section, EV, of the Vanishing Planes, is parallel to the common Section, AB, of the Original Planes.

COR. 1. The Lines, which produce the Vanishing Points of any two Original Lines, make the same Angle, at the Eye, as the Original Lines make with each other.

For, they are respectively parallel to their Originals (Def. 22.) and, consequently, they make equal Angles, respectively.†

† 6. 1. El.

Fig. 15. EX. EV is the Radial or Parallel of AB, and EY of CB.
V and Y are, therefore, the Vanishing Points of AB and CB; the Radials EV, EY being parallel, respectively, to the Originals, AB and CB.
Consequently they make the same Angle at the Eye, E, as the Original Lines make with each other; viz. VEY equal ABC. - - - 5. 7. El.
But, ABC is a Right Angle; therefore, VEY is a Right Angle.

This

Plate V.

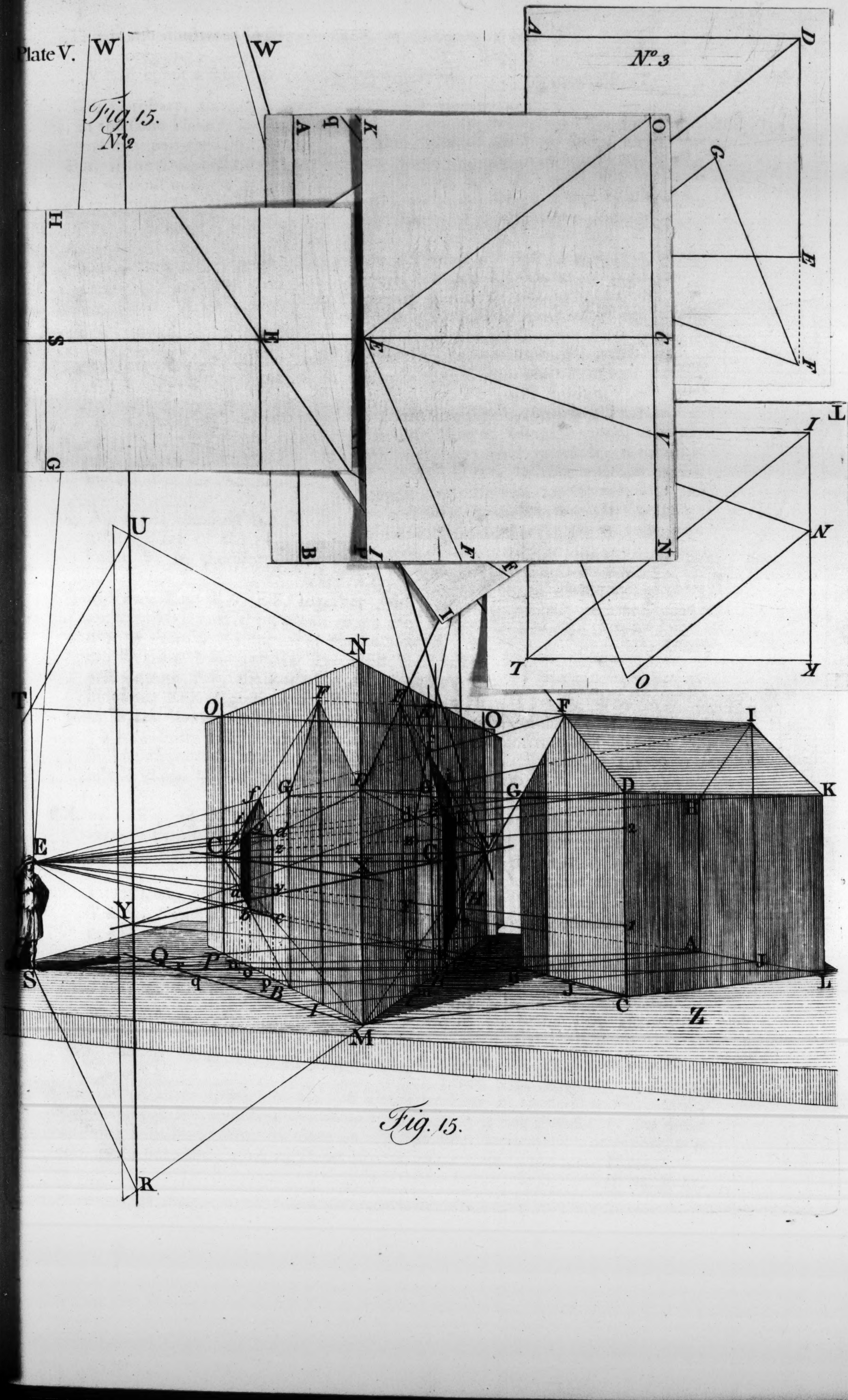


Fig. 15.

This is also illustrated in Fig. 23. and, by changing Y for M, it is described in the foregoing; save only, that ABC is not a Right Angle, consequently VEM is not a Right one, being equal to ABC.

This Corollary, though general, is applicable only to such Lines as are, or may be, in the same Plane; for, if two Lines, being produced, would cut each other, a Plane may pass through them both §. § Ax. 6.

But, if the Original Lines are so situated, that they cross, or would cross, each other, without cuting; then, the Angle, which the Radials of such Lines make with each other, is equal to the Angle that would be made, between the Originals, if either of them be moved, parallel to itself, in any direction, till it cuts the other.

COR. 2. The Vanishing Point of the common Intersection of two Original Planes, is the Point in which their Vanishing Lines intersect each other.

For, the Intersection of the parallel Planes, producing the Vanishing Lines, passes through the Eye, parallel to the common Intersection of the Original Planes (by the Theorem) and consequently, will cut the Picture in that Point where the Vanishing Lines intersect.

But, (by Def. 22.) the Vanishing Point of a Line is that Point, in which, a parallel Line, from the Eye, cuts the Picture.

Therefore, the Point of Intersection of the Vanishing Lines is the Vanishing Point of the common Intersection of the Original Planes.

EX. AB is the common Intersection of the Planes X and Y. ML is the Vanishing Line of the Plane X, and *ML* of the Plane Y; their intersecting Point, V, is, therefore, the Vanishing Point of AB. Fig. 23.

The Visual Rays EA, EB, together with the Line AB, form a plane Triangle, which cuts the Picture in ab (the Representation of AB) whose Vanishing Point is V; for, it is the Section of a Radial Plane, passing through the Original Line and the Eye, with the Picture; which must necessarily pass through EV, the Radial of AB†, and cut the Picture in JV, the whole indefinite Representation of AB; by Theorem 1st. † Ax. 5.

Mm is the Vanishing Line of the vertical Plane W, and Mq of horizontal Planes; both which, cut ML, the Vanishing Line of the inclined Plane, X, in M; therefore, M is the Vanishing Point of CB, the common Intersection of the Planes W and X, and a horizontal Plane, CBD.

EX. 2. In Fig. 15. VY is the Vanishing Line of horizontal Planes; RU, of the vertical Planes BFC and AIL; and VW†, of the inclined Plane FGHI. V, Y, and W† are, therefore, the Vanishing Points of the common Intersections of those Planes, respectively; viz. V, of GH, the common Intersection of DGHK and FGHI; and also of ABGH, whose Vanishing Line is OP. Y is the Vanishing Point of the common Intersection of horizontal Planes, and of the vertical Planes BFC and AIL; and, W is the Vanishing Point of GF and HI, the common Intersections of those vertical Planes and the inclined Plane FGHI.

† The Van. Point W is where EW parallel to FGH cuts RU produced. See Apparatus.

I have given various Examples of this Corollary, it being very essential in Practice, and cannot be too much enforced.

For, having found the Vanishing Point of a Line (in any Plane whatever) which is either the common Intersection of that Plane with some other adjoining Plane, or a Line parallel to it; the Vanishing Line of that other Plane must, necessarily, pass through that Vanishing Point; and, if the Position of the other Vanishing Line be known, its place on the Picture is determined; otherwise, it is necessary to have the Vanishing Point of some other Line in the same Plane.

T THEO-

Plate IV.

THEOREM IX.

The Radial, or Parallel, of an Original Line producing its Vaniſhing Point, makes the ſame Angle with the parallel of the Eye and Vaniſhing Line, as the Original Line makes with the Interſection and Directing Line, of the Plane that Original Line is in.

DEM. Since the Interſection and Vaniſhing Line, the Parallel of the Eye and Directing Line, of every Original Plane, are parallel amongſt themſelves; (Th. 3.) and the Radial, producing the Vaniſhing Point of an Original Line, is parallel to the Original, (Def. 22.); it neceſſarily follows, that, whatever inclination the Original Line has to the Interſection and Directing Line (which are all in the ſame Plane) the Radial of that Line has the ſame Inclination to, or makes equal Angles with, the Parallel of the Eye and Vaniſhing Line, which are all in a parallel Plane. - - - - - 4. of 1. and 7. El.

Fig. 20. and 21. EX. NO is an Original Line, which, being produced, cuts the Interſection and Directing Line, AB and GH, in equal Angles. - - 4. 1. El.

EV is the Radial of NO, producing its Vaniſhing Point, V, in the Vaniſhing Line ML, of the Plane NGH, making the Angles EVL, with the Vaniſhing Line, and VEI, with the Parallel of the Eye, equal.

And, they are alſo equal to the Angles N*I*A, N*D*H, which the Original Line, NO, makes with AB and GH, the Interſecting and Directing Lines.

Becauſe, the Original Line (NO) and its Radial (EV) are parallel (Def. 22.) and, becauſe they cut parallel Lines in parallel Planes.

2. From the ſame reaſoning, it is evident, that the Angles KE*D*, E*D*H, made by the Director, *ED*, of the Line NO, with the Parallel of the Eye and Directing Line, are equal to the Angles LV*I*, V*I*A, which the indefinite Repreſentation (IV) of the Original Line (NO) makes with the Interſection (AB) and the Vaniſhing Line (VL).

For, they are parallel Lines in parallel Planes; and they are cut by parallel Lines.

3. Hence, it is alſo manifeſt, that, the Angle, VEC, made by the Radial, EV, of the Original Line NO, and EC, the Radial of the Vaniſhing Line, VL, producing its Center (C) is equal to the Angle which the Original Line, N*ID*, makes with FD, the Interſection of the Vertical Plane, ECDF, with the Plane NGH, in which the Original Line (NO) is ſituated.

For, theſe Angles are the Complements of the Angles made by the Original Line and its Radial, with the Interſection of the Picture, and the Parallel of the Eye or Vaniſhing Line.

I would recommend it to young Students, to be very clear in this Theorem, as it is moſt eſſential in Practice. For, from the knowledge it inculcates, the Vaniſhing Points of Lines, in any Plane whatever, may be readily found; knowing the Angle which the Original Line makes with the Interſection of the Plane it is in, with the Picture; or with a Line paſſing through the Station Point, and cuting the Interſection at Right Angles; making an equal Angle, at the Eye, with the Parallel of the Eye; or, with the Radial of the Vaniſhing Line, producing its Center.

THEO-

THEOREM X.

The Representation, on the Picture, of a Line parallel to the Picture, is parallel to the Original; and, has that proportion to the Original, as the Distance of the Picture to the Distance of a Plane, passing through the Original Line, parallel to the Picture.

DEM. The Perspective Projection of every Right Line on the Picture, is a Right Line; and it is produced, in Theory, by the Intersection of a Plane, with the Picture, passing through the Eye and the Original Line. - - Theo. 1.

Wherefore, since the Original Line is, in this Case, supposed parallel to the Picture, a Plane may pass through that Line, also parallel to the Picture.

But, if two Planes being parallel, are both cut by another Plane, their Intersections are parallel. - - - - - - 8. 7. El.

But the Original Line is one of the Sections, and its Projection, on the Picture, is the other (the cuting Plane being supposed to pass through the Original Line and the Eye).

Therefore, the Representation of a Line, which is parallel to the Picture, is parallel to the Original. Q. E. D.

EX. NO is an Original Line, parallel to the Picture, AMLB; E is the Eye, in the Directing Plane, IKGH. Fig. 22.

Let ghi be supposed a Plane, passing through the Original Line, NO, parallel to the Picture; and, EN, EO, Visual Rays, from the Eye, to each extreme of the Original Line; which, are in the same Plane with NO. - - Ax. 8.

Wherefore, NEO is a Radial Plane passing through the Original Line and the Eye, and cuting the Picture in no, the Representation of NO. - - Th. 1.

But, the Plane ghi is parallel to the Picture, and NO is in that Plane; consequently, no, its Representation, which is the Intersection of the Radial Plane, NEO, with the Picture, is parallel to the Original Line, NO.

For, they are the common Sections of two Planes, which are parallel, by another Plane.

In the same manner, rs, the Representation of RS, may be proved parallel to RS.

EX. 2. AH, BG, CD, &c. are Original Lines, parallel to both Pictures, MNOP or *OP*. EB and EG are Visual Rays, from the Eye, E, to the Line BG; which are all in the same Radial Plane, cuting the Pictures in bg, and *bg*, which are, therefore, the Representations of BG, on each Picture, respectively (by Theorem 1st.) Fig. 15.

But, BG, the Original Line, is perpendicular to the Horizon or Ground Plane, SMZ; and the Pictures are Vertical Planes.

And, the Radial Plane GEB is also Vertical, seeing it passes through GB.-9.7. El.

Consequently, the common Sections of those Planes are Right Lines, perpendicular to the Ground Plane. - - - - Cor. to 9. 7. El.

and, consequently, they are parallel between themselves. - - - - 3. 7.

Therefore, bg or *bg*, the Representation of BG, is parallel to BG.

Also, cd, or *cd*, is parallel to CD; and ah, or *ah*, to AH.

After the same manner, the Representation of any other Line, BC, GD or GF, &c. in the Plane BFC, which is parallel to the Picture MNOP, only, may be proved parallel to their respective Originals.

DEM.

Plate IV. Fig. 22. N° 2.

DEM. 2. The Representation, n o (of N O) is proved parallel to N O, by the 1st Part.

Wherefore, the Triangles NEO, nEo, are similar;
and no : NO :: En : EN, or as Eo : EO. - - Cor. 3. 2. 6. El.

But, ECſ is a Right Line perpendicular to the Picture AB, and consequently, to the Plane g h i, which is parallel to the Picture, cuting the Picture in C, its Center, and the Plane g h i in ſ.

Now, becauſe ECſ is perpendicular to the Picture, and to the parallel Plane g h i, in which the Original Line, NO, is ſituated, it is the ſhorteſt Line that can be drawn to thoſe Planes, and conſequently meaſures their Diſtances. - - - - - - - Cor. to 12. 1. El.

But, if two or more Right Lines are cut by parallel Planes, they will be cut proportionally (whether they proceed from one Point or not) - - 10. 7. El. wherefore, EC : Eſ :: En : EN, or, as Eo is to EO.

But, no : NO :: En : EN. Th. no : NO :: EC : Eſ, by equality of Ratios.

That is, the Repreſentation, n o, has that Proportion to N O, its Original, as, the Diſtance of the Picture, EC, has to Eſ, the Diſtance of a Plane paſſing through the Original Line parallel to the Picture.

N. B. The ſame Demonſtration holds good if *NO* be conſidered as the Original Line, on this Side of the Picture.

COR. 1. From the former part, It is evident, that the Projections of any number of Lines, which are parallel amongſt themſelves and to the Picture, are alſo parallel amongſt themſelves and to the Originals.

Fig. 15. AH, BG and CD are parallel amongſt themſelves, and to both Pictures; their Repreſentations, on both, are therefore parallel.

Alſo, BC and GD are parallel between themſelves and to the Picture MN*OP* only; their Repreſentations, on that Picture, are therefore parallel between themſelves and to the Originals.

COR. 2. From the ſecond part of this Theorem, may be clearly deduced; that if an Original Line parallel to the Picture, be any how divided, the Repreſentations of the ſeveral Parts will have the ſame Proportion to each other, and to the whole Repreſentation, as the Parts of the Original Line have to each other, and to the whole Line.

If from the diviſions, 1 and 2, of the Line CD, the Viſual Rays E 1, E 2 be drawn, they will cut the Repreſentations, c d, on both Pictures, to which the Original, C D, is parallel, in the ſame Ratio, in the Points y and z.
For, becauſe c d is parallel to CD, the Triangles CED, c Ed are ſimilar.
And, for the ſame reaſon, CE 1, CE 2, are ſimilar to c E y, c E z, &c. C. 3. 2. 6. El.
Wherefore, c y : C 1 :: c z : C 2; i. e. as c d : C D; - - 4. 6. El.
And, conſequently, c y : y z : z d :: C 1 : 1 2 : 2 D; or as c d to C D.

COR. 3. Hence it is manifeſt, that if the Eye be moved, ſtill keeping the ſame Diſtance from the Picture; i. e. let the Eye be any where in the Directing Plane; the whole Repreſentation, and each Segment, will ſtill have the ſame proportion to the Original Line.

For, by the ſecond part of the Theorem, the Repreſentation of a Line parallel to the Picture, has the ſame Proportion to the Original, as the Diſtance of the Picture to the Diſtance of a Plane, paſſing through that Line, parallel to the Picture.

Conſequently. if the Original Line C D, (which is ſeen very oblique in reſpect of the Picture MN*OP*), was directly oppoſite to the Eye; ſo that EC produced was perpendicular to C D, the Proportion of c d, its Repreſentation, will be the ſame, in reſpect of its Original; and, conſequently, if the Eye be moved oppoſite to C D, ſtill keeping the ſame Diſtance, EC, from the Picture, or any where, ſituated in the Directing Plane, which is parallel to the

the Picture, therefore equidistant; the whole Representation and each of the Divisions have still the same Ratio to the Original.

For, the Distance of the Picture and the Distance of the Plane the Original Line is in, remain the same.

COR. 4. The Angle, which the Representations of any two Original Lines that are parallel to the Picture, make with each other, is equal to the Angle made by the Original Lines.

For, the Representations are respectively parallel to their Originals.

In Fig. 22. No. 2. Let the Original Lines, ON and SR, be produced till they intersect, in P, making an Angle OPS. Fig. 22. No. 2.

Because no is parallel to NO and rs to RS, being produced, they will, necessarily, make the Angle ops equal to OPS, made by the Original Lines. - - - - - - - - - - - 5. 7. El.

In Fig. 15. the Angle *gfd*, on the Picture *MNOP*, to which the Plane BFC is parallel, is equal to the Angle GFD; and *fgd* to FGD, &c. Fig. 15.

For, *fg* is parallel to FG, *fd* to FD, and *gd* to GD; - by the Theorem.

COR. 5. Original Figures, in Planes parallel to the Picture, have their Representations similar to the Originals.

The Plane BGFDC is parallel to the Picture MN*OP*.

By Cor. 1. *bg* and *cd* are parallel between themselves, and to the Originals, BE and CD; and so are *gf* and *fd* to their Originals.

And by Cor. 4. the Angles *bgf* and *gfd*, &c. are equal, respectively, to the Angles BGF and GFD, &c.

But, it is demonstrated, in the Theorem, that *bg* : BG : : *gf* : GF, or, *gd*: GD, &c. for, each is to its Original, as the Distance of the Picture to the Distance of the Plane they are in; consequently, they have the same Proportion between themselves.

Therefore, since the Sides are directly proportional, and the Angles contained by corresponding Sides, or Diagonals, equal; the Triangle *gfd* is similar to the Original GFD; and the whole Representation *bgfdc* to the Original BGFDC.

The Plane ghi is parallel to the Picture, AB; wherefore, the Representation op is parallel to the Original Line, OP; ps is also parallel to PS; and, if os, OS be drawn, os will be parallel to OS, - by Theo. Part 1st. Fig. 22. No. 2.

By Cor. 4. the Angle ops is equal to OPS; and by the second Part of the Theorem, op : OP : : ps : PS; for, each is as EC to Es.

Consequently, the Triangles pos, POS are equi-angular; the Angle o is equal O, and s equal S; and consequently, os : OS : : op : OP, or as ps : PS.

Therefore, the Representation, pos, is similar to the Original Triangle, POS.

For, SEOP may be supposed a Pyramid, cut by a Plane, AB, parallel to its Base, POS. E, the Eye, is the Vertex of the Pyramid.

N. B. All Lines, that can be drawn in a Plane, which is parallel to the Picture, are parallel to the Picture; and their Representations have all the same Proportion to their respective Originals.

Plate VI.

THEOREM XI.

All Right Lines, in any Original Plane, not parallel to the Picture, have their Intersecting Points in the Intersecting Line, and their Directing Points in the Directing Line of that Plane.

Also; the Vanishing Points, of Original Lines, are all in the Vanishing Line of the Planes the Original Lines are in.

DEM. For, first, if a Right Line be not parallel to the Picture, it will, if produced, cut the Picture, and also the Directing Plane, in its Intersecting and Directing Points - - - - - - - - - Def. 21 and 23.

Now, since one part of a Right Line cannot be in a Plane and another part of it out of that Plane;† consequently, the Original Line must cut the Picture, somewhere, in the Intersection of the Plane, it is in, with the Picture.

† Ax. 1.

And, for the same reason, it will cut the Directing Plane, in the Intersection of the Plane it is in with the Directing Plane, i. e. in the Directing Line of that Plane. Therefore, all Lines, &c. Q. E. D.

Fig. 23. EX. AB and CB are two Original Lines in the Plane X; which, being produced, cut the Picture (JTU*N*) in J and d, and the Directing Plane, in P and O.

NA, in the same Plane, X, cuts the Picture in *I*, and would, if produced to the Directing Plane, cut it, somewhere, in the Line PO, produced.

But, if the Plane ABC, was produced, it would cut the Picture in the Line J*I*, which is, therefore, its Intersection. - - - - - - Def. 7.

And, it would cut the Directing Plane, *GRH*, in PO (parallel to J*I*) its Directing Line. - - - - - - - - - Def. 10.

Wherefore, the intersecting Points J, d, and *I*, of the Original Lines, AB, CB, and NA, are in the Intersection, J*I*, of the Plane (ABC, or X) those Lines are in, with the Picture; and the Directing Points, of those Lines, P, O, &c. are in the Directing Line, PO, of the Plane X.

Also, J and N are the intersecting Points of AB and AD, in the Plane Y or BAD.

Wherefore, J*N* is the Intersection of the Plane Y, which, by reason of the Inclination of that Plane, falls, almost wholly, below the Ground Plane, Z, and its Intersection *F*I.

P and Q are the Directing Points of those Lines; PQ is, therefore, the Directing Line of the Plane Y which is parallel to J*N* (by Theorem 3d.)

EF is the Line in which the Plane BDFG would cut the Picture, if it was produced; *E* and *F* are, the Intersecting Points of BD and FG in that Plane.

i d is the Intersection of the Plane W, in which are the Intersecting Points, i and d, of the Lines HG, and CB, in that Plane; which Lines would, if produced, also cut the Directing Plane, in the Directing Points of those Lines.

DEM. 2. Because the Original Lines, whose Vanishing Points are required, are all in the same Plane, the Radials of those Lines, producing their Vanishing Points, being parallel to the Originals, are, consequently, in the Vanishing Plane, or Parallel of the Plane the Original Lines are in; and, since they cannot go out of that Parallel Plane†, their Intersections with the Picture, are, consequently, in the Intersection of the Vanishing Plane with the Picture.

† Ax. 1.

But, the Intersection of the Vanishing Plane, of any Original Plane with the Picture, is the Vrnishing Line of that Original Plane. - - Def. 8.

Therefore, the Vanishing Point of every Original Line, is in the Vanishing Line of the Plane that Original Line is in. Q. E. D.

EX. The

EX. The Vanishing Points V and M, of the Original Lines AB and CB, are in the Vanishing Line of the Plane, X, those Lines are in. Fig. 23.

For, the Vanishing Line, ML, of the Plane X, is produced by the Intersection of the Plane IKLM passing through the Eye, E, parallel to the Original Plane. - - - - - - - - Def. 8.

Wherefore, since the Lines, EV and EM, which produce the Vanishing Points of the Original Lines, AB and CB, are parallel to the Originals §, they are in the Vanishing Plane, IKLM, which produces the Vanishing Line of the Plane X, those Lines are in; and consequently, the Vanishing Points of those Lines are in the Vanishing Line, ML, of that Plane. § Def. 22.

COR. 1. Hence, it is manifest, that a Right Line, drawn through any two Vanishing Points, of Lines in any Original Plane, is the Vanishing Line of that Plane.

For, the Vanishing Point of every Line, in that Plane, is in its Vanishing Line; by the Theorem.

Also, a Right Line, drawn through any two Intersecting Points, is the Intersecting Line; and, a Line drawn through two Directing Points is the Directing Line of the Plane, in which the Original Lines are situated.

EX. ML, drawn through the Vanishing Points M and V, is the Vanishing Line; J*I*, drawn through the Intersecting Points J, d, and *I*, is the Intersection of the Plane, X, with the Picture; and PO, drawn through the directing Points, P and O, is the Directing Line, of the Plane X, in which, the Original Lines, AB and CB, are situated.

For, if two Points in a Right Line are given, the whole Line is determined.

COR. 2. All Original Lines, which are parallel to any Original Plane, have their Vanishing Points in the Vanishing Line of that Plane.

Because, a Plane may be drawn, through any one of the Lines, parallel to the Original Plane; and two or more Planes, being parallel, have the same Vanishing Line; by Theorem 5th.

COR. 3. The Intersecting Point of the common Intersection, of two Original Planes, is the Point in which the Intersecting Lines of those Planes, cut each other.

For, the common Intersection of two Planes is a Right Line, and is in both Planes.

Wherefore, the Intersecting Point, of that Line, is in the Intersections of both Planes; consequently, it is that Point in which the Intersections cut each other.

For, there is no other Point common to both Lines.

EX. J*I* is the Intersection of the Plane X, and J*N* of the Plane Y. J the Point of their common Intersection, is, therefore, the Intersecting Point of AB, the common Intersection of the Planes, X and Y.

J*I* is the Intersection of the Plane X; and i d of the Plane W; d is, therefore, the Intersecting Point of CB, the common Intersection of the Planes X and W.

2. FD and FG are the Intersections of the vertical Plane BFC, and the inclined Planes, FK and FH; wherefore, E and H are the intersecting Points of those Lines, on the Picture MNOP, where the vertical Intersection, BE, of the Plane BFC, is cut, by the Intersections, *DF* and *FG*, of the inclined Planes, FK and FH. Fig. 15.

Hence

Plate VI. Hence, it is evident, may alſo be deduced the 2nd Corollary to the 8th Theorem, viz. The Vaniſhing Point of the common Interſection of two Original Planes, is the Interſection of their Vaniſhing Lines. Which, being ſo very eſſential in Practice, cannot be too ſtrongly enforoed; and which, I adviſe the Student to be quite clear in; for, by that means, he will be able to determine the Vaniſhing Lines of contiguous Planes; and ſo proceed, in the delineation of Objects, from one Plane to another, with the greateſt facility.

COR. 4. And hence, it is alſo evident, that the Directing Point of the common Interſection of two Planes, is the Interſection of their Directing Lines. Which, being of ſo little uſe in Practice, is not worth the while to enforce, by more than one Example.

Fig. 23. EX. PO is the Directing Line of the Plane X, and PQ of the Plane Y; conſequently, P, the Point of their Interſection, is the Directing Point of AB the common Interſection of thoſe two Planes.

COR. 5. The Repreſentation of the common Interſection of two Original Lines, is the Point, in which the Indefinite Repreſentations interſect each other.

For, the Point B, of the Interſection of the two Original Lines AB and BC, is common to both Lines; wherefore, its Repreſentation is in the Indefinite Repreſentations of both Lines; and, conſequently, it is in the Point, b, of their mutual Interſection; for, there can be no other Point common to both. Alſo, a, the common Interſection of JV and *NV*, the indefinite Repreſentations of AB and AD is the Repreſentation of A, the Interſection of thoſe Lines.

SCHOL. I thought proper to give this Diagram in this manner, rather than to continue the Original Plane Y, in which the Original Lines, AB and AD, are, to the Picture, without being intercepted by the Horizontal Plane, Z, on which the Picture and Object are ſuppoſed to ſtand; in order to enforce the Theorem; and to ſhew, that no regard is to be had to any other Plane or Interſection whatever, lying between; but only, to the Interſection of that Plane in which the Original Lines are ſituated. And alſo, to ſhew (as it will be ſhewn in Practice) that the Interſecting and Vaniſhing Points are found or produced, by ſuppoſing the Original Line and its Radial to be produced, through every impediment, to the Picture *.

THEOREM XII. Defin. 25th.

The Indefinite Repreſentation, or Projection on the Picture, of an Original Right Line, not parallel to the Picture, is a Line drawn through its Interſecting and Vaniſhing Points.

DEM. For, every Right Line, which is not parallel to the Picture, will, if produced, cut the Picture, ſomewhere in the Interſecting Line of the Plane it is in. - - - - - - - - - - by Theorem 11.

And the Radial of every Line producing its Vaniſhing Point muſt cut the Picture, in the Vaniſhing Line of the Plane the Original Line is in; by the ſame Theorem.

* I look on this Diagram as the compleateſt in the whole Theory, for illuſtrating it generally; without regard being had to any Poſition, in reſpect of the Horizon; and I think it cannot fail of anſwering the intention I aimed at, viz. the diveſting the Reader of that partiality which moſt dablers in Perſpective have to the Horizontal Vaniſhing Line. It is, at firſt ſight, to a Novice in Perſpective, ſomewhat intricate, and ſome have affirmed it to be ſo. This I know, it coſt me more pains, to form it correctly, than any other three; and I am alſo certain, that every Perſon, who has given due attention to all the References it contains, will be convinced that it is not intricate, ſo far from being ſo, they will find it quite eaſy, being regularly analized.

Wherefore

Wherefore, a Radial Plane passing through the Original Line and the Eye, must necessarily pass through the Radial of that Original Line, and consequently through its Vanishing Point. - - - - Ax. 5.

For, the Radial of every Line is parallel to the Line. - - Def. 14.

But, the Perspective Projection of every Right Line on the Picture, is a Right Line, produced by the Intersection of a Radial Plane with the Picture. - - - - - - by Theo. 1.

And, the Radial Plane, passes through its intersecting and vanishing Points.

Consequently the Indefinite Representation of the Original Line on the Picture passes through its Intersecting and Vanishing Point. Q. E. D.

EX. Let HL be a Right Line, cuting the Picture (ACB) in I, and the Directing Plane in D; let E be the Eye; EV is the Radial of HL, cuting the Picture in its Vanishing Point, V; by Def. 22. Fig. 24.

Let a Radial Plane, NOPQ, be supposed to pass through the Original Line and the Eye, at E; consequently through the Radial EV (for it is parallel to HL) and also through I, the Intersecting Point of HL.

Now, the Points I and V are in this imaginary Plane, and they are also in the Picture; therefore, a Right Line, drawn through the Points I and V, is the Intersection of the Plane NOPQ with the Picture, ACB.

But, the Plane NOPQ passes through the Eye, E; therefore, the Eye is in that Plane; and, consequently, the whole of that Plane, and every Line in it, appears but a Right Line.

Wherefore, the Intersection IVr, of that Plane with the Picture, is the indefinite Representation of the Original Line, HL, on the Picture; seeing that, every Line, drawn, through E, to any Point, N, M, F, K, &c. of that Line (except the Point D) is in the Plane NOPQ, and will, if produced, cut the Picture, somewhere in IV, produced both ways, to g and r;

But, the Points n, m, f, &c. where the Visual Rays, or Right Lines, EN, EM, EF, &c. cut the Picture, are the Representations of the Points N, M, F, &c. in the Original Line§; consequently, the Representation of every Point, in the Line HL, except the Point D, where it cuts the Directing Plane, is in the Intersection gVr, produced indefinitely. § Persp. Art. 2.

But, the Intersecting Point, I, and, V, the Vanishing Point of the Original Line, are in that Intersection.

Therefore, the Indefinite Representation of every Right Line is a Right Line, drawn through its Intersecting and Vanishing Points.

COR. 1. The Representation of an indefinite Right Line, situate on the other Side the Picture, lies between its Intersecting and Vanishing Points

If that part of the Original Line, IN, which lies beyond its Intersecting Point, I, was produced infinitely, towards H, its whole Representation, on the Picture, is IV.

For, the Point I is its own Representation; Im is the Representation of IM, and mn of the part MN; and if the Point H be supposed at any finite Distance, its Representation h, must lie between n and V.

For, since the Radial EV, producing the Vanishing Point, V, is parallel to the Original Line, IN, the Angle VEH will always be equal to EHD.

And, if the Point H be supposed at an infinite Distance, its Representation, h, will coincide with V; the Angle VEH will not be sencible, and consequently the Point H will vanish, or be lost to sight; and, therefore, the whole indefinite Representation of IH, infinitely produced, lies between its Intersecting Point, I, and its Vanishing Point, V.

Plate VI. COR. 2. The Perspective Representation, of a finite Right Line, is part of a Line drawn from its Intersecting to its Vanishing Point.

Fig. 24. For the Visual Rays EM, EN, EH, &c, from the finite Parts of the Original Line, MN, NH, &c. to the Eye, determine the finite Representations of those Parts; and, since the whole infinite Line, from I its Intersection, is represented between its Intersection and its Vanishing Point, I and V; consequently, the Representation of every finite Part is a part of IV.

Im, represents the Part IM, mn represents MN, and, nh represents NH; the remaining Part, hV, represents all the Line beyond H, infinitely produced.

COR. 3. The Projective Representation of that Part of an Original Line, which lies between the Picture and the Directing Plane, falls on the other side of its Intersecting Point; and, its whole Representation is infinite.

I is the Intersecting and D the Directing Point of the Line HL; wherefore, ID lies between the Picture, ACB, and the Directing Plane, ED.

EX. The Point F will be projected to f, where the Visual Ray EF, produced, cuts the Picture; and the Point G to g. - - See Proj. Perf. P. 52.
If is, therefore, the Projective Representation of IF, and fg of FG.

If any Point, K, be taken, near D, its Representation will be projected at a great distance from the Intersection I; and, the nearer the Original Point is to D, the farther will it be projected, from I; for, Ek will always make an Angle with Ig equal to DEK; but, when the Point K coincides with D, it will be projected to an infinite Distance.

Therefore, the Part, ID is represented by Ig, infinitely produced; and consequently, the Directing Point D, can have no Representation.

SCHOL. It may be observed, that, as the Perspective Part, IV, of the Indefinite Representation, represents all that Part beyond its Intersection, I, to an infinite Distance, so the finite part of the Original Line, ID, between the Intersecting and Directing Point, is infinite in its Representation, from I through g.

COR. 4. All that part of the Original Line, which lies on the other side of the Directing Plane, infinitely produced from its Directing Point, is transprojected to the Picture, beyond its Vanishing Point; and the whole Rrepresentation is infinite.

EX. The Representation of the Point Q is at q, and of R at r; and, the nearer any Point, S, is taken to D, the farther will its Representation be from V; but, when S coincides with D, its Representation will be at an infinite Distance. See Transprojection, Page 52.

Also, the farther any Point, L, is taken beyond Q the nearer its Representation, C, will approach to V; but, unless the Point be at an infinite Distance, it cannot coincide with V.

SCHOL. Here, it may be observed, that an indefinite Original Line is represented by an indefinite Line, the Terms being inverted; and that, a Point, on the other Side of D, will be trans-projected to an infinite Distance, above the Vanishing Point, V. On this side, it will be projected to an infinite Distance, below the Vanishing Point; whereas, but one Point, D, lies between; and also, that the Representations of two Points, H and L, the extremes of a Right Line, at infinite Distances, both ways, from the Intersecting Point, I, coincide at V.

The

The trans-projected part of the indefinite Projection, CV, may represent a Line, in the same Plane above the Eye; from the Point C produced indefinitely, through O; which must, also, be infinite, if it be parallel to EV, before the Points O and V will coincide.

N. B. If the Original Line TV pass through the Eye, its whole Representation is in its Vanishing Point; for, its Intersecting and Vanishing Point is the same; and, the Point of Sight, E is its Directing Point.

COR. 5. The whole indefinite Representation (IV) of an Original Line, from its Intersecting Point (I) is not varied on the Picture, the Eye being in any Part of its Radial (EV) at any Distance, less than Infinite.

Suppose the Eye, at E, removed to *E*, in the Direction VE.

*E*V, being parallel to IN, the Original Line, the Vanishing Point, V, remains the same; and, I, being its Intersecting Point is invariable; consequently, the Indefinite Representation, IV, will be the same at any Distance of the Eye, from V; in VE, produced.

But, it must be observed, that, it is only so in respect of the whole of IV; for the finite Parts of the Representation are continually varied, while the Eye moves from V to *E*; m being the perspective Representation of the Original Point M, from the Eye at E; but, removed to *E*, it will appear at m^2; consequently, Im is the Representation of the finite Part IM, of the Original Line, from the first Point of View, and I*m* from the Point *E*. And so, of any other part of the whole Line.

Again, if the Eye move from E or *E* in the direction of EM or *E*M, the Representation m, or *m*, remains the same, and the indefinite Representation is varied.

For, at *E*, the indefinite Representation is IV; but, at E_2, it is In; yet I*m*, the Representation of the finite part IM, is the same, from both Points of View.

COR. 6. Hence, it is also evident, that, if the Eye be removed to any other Point in ED (the Director of the Original Line, IN) its whole Indefinite Representation, as well as the finite Parts are varied from every Station.

For, if the Eye be raised to E^3 the Radial is E^3C; consequently, IC is the whole indefinite Representation; but if it be depressed to E^2, then is E^2n the Radial, and In the whole indefinite Representation; and the finite Parts I*m*, *mn*, &c. are raised higher, or the Points, m, n, fall nearer to the Intersection, I, as the Eye is raised or depressed.

As this Theorem, and what is deducible from it, contains the whole essence of of practical, rectilinear Perspective, I would advise the young Student to make himself particularly well versed in it before he proceeds to Practice; for if this Theorem be clearly understood and retained, he will find the Practice easy to be acquired, and, at the same time, rationally accounted for.

THEO-

Plate VI.

THEOREM XIII.

The Distance between the Intersecting Point of an Original Line, and the Representation of any Point in that Line, is to the whole Indefinite Representation; as the Distance between the Original Point and the Intersecting Point, is to the Distance between the Original Point and the Directing Point, of that Line.

This Theorem will be best demonstrated in the Example.

Fig. 24. EX. MN is an Original Line produced to D; I is its Intersecting, and D its Directing Point; and V is the Vanishing Point of that Line, the Eye being at E; EV is, therefore, its Radial.

Now, if any Point, M or N, be taken in the Original Line, apart from its Intersecting Point; its Representation, on the Picture, will be, somewhere, between I, the Intersecting, and V, the Vanishing Point of the Original Line.

I say, that, the Distance of m or n, from I, the Intersecting Point, is, in proportion to the Indefinite Representation, IV; as MI or NI, the Distance of the Original Point from the Intersecting Point, is to MD or ND, the Distance of the Original Point from the Directing Point.

DEM. Having drawn the Visual Ray, EM or EN; the Representation, m or n, of the Point M or N, is where the Visual Ray cuts the Picture. Persp. Art. 2.

† Def. 22. ‡ Def. 33. Geom. || 15. 1. El.

Now, EV is parallel to NMD †, and, IV is parallel to ED. - 8. 7. El.

Consequently, IVED is a Parallelogram ‡; and the Triangles NED, NnI are similar; wherefore, In : ED : : IN : ND. - - - 4. 6. El.

But, IV is equal to ED ||; therefore, In : IV : : IN : ND. Q. E. D.

Or, it may be demonstrated thus.

Because EV is parallel to IN, the Triangles INn, nEV are similar.

Wherefore, In : nV : : IN : VE; and consequently, - -
In : In+nV (=IV) : : IN : IN+VE (=ND) i.e. In : IV : : IN : ND. } 4. 6. El.

N. B. This Proportion is invariable, whether the Eye be farther from, as at *E*, or nearer to the Picture; or whether the Eye be raised or depressed, as at *E* 3 or *E* 2; for, E 3 D being still parallel and equal to the Indefinite Representation, IC; consequently, I*n* : IC : IN : ND; and I*m* : IC : : IM : MD.

COR. 1. From this Theorem, the Indefinite Representation of a Line being given or drawn, and the Distance of any determinate Point, in the Original Line, from the Picture, known, the Representation of that Point, on the Picture, is also determinable.

For, whatever Plane the Original Line is in, is not material, the Distance of the Point, in question, from the Picture, answers the same purpose, as its Distance from the Intersecting Point of the Line it is in; or, from the Intersection of any Plane with the Picture in which that Line is situated. e. g.

Fig. 25. EX. NM is an Original Line cuting the Picture, in the Point I, in the Intersection, AB, of the Plane, NBC, that Line is in; and the Directing Plane, DEC, in its Directing Point D.

I say,

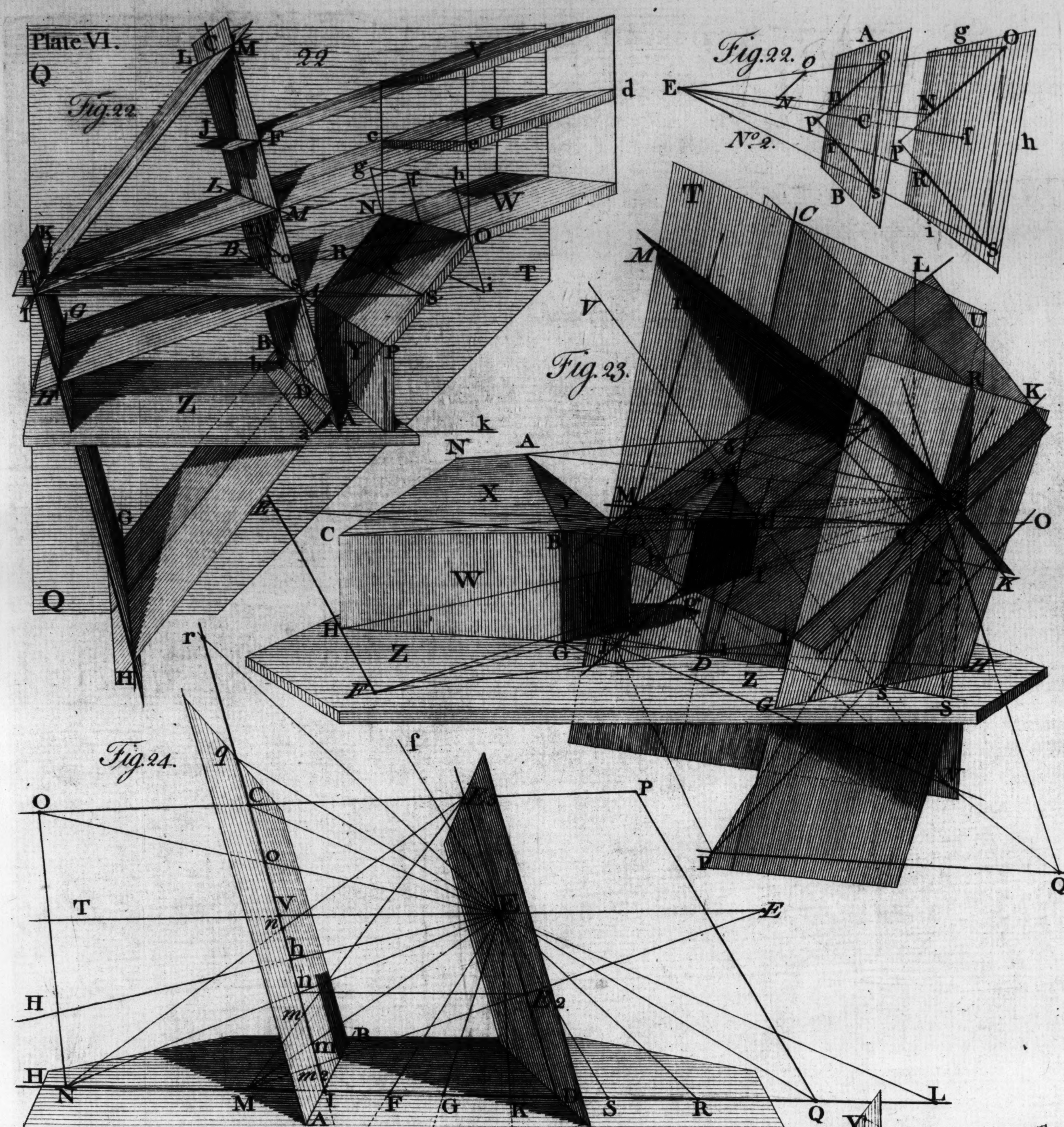


Plate VI.
Fig.22.
22
Fig.22.
No.2.
Fig.23.
Fig.24.
Fig.25.
Fig.26.

I ſay, that NF (the Diſtance of the Point N from the Picture, AVB) is to FG, the Diſtance of the Picture, as NI to ID (the Diſtance of the Point N from the Interſecting Point, to the Diſtance between the Interſecting and Directing Points, of the Line MN) or, as NA to AC, its Diſtance from the Interſection, to the Diſtance between the Interſection and Directing Line, of the Plane the Original Line is in.

DEM. For (having joined IF and DG) becauſe the Directing Plane is parallel to the Picture, IF is parallel to DG - - - - - - 8. 7. El. (for the Triangle GND is a Plane cutingthem both; Ax. 8.)

Wherefore, the Triangles GND, FNI are ſimilar; and, for the ſame reaſon, GNC, FNA, and alſo DNC, INA are ſimilar;
conſequently NF : NG : : NI : ND; or, as NA : NC. - - 4. 6. El.

Wherefore, the Diſtance of the Original Point from the Picture, and the Diſtance of the Picture being known; the Diſtance of its Repreſentation, from the Interſecting Point, is a fourth Proportional, viz. as the Diſtance of the Original Point from the Picture, added to the Diſtance of the Picture, i. e. as the Diſtance NG (of the Original Point, N, from the Directing Plane) is to NF (the Diſtance of that Point from the Picture) ſo is the Indefinite Repreſentation, IV (of the Original Line MN) to In; the Diſtance of n (the Repreſentation of the Point N) from I, the Interſecting Point of the Original Line. Q. E. D. It is, therefore, as NG : NF : : IV : In.

For, (by the Theorem) it is, inverſely, as ND : NI : : IV : In.

Or, from the Vaniſhing Point, it will be, as NG : FG : : IV : nV.

In Numbers, it is thus calculated. The Diſtance NF of the Original Point from the Picture, and the Diſtance of the Picture, FG, being added (equal NG). The Indefinite Repreſentation, IV, being multiplied by NF, and that Product divided by NG (the Diſtance of the Original Point from the Directing Plane) gives In; the Diſtance of n, the Repreſentation of the Original Point, from the Interſecting Point, of the Line it is in. For, NG : NF : : IV : In; as above.

Wherefore, the Rectangle, under NG and In, is equal to the Rectangle under NF and IV; and conſequently, NG In = NF × IV; - 9. 6. El.

It more frequently occurs, in Practice, that the Repreſentation of a certain portion of a Line is required, from ſome Point already found; the Interſecting Point of the Original Line not being within reach, nor attainable, but only the Vaniſhing Point. For, the Interſection of every Plane is neither wanted nor can be had, when the Vaniſhing Line of the Plane is abſolutely neceſſary; it is the ſame in reſpect of the Vaniſhing and Interſecting Points, of Original Lines; as it will be exemplified in the practical part of this Treatice.

COR. 2. If the whole Indefinite Repreſentation be not given, but only a part, from ſome determinate Point, in the Original Line; any other portion, of that Line, may be perſpectively proportioned, geometrically.

EX. The Repreſentation, m, of the Point M, in the Original Line, MN, being given or found, and the Indefinite Repreſentation, mV, from that Point drawn (V being the Vaniſhing Point) the Repreſentation, n, of any other Point, N, in the Original Line, will be;
as NM : MD : : mn : nV; or, as NM : ND : : mn : mV.

DEM. For, as NI : ND : : In : IV; and, as MI : MD : : Im : IV; by Theo.
But, - - ND : ID : : IV : nV; and - MD : ID : : IV : mV; 4.6.El.
for, the Triangles NED, VEn are ſimilar; DE is equal to IV, and ID to VE (becauſe NDEV is a Radial Plane, paſſing through the Original Line, NM, and the Eye (E) cuting the Picture and Directing Plane; in IV and ED; and, EV, producing the Vaniſhing Point, V, is parallel to ND; by Def. 22.)

Plate VI. Fig. 25.

Now, ND : ID : : IV : nV; alſo MD : ID : : IV : mV;
inverſely - - - - - ID : MD : : mV : IV.
Wherefore, by inordinate equality, ND : MD : : mV : nV.
Conſequently, ND—MD (=NM) - : MD : : mV—nV (=mn) : nV.
that is, - - - - - NM : MD : : mn : nV, as it was affirmed.
Wherefore, by addition, - - NM : ND : : mn : mV;
and conſequently, by inverſion, - ND : NM : : mV : mn.

Wherefore, the Diſtance of M (whoſe Repreſentation, m, is given) and the Diſtance of any other Point, N, in the Original Line, from the Picture, being known; the difference being N*m*, it will, conſequently, be, as NG : N*m* : : mV : mn; Q. E. D. For, NG : N*m* : : ND : NM.

Thus, may the Perſpective Proportion, of the known proportions of any Original Line, from any Point in it, be aſcertained, geometrically; having the Repreſentation of that Point, in the Indefinite Repreſentation given, and the Diſtance of that Point, with any other Point in the Original Line, from the Picture, known; ſeeing, that the Diſtances of the ſeveral Points, from the Picture, are in the ſame Ratio, as their Diſtances from the Interſecting Point; and the Diſtance of thoſe Points, from the Picture, may be had, when often their Diſtances from the Interſecting Point of that Line cannot; for ſeveral reaſons.

EX. The Diſtances of the Points M and N, from the Picture, being known, and the Diſtance from any other Point, in that Line, as O, required; it will be, as MN : MO or NO : : *m*N : *mo* or N*o*; O*o* being ſuppoſed parallel to M*m*.

So that, if *m*F, the Diſtance of the Point M from the Picture, be added, we have the Diſtance of the Point O from the Picture, equal *o*F.

And the Diſtance of the Picture (FG) being known, the Repreſentation, o, of the Point O, is determinable; for, it will be, as *om* : *m*G : : mo : oV;
and, by addition, - - - - - as *om* : *o*G : : mo : mV.

By which means, the Perſpective Repreſentations of ſeveral Diviſions in a Right Line may be had, geometrically, their Diſtance from each other being known.

Fig. 26.

EX. Let ABC be an Original Line, and CDEG a Radial Plane, paſſing through the Eye, E, and alſo through the Original Line; cuting the Picture in IV, their common Section.

EV, parallel to AC, is its Radial, IV is, therefore, the whole Indefinite Repreſentation of ABC; I is its Interſecting, D its Directing, and V its Vaniſhing Point.

Now, if the Repreſentation, a, of any Point, A, in the Original Line, be determined, all other Diviſions, B, C, &c. in that Line, are alſo determinable, without the Interſection I; the Diſtance of A and B, from the Picture, and the Diſtance of the Picture being known.

For, ab : aV : : AB : BD; ac : aV : : AC : CD; and alſo, bc : bV : : BC : CD.

DEM. For, (having drawn AF and BG, parallel to IV) becauſe aV is parallel to AF, ab : aV : : A*b* : AF; alſo, ac : aV : : A*c* : AF;
and, for the ſame reaſon, bc : bV : : *bc* : *b*F; and, as B*C* : BG. } Cor. to 6. 6. El.

But, the Triangles GE*C*, *C*CB are ſimilar, for EG is parallel to CB;
wherefore, - - - B*C* : *C*G : : CB : GE (equal BD) 4. 6. El.
Conſequently, by compounding, B*C* : BG : : BC : CD, equal CB+GE.
But, as B*C* : BG : : bc : bV; therefore, bc : bV : : BC : CD.

c is, therefore, the Repreſentation of the Original Point C, and b of B; which are determined from the given Repreſentation of A; the Diſtance of A and any other Original Point from the Picture, being known; no regard being had to the Diſtance of the Point A, from the Interſecting Point I, or from D, the

the Directing Point of the Original Line; seeing that, the Distance of the Original Points, from the Picture and Directing Plane, are in the same Ratio, as their Distances from the Intersecting and Directing Points of the Original Line.

In this Theorem is the Perfection of Practical Perspective. It is, at the same time, the most mathematical, and the Demonstrations the most perfect, elegant, and convictive, of the whole Theory. By it, we not only know that there is Analogy of Ratios, between the several Distances of Points, in an Indefinite Representation, from the Intersecting Point, and the whole Indefinite Representation; to the several Distances of the Original Points from the Intersecting and Directing Points, or from the Picture and Directing Plane; but, by its means, the Representations of the several portions of an Original Line, in the Indefinite Representation, are determined, with the greatest facility, accuracy, and expedition, geometrically; or they may be determined numerically, by a Scale of equal Parts; but that is seldom practiced, the method of doing it geometrically, being much readier and more accurate than it is possible to calculate, by Numbers.

THEOREM XIV.

The Perspective Projection, or Representation, of every Right Line, is parallel to its Director.

DEM. The Radial of an Original Line, producing its Vanishihg Point, is parallel to the Original Line; - - - - - - Def. 22.
wherefore, a Radial Plane may pass through the Original Line and its Radial; consequently the Eye is in that Radial Plane. - - - Ax. 5.

Now, since one part of a Right Line cannot be in a Plane, and another part of the Line out of that Plane, the Radial Plane will cut the intersecting Line (of the Plane the Original Line is in) in the intersecting Point, and the Directing Line, in the Directing Point, of the Original Line. - - Ax. 1.

But, the Radial Plane passes through the Eye; - - Def. 6.
and, consequently, through the Vanishing Point; seeing it passes through the Radial Line, producing the Vanishing Point.

Wherefore, the Section of this Radial Plane, with the Picture, is the Indefinite Representation of the Original Line; seeing, it passes through its Intersecting and Vanishing Points. - - - - - - Theorem 12.

But, it also passes through the Eye, and Directing Point, of the Original Line; consequently, it cuts the Directing Plane in the Director of the Original Line. - - - - - - - - - Def. 12.

But, the Directing Plane is parallel to the Picture. - - Def. 4.

Wherefore, the Sections of the Radial Plane, with the Picture and Directing Plane, are parallel to one another. - - - - - - 8. 7. El.

But, its Section with the Picture is the Indefinite Representation; and, its Section with the Directing Plane is the Director of the Original Line; as above.

Therefore, the Perspective Projection of an Original Line, being a part of its indefinite Representation, is parallel to its Director. Q. E. D.

EX. Imagine a Plane, N*D*EV, passing through the Eye and the Original Line, NO. This Radial Plane must necessarily cut the Picture and Directing Plane, in *I*V and E*D*; for EV is parallel to *DI*N, i. e. to NO; wherefore, EV*ID* is a part of the Radial Plane; which, being produced, would pass through NO, the Original Line. Fig. 20.

But

Plate IV. But, I is the Intersecting, and, V is the Vanishing Point of NO; Def. 21. and 22.
wherefore, IV is the Indefinite Representation of NO; - - Def. 25.
and, since the Radial Plane passes also through the Eye and Directing Point,
‡ Def. 12. E and D, its Section with the Directing Plane, ED, is the Director of N O‡.
Consequently, IV, or n o, a part of IV, is parallel to ED. - 8. 7. El.

Fig. 20 and 21. In Fig. 21. this is more perfectly illustrated, by means of Visual Rays, from the Extremes, N and O, of the Original Line, N O.

For, NO being produced to the Directing Plane, D is its Directing Point; and E, D, being joined, is the Director of NO, as before.
But END is a Triangle, consequently it is a Plane. - - - Ax. 8.
which cuts the Picture in In, or IV, the Indefinite Representation of NO; and, consequently, it is parallel to ED, its Director.

COR. 1. If two, or more Right Lines cut the Directing Plane in the same Point, they will have parallel Representations.
For they have the same Director, to which they are all parallel.

EX. PQ cuts the Picture in P, and the Directing Plane in D, the Directing Point of NO; and, because it is perpendicular to AB, the Intersection of the Picture, C is, therefore, its Vanishing Point. (EC being perpendicular to LM, is therefore parallel to PQ, being in parallel Planes.)
Consequently, PC is the indefinite Representation of PQ; which is parallel to IV, the indefinite Representation of NO; by Theorem, and, 4. 7. El.
And also, by Theo. 7; for, ED, the Director of both, NO and PQ, is the common Intersection of the Radial Planes, $DEVI$ and $DECP$; passing through both Lines. Therefore, their Intersections, IV and PC, are parallel.

N. B. This Corollary is true if the Lines, NO and PQ, are not in the same Plane.

COR. 2. All Lines, not having the same Directing Point, but, which cut the same Director, indefinitely produced (through the Eye) both ways, have parallel Representations.

For, if two Original Lines, NO, or PQ, and RS, cut the Directing Plane in the two Points D and d, in such wise, that a Right Line drawn through the Eye and the Directing Point of one, shall also pass through the Directing Point of the other, their Representations, n o and pq, are parallel.
Because they have the same Director, DEd.

EV is the Radial or Parallel of NO, V is therefore its Vanishing Point; and E v is parallel to RS; therefore v is its Vanishing Point; and, S v its Indefinite Representation.

COR. 3. All Lines cuting the Directing Plane in EF, the Intersection of the Vertical and Directing Planes, will have their Representations parallel to the Vertical Line, CD.

Because, they all have the same Director, EF, to which the Vertical Line, CD, is parallel; being producd by the Section of the Vertical Plane, ECDF, with the Picture and Directing Plane. - - - - 8, 7. El.
As PY, parallel to RS, cuting EF, the Prime Director, in f.
E v is its Radial and Pv its indefinite Representation.

COR. 4. All Lines which cut the Parallel of the Eye, of any Original Plane, have their Representations parallel to the Vanishing Line of that Plane.

Because, the Parallel of the Eye is the Director of all such Lines, and it is parallel to the Vanishing Line; by Theorem 3d.

TU

TU is an Original Line, cuting IK, the Parallel of the Eye, of the Original Plane NBH, in *k*; Ex is its Radial, x is therefore its Vanishing Point, and Ux its indefinite Representation; which is parallel to LM, the Vanishing Line of the Plane NBH.

For, IK is parallel to LM†, and it is the common Section of the Plane, IKLM and E*k*Ux; therefore, Ux is parallel to LM, by Theorem 7. † Theo. 3.

COR. 5. The Representation of any Original Line makes the same Angles with the Intersection and Vanishing Line, of the Plane it is in, as the Director, of that Line, makes with the Parallel of the Eye and the Directing Line of that Plane.

Because the Representation is parallel to its Director, by the Theorem; and because they cut parallel Lines in parallel Planes.

COR. 6. If the Representations of any two Lines are parallel, the Originals are either parallel between themselves and to the Picture, or they have the same Director.

For, if the Original Lines are parallel between themselves and to the Picture, their Representations will be parallel; but, if they are not parallel to the Picture, they must have the same Director; seeing, there can be but one Line drawn in the Directing Plane, parallel to both Representations.

N. B. Lines, which are parallel to the Picture, have no Directing Point, but the Director of every such Line, is a Line drawn through the Eye parallel to the Original Line.

For, the Representations are parallel to the Originals; by Theorem 10.

This last Theorem (and the Corollaries deducible from it) contains the whole Theory of the Directing Plane; as, in it and the thirteen preceding Theorems, is contained the whole knowledge of rectilinear Perspective; or, at least, all that I conceive to be really useful. It has been my aim, not, merely, to amuse or to shew my knowledge in it, but to give useful Instruction; and, I dare venture to affirm, that, if the whole of this Theory be clearly understood, the Student will seldom be at a loss in Practice.

It is a mistaken notion which many entertain of Perspective, that, the Theory is unnecessary to a Practitioner. It is certainly possible to practice Perspective, in all common Cases, without being able to account, or give a reason for any rule that is followed; for, the Rules, being deduced from the Theory, will, undoubtedly, if strictly followed, produce certain effects though we are not able to account for it: as there are many Persons very acute in Mensuration, Gauging, Surveying, &c. who know nothing of Geometry, the foundation of the whole. The case is very different in Perspective; for I am well convinced that it is of great use to understand the Theory well, in the first place; and, that the Practice will, by that means, be sooner acquired, and more securely retained. For want of Theory, the Pupil is frequently bewildered, and knows not what he is about; every different Example appearing difficult and strange, though, perhaps, founded on the same invariable Principles. In short, the nearest and most certain road, to Perspective, is to go through the Theory to Practice; and, I will venture to stake all my knowledge in it, that when acquired, the loss of Time (if it be any) will never be regretted.

I shall give three more Theorems, on Circles and spherical Bodies, and then proceed to Practice. If any Person require further knowledge, or a more extensive Theory, I refer him to the elaborate Work of Mr. Hamilton which, is deserving of the highest encomiums, if it was as useful as it is ingenious and learned in the Science; for, he has certainly said all that can be said of it, in Theory; and, I am persuaded, more than any other Person would ever have thought on, and much more than is of real use; for, I think I have omitted nothing that can be useful or necessary to be known, by any Practitioner, whatever.

SECTION V.

Plate VII. **Of the THEORY of CURVILINEAR PERSPECTIVE.**

IN this Section, I shall chiefly consider the Theory of Perspective relative to circular Objects; which are the most common, and most useful of all curve lined Figures. Other Curves cannot be comprised in any certain Theory, by which their perspective Representations can, with certainty, be ascertained; or if they could, it would answer no purpose to an Artist, seeing that, irregular curved Figures or Objects but seldom occur, in Practice. I would not be understood to mean the curves of the apparent Contours of human or other Figures (endowed with Life or not) which occur in almost every Picture, but which, can never be reduced to Rules, for Practice, from an established Theory; but, irregular curved Figures, in Planes or other Surfaces. Winding or serpentine Rivers, Rocks, Mountains, Trees, &c. which are composed of, or bounded by, irregular curved Lines or Surfaces, cannot be reduced to Practice, in delineating them, by any Theory in the Science of Perspective. Notwithstanding they may be delineated with great accuracy, by any Person, who is a little accustomed to sketch by sight only, by means of an Apparatus, which I shall describe in the Appendix to this Work.

To treat, at large, of the various Curves which the Representation of a Circle may take, such as the Parabola or Hyperbola, is foreign to my Design; as it so rarely assumes those forms. Nor is the knowledge thereof of any real use in delineating; seeing that, the small part of the Representations of such Circles as are or can be represented, when they do assume either, could not readily be distinguished from a portion of an Ellipsis; which Curve, as it is the most general and useful, so it is the easiest to describe, and the only one of real use, in Perspective.

In order to a clear understanding of the nature of an Ellipsis and its Properties, it is necessary to be acquainted with the Conic Sections; since every Representation of a Circle or Sphere, in Perspective, is either one or other of the Sections of a Cone. But, as a thorough investigation of it is not necessary, here, I shall refer the Reader, who desires to be perfectly acquainted with the Conic Sections, to Mr. Steel's, or to a later Work, by Mr. Emerson.

Nevertheless, I find it impossible to treat the Theory of the Circle, in Perspective, without having recourse to them, in some degree; therefore I shall, in the first place, define what is a Cone, and the difference between a Right and a Sealene Cone; for, without that knowledge, all that can be said of it would be to little or no purpose.

The methods of describing an Ellipsis, and all which appertains to it, are treated fully, yet briefly, in six Problems in an Appendix to the practical Part (Book 1st) of my Treatice on Geometry; together with a concise Theory, of its most essential Properties; to which I refer the Reader; I shall consider it, here, only as being the Perspective Representation of a Circle or Sphere.

DEFINITIONS.

Fig. 27. No. 1 & 2. A CONE is a geometrical Solid, whose Base is a Circle, which terminates in a Point, called its VERTEX; and, a Right Line, passing through the Vertex and the center of its Base, is called its AXE or AXIS *.

It may be considered as a Pyramid whose Base is a Polygon of an infinite number of Sides; every Section of which, by a Plane parallel to its Base, will, consequently, be a similar Figure; wherefore, the Section of a Cone, parallel to its Base, is a Circle.

* The Axis of any thing is either a real or imaginary Right Line passing through its middle, in a certain and determined Position.

If the Object be a Plane Figure, its Axe may be either perpendicular to, or in the Plane of the Figure. If the Axe be in the Plane of the Figure, it is divided, by the Axe, into two equal and similar Figures. Any Diameter of a Circle may be its Axe, an Ellipsis has but two Diameters which are Axes, viz. the Transverse and its Conjugate, at Right Angles with each other.

2. A

Plate VII.

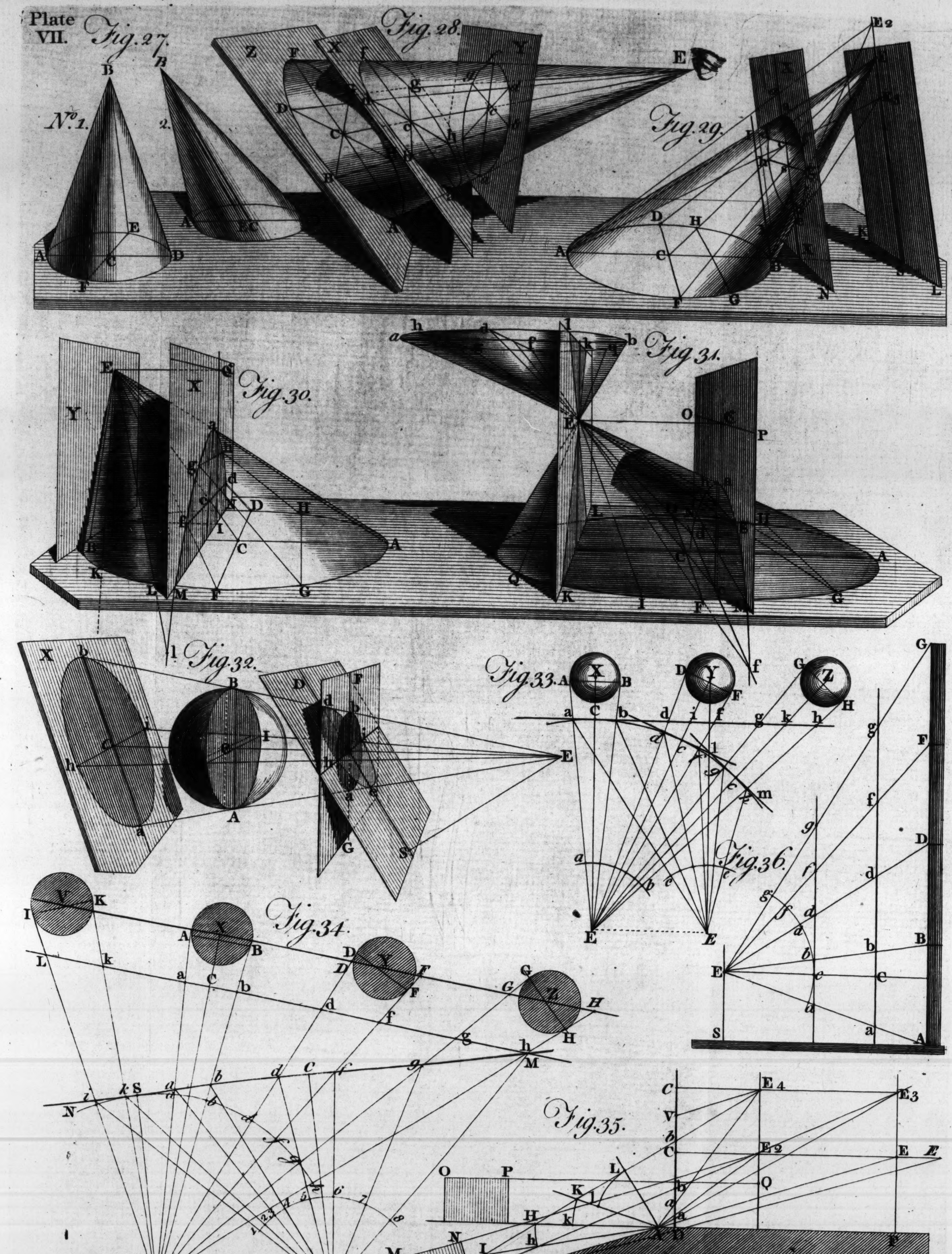


Fig. 27
No. 1.
2.
Fig. 28
Fig. 29
Fig. 30
Fig. 31
Fig. 32
Fig. 33
Fig. 34
Fig. 35
Fig. 36

2. A RIGHT CONE is that which is formed, or supposed to be generated, by the Revolution of a Right Angled Triangle, on one of its Legs.

ABC is a right angled Triangle; C is the Right Angle; BC is, therefore, perpendicular to AC, its Base. Fig. 27. No. 1.

If you suppose the Triangle ABC to be revolved quite around, on BC, as an Axis, its Base, AC, will describe the Circle AEDF, which is the Base of the Cone ABD; and the Hypothenuse, AB, will have described the Surface of the Cone; which, in a Right Cone, is every where equal, from the Vertex B to the Periphery of its Base (as, BA, BE, BD, &c.) while the Perpendicular, BC, which is the Axe of the Cone, remains at rest; one extreme in B, the Vertex, the other in C, the Center of its Base.

3. A SCALENE CONE has, also, a Circle for its Base, but its Axe is inclined to its Base, as *B*C to AD. No. 2.

As if the Vertex (B) of a Right Cone, was drag'd on one side, out of its perpendicular position; consequently, the real Axe, of such a Cone, does not pass through the center of its Base, yet *B*C is called its Axe.

A Right Line bisecting any Angle of a Triangle is called its Axe; wherefore, *B*E, bisecting the Angle A*B*D, is the Axe of the Triangle A*B*D, and consequently of the Cone; for it passes through the middle of such a Solid, whose rotundity is elliptical, seeing, its Dimensions, perpendicular to its Axe, are unequal; and, the more the Axe is inclined to the Base, the more its Dimensions differ (its Base remaining the same) and the real Axe is removed further from the Center, C, towards A.

If a Section of a Scalene Cone be made, by a Plane, perpendicular to its real Axe, *B*E, it is considered as an oblique Section of it; which Section is an Ellipsis, and *B*E would pass through its Center; consequently, the Solid would revolve regularly on *B*E, its real Axe. Whereas, on *B*C it would revolve very irregularly and unequal; but the Cone, A*B*D, would be equally poised on BC, in a horizontal Position; wherefore, every Section of a Cone, through the Axe *B*C, bisects the Cone; for the Triangles, A*B*C, C*B*D, are equal, in every Section, through BC; by Prop. 18. 1. El.

THEOREM I.

The Representation, or perspective Projection, of every Circle, in a Plane to which the Picture is parallel, is a Circle.

The scenographic Projection or perspective Representation of an Object, is the Section of the Optic Cone or Pyramid of Rays, by a Plane, passing between the Eye and the Object. (See Scenography, p. 49.)

DEM. Let ADGH be a Circle, in any Original Plane, Z; E is the Eye, and EA, EB, &c. Visual Rays, from every Point in the Circumference, to the Eye, forming a Scalene Cone, AEF. Fig. 28.

Now, if the Cone of Rays be cut by a Plane, X, parallel to the Plane Z, in which is the Original Circle; adgh, the Section of the Rays, by that Plane, is a Circle.

For, it is the section of a Cone, by a Plane, parallel to its Base; and, whether it be a Right or Scalene Cone, every such Section is a Circle; seeing that, the Cone, aEf, cut off by the Plane X, is similar to the larger Cone, AEF.

This

Plate VII. This is otherwise demonstrable, from Theo. 10, and Corollaries; in which it is demonstrated, that the Representation of every Plane Figure, parallel to the Picture, is similar to the Original.

Every Diameter in the Original Circle AE, DH, &c. being equal, the Representations af, dh, &c. have that proportion to their Originals, as the Distance of the Picture to the Distance of the Plane of the Original Circle; consequently they are also equal; therefore, the Representation, adgh, is a Circle. Q. E. D. For, all Circles are similar Figures.

COR. Hence it is manifest, that the Representations, in Perspective, of various Circles, in the same Plane, parallel to the Picture, have all the same Ratio amongst themselves, and to each other, as the Original Circles.

THEOREM II.

The Representation of a Circle, in a Plane not parallel to the Picture is an Ellipsis: except in one certain Point of View, in which, its Representation is, also, a Circle.

Fig. 28. Let ADG be an Original Circle in the Plane Z; E is the Eye, and EA, EB, &c. Visual Rays forming (as before) a Scalene Cone. C is the Center of the Circle.

DEM. If the Cone of Rays be cut by a Plane, Y, which is considered as the Picture, passing through both Sides of the Cone, not parallel to the Base, it is an oblique Section; and, consequently, every such Section, of a Cone, is an Ellipsis. - - - - - - Prop. 90. P. 73. Em. Con. Sec.

Wherefore, the Representation, adg, on the Plane Y, (which is an oblique Section of the Cone of Rays EA, EB, ED, &c.) is an Ellipsis. Q. E. D.

Every Circle, which is visible, appears an Ellipsis, except when the Axe of the Eye is perpendicular to the Plane of the Circle, and passes through its Center; for, in that Case, only, the Visual Rays, from the Eye to every Point in the Circumference, are equal, and consequently they generate a Right Cone, the Axe of the Eye being the Axe of the Cone. In every other Position, whatever, they must necessarily form a Scalene Cone, seeing that, the Axe of the Eye (which is always the Axe of the Cone) must be inclined to the Plane of the Circle, if it be not perpendicular; and consequently, the Visual Rays, forming the surface of the Cone, have various Inclinations to the Plane of the Circle, and therefore they are unequal.

Wherefore, since the section of a Scalene Cone, perpendicular to its central Axe, is an Ellipsis, and the more the Axe is inclined to the Base, the more excentric is the Section; seeing that, if the Eye be nearly in the Plane of the Circle, the Section approaches nearly to a Right Line; and, the Section, made by a Plane in that Position, is the only true Appearance of the Circle; or rather, by a spherical Surface, perpendicular to the Axe, which truly measures the Optic Angle (the Radius being equal to the Distance) under which, the solid Dimensions of the Cone, every way, are seen.

N. B. If the height of the Eye, or its Distance from the Directing Line, be taken a mean Proportional, between the distance of the nearest convex part of the Circumference and the distance of the farthest concave part, from the Directing Line, the Representation will then be a Circle.

Fig. 29. Let AB be a Diameter of the Circle, ADG, perpendicular to the Directing Line, KL; BS is the distance of the Circle from it.

Make SE : SA : : SB : SE; i. e. let SB : SE : : SE : SA. by Pr. 30. Geo.

At

At E, as the Point of View, the Representation of the Circle ADG, on any Plane, X, parallel to the Plane KEL, will be a Circle, and in no other Point whatever, on that Plane, and at the distance BS.

DEM. Draw BI parallel to SE; and, suppose AEB a Section, by a vertical Plane, through CE, the Axe of the Cone, and the Diameter AB, of its Base; and passing through S.

Then, because BI is parallel to SE, the Triangles AIB, AES are similar;

Wherefore, - - - - - - AB : BI :: AS : SE.

But, by Construction, - - - AS : SE :: SE : SB.

Wherefore, - - - - - - - - SE : SB :: AB : BI;

and therefore, the Triangles AIB, BES are similar †; for the Angle ABI is equal to ESB ‖; EBS is equal AIB, and IAB equal BES; seeing that, the Sides which subtend those Angles are proportional. † 5. 6. El. ‖ 4. 1.

But, the Angle BES is equal EBI ‡, for they are alternate; wherefore, EBI is equal IAB (of the Cone AEB) and consequently, the lesser Cone aEb, cut off by the Plane X (parallel to BI, and the Plane KEL) is similar to the Cone BEA; for, the Angle E, at the Vertex, is common to both; the Angle Eba (equal EBI) is equal EAB, consequently, Eab is equal EBA §, and therefore, the Cone AEB is cut sub-contrary, by the Plane X. ‡ 4. 1. El. § C. 5. 10. 1.

But, if a Scalene Cone be cut sub-contrary, the Section is a Circle.

Therefore the Representation, ahbg, on the Plane X, of the Original Circle ADBF, is a Circle. Q. E. D. Prop. 89. P. 73. Em. Con. Sec.

N. B. The Center, s, of the Representation, is not the Representation of the Center, C, of the Original Circle; for it is at c, where the Visual Ray EC cuts ab, the Representation of the Diameter AB. Wherefore, CE is not the real Axe; seeing that, it cannot be the Axe of the Cone AEB, and also, of the lesser Cone aEb, which is cut sub-contrary, and is therefore similar to AEB; consequently, the true Axe is common to both Cones.

So likewise, df, the Representation of the Diameter, DF, in the Original Circle, is not a Diameter of the Representation; but, gh, the Representation of the Chord GH, (which bisects AB) is its Diameter perpendicular to AB.

Wherefore, the Representation of the Segment GBH is the lower Semicircle gbh; and, gah, the upper Semicircle, represents the large Segment GAH.

It is evident, that, if the Eye be raised, as at E^2, the Diameter ab, in the Representation, will be lengthened; seeing, that the Visual Rays, E^2A, E^2B, cut the Picture more oblique; whilst the other Diameter, gh, remains of the same length §; and consequently, the Representation of the Circle, from that Point of View, will be an Ellipsis, and ab its Transverse Diameter, or Axe. § Cor. 3. Theo. 10.

But, if the Eye be lowered, to E^3, the Diameter ab (now ce) will be shorter than the Diameter gh (which, being parallel to the Intersection of the Picture, will have the same length, seen from any Point in the Directing Plane, as above) and it will then be the Transverse Axe; and ce (the Conjugate to it) is the shortest Diameter of the Ellipsis.

If the Eye be removed on either Side of the Point E, the Representation will be an Ellipsis; for, the Section will not, then, be sub-contrary (the Picture remaining as before). The Representation, ab, of the Diameter AB, will still be a Diameter of the Ellipsis, because it will pass through its Center; but it will be neither the Transverse nor the Conjugate Axe, for they are always at Right Angles with each other.

Hence, the Point of View, from which the Representation of a given Circle, on any Picture, however situated, will be a Circle, is easily determined.

The Original Plane, KAL, in which the Circle is situated, being produced if necessary, cuts the Picture in MN, its Intersection.

Aa Having

Plate VII. Having fixed on the Distance, or Station Point, S (at pleasure) in the Diameter AB, produced, which is perpendicular to the Intersection of the Picture; let KL be drawn, through S, parallel to MN; KL is the Directing Line. - Def. 10.

Draw SE perpendicular to KL, and parallel to the Picture.

Make SE a Mean Proportional, between SA and SB; - - - Pr. 30. Geo.

E is the place of the Eye, or the Point required; from which, a Circle, agbh, (on the Plane X) whose Center is c, and Diameter ab, will truly represent the Original Circle ADBF, on the Plane AKL.

N. B. The distance of the Picture, in this Case, is not material; for, the Point E being fixed (as above) every Section of the Cone of Rays EA, EF, EB, &c. parallel to the Plane X, or to the Directing Plane KEL, being cut subcontrary, will, consequently, be a Circle.

From what has been advanced, it is evident, that the Representation of every Circle, in Perspective, is some one or other of the Conic Sections; which, being of itself, a distinct Science, would not be proper to enter on here; but, from that consideration and the preceding Theorems, the two following Corollaries may be deduced.

COR. 1. If the Circumference of the Original Circle touch the Directing Line in a Point only, and is not cut by it, the Representation of that Circle will be a Parabola.

Fig. 30. For, whether the Original Circle, ADBF, touch the Directing Line, in S, the Station Point, or in any other Point, B, it is the same; since that Point, and also the Eye, E, which is the Vertex of the Optic Cone of Rays, are both in the Directing Plane, Y; it is evident, that the Directing Plane touches the Cone in the Right Line EB, from its Vertex, E, to its Base, at B.

‡ Def. 4. Wherefore, since the Picture, X, is always parallel to the Directing Plane ‡, the Cone, AEB, is cut by a Plane parallel to its Side, EB; the Curve produced by every such Section is a Parabola. Prop. 76. P. 223. Em. Con. Sec.

That part of the Original Circle, MGAN, which lies beyond the Picture, is represented perspectively, and falls above the Intersection MN, as MaN; the remainder of the Circle, lying between the Picture and the Directing Line, is projected below the Intersection; as l is the representation of the Original Point L. Every other Point, as K, is projected further from the Intersection as it lies nearer to the Directing Line, BS; and, except the Point B, in which it touches the Directing Line, may be supposed to have a Representation, on the Picture, though at an immense distance; but, the Point B can have none; because, the Line EB, which should produce it, is parallel to the Picture, seeing, it lies in the Directing Plane, and therefore can never cut the Picture. The Representation of which Point, only, is wanting to compleat the Figure and form an Ellipsis; but, for want of that Point, in the Representation, it is kept open, and falls off in Right Lines, nearly, at a Distance, to all sense, infinite.

Hence it is plain, that, on Account of the Distance that part of the Curve falls below the Intersection, a small portion of it, only, can be represented, in a Picture; and the part which can, together with the Part which lies beyond the Picture, differs so little, in its Representation, from an elliptic Curve, that, in delineating, it would be needless to deviate from it; and I question that ever a distinction was made, in Practice. Nor, can it ever be of use, but in delineating the inside of a large Rotunda, Circus, or circular Area; when the Spectator may be supposed to stand on the hither part of the Circumference, or in a Line which is a Tangent to it, and parallel to the Picture; in which Case, only the farther, concave part of the Curve can, properly, be represented.

COR.

COR. 2. If the Directing Line (KL) cuts the Original Circle in two Parts; that part (KFAHL) which is on the same side with the Picture, will, in its Representation, form an Hyperbola, below the Vanishing Line (OP) of the Plane of the Circle; the other part (KBL) of the Circle (if it be represented on the same Picture) will be transprojected, and form an opposite Hyperbola, above the Vanishing Line, which is equal and similar to the other. Fig. 31.

For, if the Directing Line passes through the Center of the Original Circle, and consequently, divides the Circumference equally in two; the Eye being always supposed the Vertex of the Cone, is cut by the Directing Plane; in which Case, the Section, made by the Picture, is parallel to the Axe of the Cone, and the opposite Hyperbolas are equal, and equally distant from the Vanishing Line OP.

But, when the Circle is cut unequally, by the Directing Line, KL, the Cone, BEAF, is also unequally cut by the Directing Plane, KEL; the Section of which, with the Cone, is the Triangle KEL; the Picture, NOPM, being parallel to the Directing Plane, the larger Segment of the Circle, in MN, and cuts also the Cone, in the Curve f g a N, which is an Hyperbola. Prop. 104. P. 160. Em. C. Sec.

If the Sides AE, FE, BE, &c. of the Cone BEA, are produced through the Vertex, E, forming an opposite Cone, a E b d; the Picture being produced, will also cut that Cone produced; its Section with it is the opposite Hyperbola, or transprojected Representation of that part of the Circumference of the Circle, KBL, which lies on the other Side of the Directing Line, by means of the Rays BEb, QEq, &c. produced to the Picture; which, notwithstanding it is the Representation of the lesser Segment of the Circle, it is generated by a similar Cone, a E b; and although the Section is made at a greater distance from their common Vertex, E, it is equal and similar to the perspective and projective Representation (f M g a N d) of the larger Segment.

This Curve, like the Parabola, can never be generated, or of use, but when the Area of the Circle is so large, that the Spectator is supposed to stand within it; and that Segment, MGAHN, which lies beyond the Picture, only, is required; and which differs so little from an elliptical Curve, in its Representation, M g a h N, that the distinction is not very obvious, and seldom, if ever, regarded.

These are all the variety of Curves which a Circle in Perspective can assume, however situated in respect of the Picture, or of the Eye; the chief of which is the Ellipsis. Every Circle, which the Eye is capable of taking, in at one View, has the Appearance of an Ellipsis; except, when the Axe of the Eye is perpendicular to the Plane of the Circle, and passes through its Center; in that position only it can appear, to the Eye, a true Circle; although all Representations of Circles, in Planes parallel to the Picture, are Circles †; but, being seen oblique, when the Eye is in the true Point of View, they have the Appearance of Ellipses. † Theo. I.

THEOREM III.

The Representation, in Perspective, of a Globe, or Sphere, is an Ellipsis*; except when the Center of the Sphere coincides with the Center of the Picture; in which Case, only, it is a Circle.

The Cone of Rays from the Eye, as its Vertex, to the apparent Circumference of a Sphere, is always a Right Cone. For, a Sphere can have but one Position, either to a Plane, Line or Point; and consequently, the Diameter, every way, always presents itself to the Eye, and always forms a Right Cone.

* I have found it very difficult, nay almost impossible, to convince some Persons of this plain and well known truth; because, as they truly observe, the Diameters of a Sphere always appear equal; which is as much as to say, that the Visual Rays, under which a Globe is supposed to be seen, always form a Right Cone, i. e. whose Axe is perpendicular to its Base; which is not so with Circles, but in one Position only. But, such Persons seem to forget, that the Representation is on a Plane, and that Plane is considered as the Plane of the Section; which must, consequently, cut every Cone oblique, but that which has its Axe perpendicular to the Picture.

Now,

Plate VII. Now, it is evident, that, when a Sphere is so situated, in respect of the Picture and the Eye, that, the Direct Radial coincides with the Axe of the Cone, i. e. when it passes through the Center of the Sphere; the Section of the Cone, made by the Picture, is parallel to every apparent Diameter of the Sphere, which is the Base of the Cone, and being equal every way, it is consequently a Circle.

But, the Direct Radial is the Axe of the Cone, which passes through the Center of its Base, and also through the Center of the Picture.

Consequently, the Center of the Picture coincides with the Center of the Sphere, in this Section; and therefore, the Representation is a Circle. See Theorem 1st.

For it is the Section of a Cone, parallel to its Base.

2. If a Sphere be so situated that the Direct Radial does not pass through its Center, the Axe of the Cone must be inclined to the Picture; which, being the Plane of the Section, the Cone of Rays are cut obliquely by the Picture.

But the oblique Section of a Cone, through both its Sides, is an Ellipsis. Pr. 90. Em.

Therefore, the Representation of a Sphere, whose Center is not the Center of the Picture, is an Ellipsis. Q. E. D. For it is an oblique Section of a Cone.

Fig. 32. EX. Let AB be a Sphere and C its Center, supposed to be seen. Let E be supposed the Eye of a Spectator, and EA, EB, Visual Rays, from the Eye, E, to the apparent Diameter, AB*. AEB may, therefore, be supposed a Section of a Right Cone, through its Axe, EC, which is an Isosceles Triangle; the Sides, EA, EB, &c. of a Right Cone being equal‡.

‡ Def. 2.

Now, if FG be the Picture (which is a Plane) parallel to AHBI, the Base of the Cone, the Line ab, in which the Plane AEB cuts that Plane, is a Diameter of that Section; which Section, being every way equal, is, consequently, a Circle.

But, EC, the Axe of the Cone, being perpendicular to its Base, is perpendicular to the Picture§, and to its Section, FG, with the Triangle AEB; the Point, c, where the Picture is cut by the Axe of the Cone, EC, is the Representation of C, the Center of the Sphere.

§ 6. 7. El.

And, because AB is a Diameter of the Base of the Cone, and C its Center, AB is bisected in C; consequently, ab is also bisected in c; for, the Triangles CEA, CEB are congruous‖; and, because ab is parallel to AB, the Triangles aEc, AEC, and cEb, CEB are all similar‡.

‖ 7. 1. El.

‡ 2. 6. El.

But, E is supposed the Eye of a Spectator, and, Ec, being perpendicular to FG, the Picture, is the Direct Radial¶; and the Point c, where it cuts the Picture, is the Center of the Picture§, which is also the Center of the Representation of the Sphere, on that Picture; therefore it is a Circle. Th. 1.

¶ Def. 15.

§ Def. 17.

Now, if the Cone AEB be supposed to be cut by any other Plane, passing through the Diameter hi, and consequently through c, the Center of the Picture; that Section will be an Ellipsis.

Let SD be a Section of another Plane or Picture, with the Triangle AEB, cuting the Rays EA, EB, in e and d; then is ed, in that Section, the Representation of the apparent Diameter of the Sphere, and c is the representation of its Center, as before, in the Section ab; but, ed is not bisected in c.

DEM. Let ace be an Isosceles Triangle; ac=ec; and, ac=cb; th. ec=cb Ax. 3. El.

But, because bcE, in the Triangle bEc, is a Right Angle, cbE is acute†, and, the Angle cbd, in the Triangle dcb, is, consequently, obtuse. C. 2. 1. 1.

† C. 3. 10. 1. El.

Wherefore, cd, subtending the obtuse Angle, is greater than cb; 12. 1. El.

And consequently, ec + cd, equal ed, is greater than ac + cb, equal ab.

But, the Section HEI, which is vertical to AEB, cuts both Pictures in the same Line, hi, equal to ab; as above.

* See the Note to Art. 4. Page 13, on Direct Vision.

There-

Therefore, the Section, e h d i, is an Ellipsis; for, the Diameter e d is larger than any other, in that Section.

But, ES, perpendicular to SD, is the Direct Radial; wherefore, S is the Center of that Picture ‖; and it does not coincide with the Center of the Sphere (the Eye being at E) consequently, the Representation of a Sphere cannot be a Circle, except when the Center of the Picture coincides with the Center of the Sphere, in the Center of its Representation. ‖ Def. 17.

EX. 2. To illustrate this further. Let X, Y, and Z be three Globes, whose Centers are all in the same Right Line, parallel to a h, the Intersection of the Picture; whose Center is C, and E C its Distance. Fig. 33.

Draw the Tangents EA, EB, ED, &c. to the three Globes, the Chords, AB, DF and GH, of those Tangents, are the apparent Diameters of each; which, it may be observed, is still turned towards the Eye; and are considered as the Diameters of the Bases of the three Cones AEB, DEF, and GEH, which are all Right Cones.

Now, since they are all cut by the same Plane (of which a h is a Section) each Cone, except the first, AEB (whose Axe, EX, coincides with the Direct Radial, EC) is cut oblique; and, consequently, one Diameter, of the Section, is larger, as the Globe is farther from the perpendicular EC; as ab, df, and gh, the Representations of a Diameter of each Globe, X, Y, and Z; d f being larger than a b, and g h still larger than d f.

But, the other Diameters, perpendicular to these, are equal, in all §, for they are supposed parallel to the Picture and equally distant from it; consequently, the Representation of the Globe X, only, which is in the middle of the Picture, or, the Center, C, of its Representation, in the Center of the Picture is a Circle; because, the Diameters perpendicular to each other are equal: all others, as Y and Z are Ellipses, because they are oblique Sections of Cones, whose Diameters are proved to be unequal. § Cor. 3. Th. 10.

N. B. It is the same however the Globes are situated; whether above, below, or sideways of the Center of the Picture. If they are equally removed from the Perpendicular EC, and equally distant from the Picture, the Globes being equal, their Representations are equal and perfectly similar, though differently situated; and the farther they are remote, from the Perpendicular, the more excentric is the Ellipsis.

It is also obvious and demonstrable, that ab, the representation of the Diameter AB, is bisected in C, the Center of the Picture, and the representation of the Center of the Globe; for, the Triangle a E b is Isosceles. But, d f, and gh are not bisected, in i and k, the representations of the Centers of the Globes Y and Z.

For, because the Right Lines EY and EZ, from the Eye to the Centers of the Globes, are perpendicular to the Chords, DF and GH, the Angles DEF, GEH are bisected by those Lines; because the Chords are bisected. C. 9. 1. El.

But, in the Triangles d E f and gEh, because the Angles, at E, are bisected, by the Right Lines Ei, Ek; df and gh are cut, by those Lines, in the Ratio of the other sides of the Triangles; - - - - - - 3. 6. El.
and consequently, d i : i f : : E d : E f; and g k : k h : : E g : E h.

But E g is less than E h †; because, the Angle g h E is acute, and E g h is obtuse; wherefore, gk is less than kh; and also, di than i f. † 12. 1. El.

Therefore, d f and gh are not bisected, by the Lines EY and E Z.

Plate VII.

SECTION VI.

Containing a full refutation of ſeveral Errors and abſurd Opinions, which many Artiſts entertain of Perſpective; and, therefore, look on it as an imperfect and fallacious Science.

I Shall, in this Section, in the firſt place, explain the reaſon why the Repreſentations of the Diameters of Columns, on a Picture which is parallel, or nearly ſo, to the Columns, are continually larger the farther they are removed from the Center of the Picture, and conſequently from the Eye.

As this is a particular circumſtance, which many Perſons ſeem inclined to diſpute, or, if it be admitted, they look on it as an imperfection in Perſpective, I ſhall endeavour, and doubt not, to make it appear conſonant to reaſon and Perſpective, to their entire ſatisfaction. It has ſo near Affinity to what has been ſaid, in reſpect of a Sphere, that the ſame Diagram might have done for both; but, in order to avoid miſtakes, and to keep the Ideas diſtinct and ſeparate, I have given another.

Fig. 34. Let V, X, Y, and Z be the Sections of four Columns, by a horizontal Plane, in which is the Eye, at E. Let L M be a Section of the Picture, parallel to the Columns, and EI, EK, EA, EB, &c. Viſual Rays, from the Eye to the apparent Diameters of the Columns, which are ſtill turned towards the Eye, as Globes.

It is evident, that the Viſual Rays cut the Picture more oblique, the farther the Columns are from the Perpendicular EC; and, notwithſtanding the Optic Angles DEF, GEH are leſs, as the Columns are continued, their apparent Diameters, df, and gh, intercepted between the Viſual Rays, continually increaſe; and would, if the Columns were continued, till the Interval between them was loſt; the Repreſentations of their Diameters ſtill increaſing till they touch and cut each other. For, the Space from Center to Center, of the Columns, are equal, and are, conſequently, repreſented ſo on a parallel Picture, if continued infinitely; what, then, can become of the Space between the Columns, if it be not added to their Diameters, or Repreſentations on the Picture?

I preſume, no Perſon will ſay that there is any imperfection in Perſpective, in this Caſe; I do affirm there is none in Perſpective, the buſineſs, of which, is to repreſent Objects, truly, on a Plane; according to their Magnitudes, Diſtances, and Situations in reſpect of each other, of the Eye and of the Picture; where, then, is the imperfection in this?

If a Perſon, not knowing how to chooſe a proper Diſtance, take, into the Picture, more than the Eye is capable of taking in at one View; or if, through ignorance, the Picture be abſurdly ſituated, in reſpect of the Object, is the fault in Peſpective, or in his Judgment? In Perſpective there is not, nor can be, on the Principles here laid down, any, the leaſt error, if the Elements of Euclid are to be depended on, upon which the whole Fabric is erected; if one falls, the other falls with it.

The Art of Perſpective is to repreſent Objects on a Plane, exactly as they appear, according to their Situations, &c. (See Perſpective, Page 50.)

It is well known (or ought to be) that no Perſpective Repreſentation can appear perfect, i. e. it cannot truly repreſent the Original Object, though ever ſo accurately delineated, but when the Eye is in the true Point of View.

Suppoſe, then, E to be that Point; and AEB, DEF, &c. the Optic Angles under which the Columns X, Y, Z (being equal) are ſeen.

It is, alſo, I preſume, allowed, that Objects appear to have the ſame proportion to each other, reſpectively, as the Angles under which they are ſeen. Th. 1. Sec. 3. D. Viſion.

But, the Angle DEF is leſs than the Angle A EB; becauſe their Subtenſes are equal, AB=DF (ſuppoſing the full Diameter of a Column to be ſeen) and the Viſual Rays EB, ED, EF, &c. the Sides of thoſe Angles, ſtill longer, the farther they are from the Perpendicular, EC†. How comes it then, that the Diameter of that

† C. 14. 1. El.

that Column (on the Picture) is the largest, which is seen under the least Angle? The reason is obvious; because, the Picture, LM, cuts those Rays most oblique, where the Angle is the least; in the Points g and h, &c.

DEM. If EF and EH are produced till they cut a Right Line drawn through the Centers of the Columns (which is parallel to the Picture) in *F* and *H*; ED and EG cut that Line in *D* and *G*; all which are beyond the Circumferences of the Circles; wherefore, *DF* and *GH* are, each larger than a Diameter; and, if it was not sufficiently obvious, it would be easy to prove that *GH* is larger than *DF*. But, AB is less than a Diameter; consequently, a b is less than d f, and d f than g h; for a h is parallel to A*H*. - - - 4. 6. El.

Now, if the Eye, at E, be turned towards the Column V; or, if the Situation of the Picture be changed to NM, perpendicular to ES; then is S, where ES cuts NM, the Center of that Picture; on which, it is evident, that, i k, the representative Diameter of the Column V, being nearest the Center, is the least; which, on the Picture LM is equal to that of Y; for, they are equally distant from the Center of that Picture; and *g* h, on the Picture MN, the representation of the Diameter of the Column Z, is considerably larger than g h on the other. Yet, to the Eye, at E, both these Pictures truly represent the Diameters of the four Columns V, X, Y, and Z; V and Y appear equal on both, they being equally distant from the Eye; X, the nearest, will appear the largest, and Z, the farthest, from the Eye, will appear the least.

DEM. With any radius, as ES, on E as a Center, describe an Ark of a Circle, cuting all the Visual Rays EI, EK, &c. from the four Columns.

The parts, *ab*, *df*, &c. intercepted between the Rays, are the true proportions of the apparent Diameters of the Columns, and consequently, of their Representations on both Pictures.

But the Ark *ab* is the greatest, *ik* is equal to *df*, and *gh* is the least.

Wherefore, the Angle a Eb is greater than d E f (equal i E k) and g E h is the least; and consequently, the apparent Magnitudes of the Columns, X, Y, and Z, are in the same Ratio. - - - Theo. 1. Direct Vision.

It is almost unnecessary to enforce this, by dwelling longer on it; as it is certain, if either of these Pictures be viewed from any other Station, they could not represent the four Columns V, X, Y, and Z in the Position and Situation they are in.

Suppose the Eye removed to E^2 (the Point of View ought always to be opposite to the middle) and, from that Station, to view the Picture NM.

Draw the Visual Rays E^2a, E^2b, &c. and on E^2 describe an Ark of a Circle, cuting them, in 1, 2, 3, &c. The slightest glance of the disproportion of their Appearance from that Station is sufficient conviction; for *ab* (which appears the largest from the true Point of View) whose apparent Magnitude is the Ark 34, does not appear half so large as *df*; and *gh* appears larger than *df* (which ought to appear the least) as the Arks 34, 56, and 78 sufficiently evinces.

That the true, apparent magnitudes of the Columns V, X, Y, and Z from the Point of View, E, is the portions *a b*, *df*, &c. of the Ark *i d h*, is manifest, when we consider, that the Picture, on which the Columns are represented, is a Plane; but, that their true Appearances can only be represented on a spherical Surface; i. e. on the Surface of a Sphere, the Representation and the Appearance are the same, the Eye being in its Center.

Does any one imagine that there is a real Arch in the Heavens, which has that Appearance? in which, the Stars, &c. appear; equally distant from the Eye in its Center. The Celestial Globe, for instance, is a Picture of the Heavens, Planets, Stars, &c. each Representation of a Star, on its Surface, if they are truly depictured, would, to an Eye in the Center of the Sphere, exactly coincide, and be in the same Right Line with its Original in the Heavens; and, their apparent Distances, from each other, are measured by an Ark of the Sphere; whereas, their real Distances from the Eye and from each other, respectively, have not the least affinity

Plate VII. nity to their Diſtances, as repreſented on the Surface of the Globe or from its Center. Wherefore, to an Eye at E, the true Point of View for either Picture (L M or M N) each Diameter, being ſeen under its true Angle, and the ſame as its Original, will appear leſs and leſs, the farther they are diſtant from the Eye, or from the Center of the Picture; although their Repreſentations are continually larger and larger, on thoſe Pictures, as the Original Columns recede.

And this will ever be the Caſe, on a Picture parallel to the Columns, in ſome degree, at any diſtance of the Picture; but, at a proper Diſtance, for taking in the whole, the difference is ſo little, and that ſtill leſs as the Diſtance is increaſed, that, it is and ought to be diſpenſed with, by making them equal: but, I muſt obſerve, it is not, then, true Perſpective.

2. Methinks I hear ſome carping Critic ſay, I muſt allow, then, that Perſpective is ſomewhat defective; by no means; I have not yet given up the Point in debate. I ſay, that, although, at any Diſtance, there muſt and will be a difference, though ſcarce perceptible, at a proper Diſtance, yet I would never adviſe a Perſon, who would repreſent a row of Columns, in full Front, to make the leaſt difference in their Diameters; for, ſince they ſupport an Entablature, which is repreſented perfectly horizontal, their Pedeſtals or Baſes the ſame, conſequently parallel, it would be very improper to make the Columns differ in width when they are equal in height, for this reaſon; becauſe it is impoſſible to confine the Eye to the true Point of View, always; from which if you deviate, ever ſo little, the whole Repreſentation is diſtorted and imperfect.

But, if it was poſſible to confine the Eye, I would not ſtep the leaſt aſide from Perſpective, on any account; let the Repreſentation be ever ſo diſtorted and prepoſterous, it will, and muſt, if truly repreſented (by the Rules hereafter preſcribed) appear to the Eye, in the Point of View, as the Original. If we may be allowed to take liberties, in any Caſe, where ſhall we draw the line between the perfect and imperfect Repreſentation? for the whole is more or leſs diſtorted; conſequently, the Rules of Perſpective are not to be depended on at all. To what, then, muſt we have recourſe? the Eye is not, in many Caſes, a competent judge; we ſhould, if we follow its dictates, implicitly, have as many Points of View, in a Picture, as Objects; becauſe, if every Object was to be repreſented exactly as it appears to the Eye, on the ſame Plane or Picture, there could be no point of View for the whole; and conſequently, in a long, connected, and continued Object, compoſed of Planes and Right Lines (as in Buildings of any kind, or regular piece of Architecture) it would be beyond the power of Art to connect the ſeveral detached pieces or projectures ſo, together, as to compoſe one entire and uniform Picture of the whole.

3. Notwithſtanding an ingenious Author has treated it wantonly, or ludicrouſly, in a ſuppoſed Dialogue between a Lady and an Artiſt, who was determined to abide by the Rules of Perſpective, his Argument, has not the leaſt weight, and muſt be imputed to him, as not having a right notion of Perſpective. The Eye is conſidered as a Point; therefore, whether we ſuppoſe it confined to a Pin-hole or not 'tis the ſame; for he muſt allow (or he was unfit to write on Perſpective) that there can be but one Point of View for a Picture, in which it can be perfectly ſeen, as intended by the Artiſt; conſequently, when the Eye is not in that Point, the Objects muſt neceſſarily appear more or leſs diſtorted, according as they are ſituated nearer or farther from the Center of the Picture, or that Point which is oppoſite to the Eye.

4. I ſhall, however, for the ſake of the Argument, allow each Object to be truly repreſented, as they appear to the Eye, on a Plane Picture of a tolerable length, as a h:
Fig. 33. Globes are the fitteſt Subject to expatiate on, becauſe they are every way the ſame.

Let C be the Center of the Picture, the Eye being at E; and therefore, the Picture may be ſuppoſed to be extended equally towards a as to h.

Now it is certain, that the three Globes appear round, let the Eye be ſituated where it may; but they cannot, or ought not to be ſo repreſented, on the Plane a h, to be viewed by the Eye at E; for if they were, they would not appear round, but gibbous, or Egg like,

like, standing erect, all but the Globe X, in the Center; and the more so, as they are farther removed from it; but the Eye must be removed opposite to each, and consequently, there would be as many Points of View as there are Objects, which is an absurd Hypothesis, in one Picture.

Suppose, from the Point of View E, I would represent the three Globes, X, Y, and Z, as they appear; that is, the outline of each to be a Circle; the Eye, and its Axe, EC, must be turned towards Y and Z, as EY, EZ; and consequently, the Picture is turned with it, into the Position bl, and lm. For, the Picture must be perpendicular to the Axe of the Eye, if the Object be represented as it appears; in which Case, there are three distinct Pictures, viz. ab, bl, and lm; each having a distinct Center, and the same Distance, EC or Ec; on which Pictures, each Globe is represented by a Circle, and the farthest from the Eye (Z) is the least in its Representation.

If the Objects, X, Y, and Z, were represented, truly as they appear, on one Plane (of which ah is supposed a Section, and C its Center) they would be under a worse predicament than those respresented strictly perspectively: such a Picture could not appear true in any Point of View, whatever, every Object having a distinct and separate one.

For, suppose the three Globes, X, Y, and Z, to be represented, on the Plane, ah, as they appear to the Eye, at E, i. e. round; the Representation of the Globe Y less than that of X, and, of Z, less than Y, as they are represented on hl and lm; would they appear in the proportion they are represented, at E, or in any other Point? No, certainly; for, at E, the Representations would appear much less than the Original Objects, and not round (except X, only) Y would appear more round than Z, and Z would also appear rounder than any other, more remote from C; but they would, all, except X, appear eliptical Spheroids, and not Globes (which is obvious, to any Person tolerably acquainted with Optics or Direct Vision) the farthest, from X, still more so than the last.

Now, let the Eye be removed to *E*, or opposite to Z; Y, or Z, would then appear round, but not of the same proportion, as from the Point of View E; but, at *E*, neither X nor Z would appear round; and, although equally distant from the Eye, X would appear much the largest; where, then, in this Case, must the Eye be placed to see the three Globes X, Y, and Z, such as they really appear? There cannot be a Point determined, for each Representation has a separate Point of View. Can, then, the Picture ah, in this Case, be a true perspective Representation of the three Globes X, Y, and Z, as they appear to the Eye? certainly no, but each Representation, on the Plane ah, is as much a distinct and separate Picture, as the three Pictures, ab, bl, and lm; the difference is only, that the three Pictures have but one Point of View, and the Picture ah has three, equally distant from it; supposing the three Pictures placed in a Right Line, ah.

5. If what I have here advanced be not sufficient to divest those Artists of their absurd notions of Perspective, I shall give them one observation more, which they have not, perhaps, considered with that attention it requires; and then leave them to pursue their own way, if it appears to them more eligible and reasonable, and will produce a better and more agreeable Representation, of any Object whatever.

They quarrel and find fault with Perspective, but without reason; because, it is an infallible and most perfect Science. They would have all Objects represented, in Perspective, exactly as they appear to the Eye; there is no such thing to be done; 'tis not in the power either of Art or Science to represent, on a Plane, any single Object, except a Sphere, or a regular Plane Figure, having the Eye opposite to its Center, and the Picture parallel to it, as it appears; and yet, Perspective will give a true and just Representation of every regular Object.

The cause of all their errors and false notions of Perspective is their not rightly distinguishing between the Representation of an Object, on a Plane, and the true Appearance of it; two distinct things, which can never be united, on a Plane Surface or Picture.

The Repreſentation of any Object, on a Plane, is the Section of the Cone or Pyramid of Rays, by a Plane; and the Appearance of an Object is the Section of the Rays by the Surface of a Sphere, only; to which, every Viſual Ray, from the Eye to the Object, muſt be Perpendicular; conſequently, the portions of the Arks, intercepted between the Viſual Rays, meaſure the Optic Angles, under which, every part of the Repreſentation is ſeen (the Ark a, or *a*, *dh*, Fig. 33 and 34, may be ſuppoſed a Section of a Sphere, cuting the Viſual Rays EA, EB, &c.) conſequently, the true Repreſentation, and, at the ſame time, the true Appearance, can only be repreſented on a ſpherical Surface, the Eye being in its Center; but that is not a perſpective Repreſentation.

6. I expect it will, again, by ſome Perſons, be alledged, here, that Perſpective, then, is not ſufficient, to repreſent Objects as they appear to the Eye. I affirm that it is. Let us, therefore, once more enquire, candidly, what is meant by Perſpective, and what effects it is expected to produce.

Perſpective is a repreſentation, on a Plane, of an Object, or Objects, in a fixed and determined Poſition and Point of View. (See Perſpective, Page 50.)

I have already ſhewn the bad effect of viewing a perſpective Picture, out of the true Point of View; from which, if we do deviate, we cannot expect that the ſeveral parts of a Picture can vary their Bearings and Proportions, to each other, as the real Objects; no, certainly, that muſt be a reality, not a Perſpective; which is but a Deception, a Repreſentation of a real Object, on a Plane; and, which can never repreſent the Object truly, from any other Point of View, but that for which it was delineated.

I have alſo ſhewn, to occular conviction, by the Apparatus, that there may be as many various Repreſentations of the ſame Object, from the ſame Station, or Point of View, as there can be poſitions of the Picture; all which, are true Perſpective, and will affect the Eye alike in the true Point of View; which is the Vertex of the Pyramid of Rays; ſeeing that, every correſponding Line, on each Picture, is ſeen under the ſame Plane Angle as the Original, and every Surface, as well as the whole Object, is ſeen under the ſame ſolid Angle, or Pyramid of Rays, as the correſponding Original Surfaces, or as the Original Object.

What, then, is it we are caviling about? would ye have a real, ſolid Object on a Plane, or a Repreſentation of it only, in a certain Poſition? Certainly then, if the Eye be not in the true Point of View, the Picture does not truly exhibit an appearance of the intended Object, at the fixed Station; and although it may be a juſt, perſpective Repreſentation, it may, neverthleſs, be a very diſtorted and diſagreeable Picture; not owing to any fault, or imperfection in Perſpective, but, to the choice of the Situation of the Picture, or the Diſtance and Poſition of the Object and the Picture.

Does not almoſt every Object, except a Sphere, appear different from every different Point of View? and can any Perſon be ſo unreaſonable as to expect, that a Repreſentation of a ſolid Object, on a Plane Surface, can appear truly to repreſent the Original in any other Point of View, but in the Vertex of the Pyramid of the Viſual Rays, under which the Object itſelf is ſuppoſed to be ſeen? the very ſuppoſition of it is abſurd to the laſt degree; becauſe, no two parts of the Repreſentation can, at the ſame time, be ſeen under the true Optic Angle, in any other Point; conſequently, the Repreſentation, muſt appear erroneous.

7. Now, although the true Repreſentation, and alſo the true Appearance are depictured on a ſpherical Surface, yet, I affirm that ſuch a Picture is ſubject to much greater imperfection than a true Perſpective, on a Plane Surface; becauſe, if the Eye be the leaſt removed out of the Center, the whole Appearance and Effect is deſtroyed, and exhibits a much worſe Image of the Object, than a Perſpective Repreſentation, on a Plane Picture, can poſſibly exhibit, in any Point of View; which is ſo very obvious, that it is needleſs to point out the reaſon. For, ſuppoſe the Eye, at *E*, viewing the ſpherical Picture, a*dh*; the Viſual Rays, *E* a, *Ed*, &c. ſhew, at firſt ſight

ſight, how much worſe ſuch a Picture muſt appear, than that on the Plane, a h, from the ſame, though falſe Point of View; not one of them will appear round, and the Appearance, of all, is prepoſterous. Fig. 33.

8. From the circumſtance I have mentioned, in reſpect of the true Repreſentation and Appearance being depictured, at once, on a ſpherical Surface, ſome Artiſts imagine, that the Repreſentation on a Plane ought to be ſo delineated; it cannot be; 'tis impoſſible, in the nature of things. Suppoſe a true Repreſentation of a long Building, in full front, delineated on a ſpherical Surface, and it were poſſible, afterwards, to reduce the ſpherical Surface to a Plane; is any Perſon ſo weak as to ſuppoſe, that ſuch a Repreſentation would appear like the Original, in any Point of View? he muſt be weak, indeed, and have ſtrange miſtaken notions of Perſpective, who can; and yet I have heard this Point ſtrenuouſly ſupported, or rather argued for, (ſupported it could not be) for any thinking Perſon (who can think with propriety about it) muſt be ſenſible, that, what ſhould repreſent Right Lines will be curved, and, the whole, will give the Idea of a Rotunda, or externally round Building; ſeeing that, the extremes would fall off, not in Right Lines but curved, and they would appear leſs than the real Object; to ſay nothing of the almoſt impoſſibility of producing or delineating ſuch a Picture, at all, or by any means; I ſhould be glad to be informed, how, or by what Rules.

9. I have juſt bethought me of one Circumſtance, which, I think, muſt convince an Atheiſt in Perſpective. I am perſuaded, no Perſon will deny, that, if the Eye could be fixed in a Point, at a proper Diſtance from a tranſparent Plane, placed between the Eye and an Object, whilſt the Hand traced, accurately, every Line of the Object, as it appeared on the tranſparent Plane; ſuch a delineation, all muſt allow, would be a true one. Let thoſe, who are not otherwiſe to be convinced, try the experiment. I will ſtake all my knowledge in Perſpective, that every Repreſentation of a Right Line is a Right Line, on the Plane; that Columns or Cylinders of equal magnitude, and parallel to the Plane, will be larger as they are more remote from that Point, on the Plane, to which the Eye is oppoſite; that the Repreſentation of a Circle or Sphere, ſeen oblique, is an Ellipſis; that Objects of equal magnitude, and equally diſtant from the Picture parallel to them, however otherwiſe ſituated or elevated will be repreſented equal; with various other circumſtances; all which may be fully proved to occular conviction, which will not admit of the leaſt doubt. Surely then, if Perſpective performs the very ſame thing, which it certainly will, in every reſpect, it muſt exhibit a true Repreſentation of Objects.

10. It is the buſineſs of Perſpective to produce the Figure of a Section of the Cone or Pyramid of Rays, from the Eye to the Object, by a Plane, in any determined Poſition; which, if the Rules it preſcribes be truly followed, it will moſt certainly effect, without any ſenſible Error. For, wherever any Point, or Angle of an Object, appears on a Plane, or other Surface, between the Eye and the Object, there the Viſual Ray would cut and paſs through the Plane, to the Eye; but when the diſtance of the Eye is ſuch, that the Viſual Rays, from the Eye to the Object, cut the Plane very oblique, or in Angles, nearly, or perhaps leſs than, half Right ones, the Repreſentation will conſequently be diſtorted and prepoſterous, and in other Points of View, will have a diſagreeable and unnatural Appearance.

11. Here, then, lies the miſtake, which, through ignorance or inadvertency, is attributed to Perſpective, and ſuppoſed to be a deficiency or imperfection in it. 'Tis, generally, in the Point of View, the Situation, &c. of the Picture or Object; which, by being too near the Eye, occaſions that Diſtortion and prepoſterous Repreſentation we perceive in ſeveral Pictures; for, if the Optic Angle, under which the whole Picture is ſeen, exceeds 50, or, at the moſt, 60 Degrees, the Diſtance is not ſufficient; as, the Viſual Rays will cut the Picture very oblique, near its Extremes, and occaſion a diſagreeable diſtortion of the Objects on the extreme parts of it. Yet, as I have obſerved, at any Diſtance, the repreſentations of Columns, or other cylindrical Objects, on a Picture parallel to them, will, in true Perſpective, ever be the leaſt which are neareſt to the Center.

2. I ſhall

Plate VII. 12. I shall now take notice of another great difficulty, which seems to be a stumbling Block to many Artists; who, one would imagine, would not hesitate one Moment, to determine about it with propriety; which is, to represent, on a vertical Picture, the appearance of a direct Descent; which, some have affirmed impossible, in the Nature of things, to be done; that it is a strong instance of the insufficiency of Perspective, and that, we must have recourse to experience, only, in such Cases; intimating, that it is not possible, by the Rules of Perspective, to give the Representation of an inclining Plane; which is so ridiculous an Assertion, that, any Person, who understands Geometry tolerably, will easily be convinced of the contrary.

For, first, we are to consider whether the Descent (which I shall suppose a Plane) is perceivable or not. If this descending Plane can be seen, at all, from any fixed Station, it may, undoubtedly, be represented on the Picture, from that Station, by the strict Rules of Perspective, or there is no truth in Perspective; either it is a perfect and infallible Rule, or it is no Rule at all. If the Plane can be seen, it is a subject of Perspective; if it cannot be seen, it is no subject for a Picture; which needs no Demonstration. To tell us, what descends, and we actually know to go down-hill in Nature, will, if ever so correctly drawn, appear to rise upwards on the Picture, is saying nothing to the purpose, the expression is vague and nugatory; for, if the Plane descended so much as not to appear to rise on the Picture, it could have no place or representation thereon; but if it can be seen at all, it must necessarily and unavoidably appear to rise; or rather, it must, really, rise on the Picture, for, the Appearance is to descend.

To draw two parallel and horizontal Lines across the Picture, and to give an Idea, that the space, between them, represents a descending Plane (of a certain length) without shape, bounds or limits, sideways, or any Object situated on the inclination is indeed impossible; but that is giving too great latitude to the meaning of the expression. And yet I question if a skilful and ingenious Painter, in aireal Perspective, might not, simply, by the effect of Colour, even in this Case, deceive the Eye, and give the appearance of a Descent. But, if there are Objects situated on the inclined Plane, or, if the shape or figure of the Plane, itself, is to be described; whatever can be seen of such Objects (whether Tops or Bottoms it matters not) they may, and can be represented, truly and exactly, as they appear, by the infallible Rules of Perspective; and that, on the same invariable Principles, as the most common and ordinary Cases, whatever, are subject to.

13. To illustrate what I have advanced, by a simple geometrical Scheme, will not be very difficult; which may be considered as a vertical Section through the whole. Each Plane is, therefore, represented by a Right Line, making the true Angle of the Inclination of the Planes (as I shall call them) with each other.

Fig. 35. Let AF represent the Plane of the Horizon, and AB a Descent, as a slope Bank, or any other Declivity; making the Angle ABG with the Horizon. Let CD be a Section of the Picture, E the place of the Eye, and EF its Altitude above the Horizon, at the Distance EC from the Picture, whose Center is at C.

Now, if the Eye, at E, be so situated, that a continuation of the inclined Plane would pass through the Eye, as BAE, it is evident that the Plane, AB, from that Station, is totally lost to sight; its whole Representation being the Line of its Intersection with the Picture†. Consequently, if the Eye be moved further back, as at E, its appearance, or place on the Picture, would descend; which cannot be, unless the Plane be considered simply, a Plane, without substance, and seen on the under side; its apparent width from that Station is a D; but if the Eye be removed, nearer to the Picture, in a horizontal Direction (or in any direction above BAE) the Representation of the Plane rises on the Picture. At E^2, its width on the Picture is a b, as the Visual Rays, E^2 A, E^2 B, evinces.

† See Cor. 2. Theo. 3.

If

If the Eye be moved back again, in the direction BE², to E3, it is plain that, b, the apparent height of B, remains the same, wherever the Eye is situated in that Line; but the representative width, a b, of AB, will be increased, towards D, as it recedes, if the Picture be at any distance from the descending Plane. Suppose the Eye brought forward again to E4; the representation of the Slope AB from that Station is *ab*; and, if the Inclined Plane was continued indefinitely, aV is its whole indefinite Representation; for, E4V, parallel to AB, determines the distance of its Vanishing Line, from its Intersection, equal aV.

14. I presume, this is intelligible and clear, hitherto. I will, next, shew that 'tis impossible for the Eye to judge, merely from the width of its Representation on the Picture, whether the Plane be descending or ascending, or perfectly on a level.

From the situation of the Eye at E4, the representative width, on the Picture, of the Slope, AB, is *ab*; whereas, if the Plane was more inclined to the Horizon, i. e. if the Descent was more gradual, as AI, it would represent a less space; if it was horizontal, the same *ab* represents the length AH, only; but, if the Plane ascended from the Picture, as AK, it still represents a shorter length (AK) on that Plane; the Arks I i, H h, &c. shew how much each is shorter than the other; and AL, making right Angles, BLA, AL E4, with the Visual Ray E4B is the shortest length, from the Point A, that can be represented, by *ab*, from that Station, situation, and position of the Picture.

15. Now, if any Object, as X, be situated on the Inclination AB; it is evident, if the continuation of the Plane of its Top, MN, pass above or through the Eye, at E2, it cannot be seen from that Point; whereas, if the same, or an equal and similar Object was situated on the Horizontal Plane AH, the top, OP, may be seen from that Station, notwithstanding the Object is more elevated; because, a continuation of the Plane of the Top, OPQ, falls below the Eye, at E².

The figure of the inclined Plane, itself, or the figure of any Object situated upon it, is described, perspectively, in the same manner, and on the same Principles as on a horizontal Plane; which is exemplified in the practical part of this Work.

16. In the last place, I shall demonstrate, that the representations of Objects, which are elevated perpendicularly, above the Horizon, have the same proportion*, on a vertical Picture, as those of the same Magnitude, situated on or near the Horizon; the Object being parallel to the Picture.

It is a mistaken notion which several Persons entertain, that the parts of a Building, which are elevated high above the Horizon, appear to diminish in a greater Ratio than those which are extended horizontally; such an opinion may be easily refuted. If the second Part of the 10th Theorem be well considered, it is sufficient refutation.

Let AG be supposed a high Obelisk, and AB, BD, &c. several equal Divisions thereon. Let ES be a Spectator, E the Eye, and a g, or *ag*, a Section of the Picture; which, being vertical, is parallel to the Object AG. Fig. 36.

Now, if the Visual Rays EA, EB, &c. be drawn, they will cut the Picture in a, b, d, &c. then is fg, the representation of FG, equal to ab, the representation of AB, or to b d, the representation of BD†.

For, since the parts AB, BD, &c. of the Original Object, AG, are equal, the representations of them, on the Picture, being parallel to the Object, are also equal. † 4. 6. El.

But, AB is an Object, direct before the Eye, at E, on the Horizon AS; and, FG is one of equal length, elevated greatly above it; therefore, the Representations of equal Objects at an equal Distance from the Picture, and parallel to the Picture, are equal. Q. E. D.

The same Object may be considered as a Building, extended horizontally, and AEG a horizontal Plane, in which is the Eye, and also the Visual Rays EA, EB, &c. ag, or *ag*, is a Section of the Picture, as before. The Intersections of the Visual Rays with the Picture, it is evident, are the same.

* What I would signify by the proportion of Objects, in this place, is, simply, length and breadth.

17. There is yet another Point of controversy I have some times been entertained with; which is, that, when we stand, opposite the middle, near a long Building, or range of Buildings in a Right Line, the horizontal Lines, in the Cornice, &c. seem to decline towards either end, yet make no Angle; and therefore, they imagine that the Representations of those Lines, in such Case, will be curved. What a poor Idea must a Person have of Perspective, who advances and is really prepossessed of such an Opinion.

I shall say very little on this Point, because, the first Theorem is full and perfect Demonstration, that the Representation (on the Picture) of every Right Line, is a Right Line; and, it is so if extended infinitely. Because, a Plane may be supposed to pass through that Line and the Eye; and, the Intersection of this imaginary Plane, with the Picture, is the indefinite Representation of every Line it passes through; for the whole Plane is lost to sight, the Eye being in a continuation of it †. Consequently, since the Picture is a Plane, the Line in which it cuts the Picture is a Right Line ‡; and it is the whole Representation of the Plane, and of every Line in the Plane. Therefore, the Representation of every Right Line is a Right Line.

† Art. 3. of a Plane.
‡ 1. 7. El.

18. The truth, of which, any Person may soon be convinced of, by applying a perfectly streight Ruler, before his Eye, parallel, or otherwise, to a Right Line, in an Object, of any length, and imagine the edge of the Ruler to cut the Picture, which is a Plane; then certainly, if the Ruler coincides with the Original Line, from one end to the other (which it undoubtedly will) the Right Line, in which it cuts the Picture, is the Representation of the Original Line.

Or, if a transparent Plane be placed between the Eye and the Object, and any Right Line, in the Object, traced on it, exactly, whilst the Eye remains fixed in a Point; the Line, so described, will be a Right Line.

Those Persons never consider (but 'tis plain they are not furnished with the means) that the Picture, being placed in the true Point of View (consequently at its proper Distance) will appear the same as the Original; for the Ratio of the Parts is always in proportion to the Distance. Consequently, every part of the Picture, being seen under the same Angle as the Original, will have the same Appearance, in every respect; and consequently, the Right Lines, on it, will appear to decline either way, or both ways, the same as in the Object.

I have now, I hope, fully refuted those truly ridiculous and absurd Opinions, of Perspective, which many have imbibed, and are not easily divested of; they, rather, obstinately persist in them, without being able to give a solid reason for their Opinions, and are determined not to give up their prejudices, at any rate, right or wrong. What strange infatuation must possess that Person, who, having no argument of weight, to support his false notions of Things, has recourse only to sophistry; and, because he cannot come at sterling truth, himself, imagines there is no such thing to be found. In Points of natural Philosophy, &c. where no certain criterion can be obtained, to fix our assent, it is no wonder that we meet with so many, widely different, Opinions; and, though not one argument advanced has the least foundation in Reason, 'tis amazing with what eagerness and warmth each assailant attacks his opponent. But, in Sciences, purely mathematical, all must agree; when truth appears, there is no resistance can be made; we cannot with-hold our assent, so prevalent is her influence.

To such as are open to conviction, and are desirous of coming at truth, I think I have said enough for their conviction, on the Points debated; but, if they are not, all that can be said is to no purpose, 'tis waste of time and words. I shall, therefore, leave them to enjoy their Opinions, and please themselves with their great Sagacity; and proceed to lay down certain and infallible Rules for the Practice of Perspective, deduced from a perfect and well founded Theory; which, if truly followed, and proper attention be given to the Lessons, contained in the Introduction, will most certainly produce Harmony and true Effect, of any regular Object, so far as comes within the province of linear Perspective.

BOOK III.

Of the Practice of PERSPECTIVE.

SECTION I.

An INTRODUCTORY PREFACE.

I Come, now, to the practical and, in that, the useful part of Perspective; to which, the foregoing Book is as a Key or Introduction, only, but a very necessary Introduction; insomuch, that, without the knowledge inculcated by it, we should proceed in ignorance and uncertainty. Nevertheless, those Persons who have not studied Geometry, and have not, now, perhaps, either leisure or inclination to study it (though, in my Treatise of Geometry, they will find it neither so abstruse, tedious, or dry a Study, as many look on it to be, having treated that most useful branch of Science in a more familiar and intelligible manner, than has been done heretofore) I say, that, without a sufficient fund of Geometry, to go through the Demonstrations, if they will but treasure up, in memory, the Theorems and Corollaries, and take all for granted (as they may depend on the truths contained in them) they will find the great advantage of it, in Practice.

Every branch of Science is in two parts, viz. theoretic and practical. Theory teaches the knowledge of all that is necessary for Practice, in Speculation; the other applies that Knowledge to real use. It is necessary, first, to know how, before we can perform any thing; but notwithstanding, many Persons may be said to know Perspective (as a mathematical Science) yet know not how to apply it to Practice, with success. So, every Art, dependant on Science, may be acquired without the Theory of it, by custom or habit; as in mechanic Trades, derived from Geometry and Mechanics; all which, require time and application, to become familiar to us. So likewise, Perspective may be practiced, without being said, properly, that we understand it; seeing, we may not be able to give a sufficient or satisfactory reason for the effects of it, in any instance. And, being well versed in the Theory, we shall nevertheless find, that it will require time and assiduity to apply it, in all Cases that may occur, in Practice, although founded on the same invariable Principles, without familiar Lessons, in every Case, being given. But we shall, certainly, with the assistance of Theory, be able to comprehend, and to understand the various methods of applying the Rules, better than without it; as very little reflection, when we are at a loss, will set us right again, and enforce the universality of its Principles.

It is not, to be wondered at, that the best of Treatises on Perspective has been the least understood; viz. that, by Dr. Brook Taylor; because it is very obscure. I call it the best, on account of his general Principles, not from its real utility to Artists, in respect of practicing from its Rules. It can scarce be said that Perspective existed before; it certainly was not understood, as a Science. How insipid and imperfect are all the Books on the Subject before his, except Gravesande's, which seems to touch on the same Principles, but is far short of being a perfect System.

There is, in that small Treatise, Rules sufficient, in almost all Cases, for Plane Objects; but, various Examples requiring various ways of applying them, he has not made it so useful as it would have been, had he, instead of refering to former Problems, shewn how to apply it there. As, in Example 3d, P. 24, Book the first, Fig. 21. We are told to make CB represent a Line equal to that which is repre-

represented by CA (by Prop. 15.) Now, I question that one in fifty (who understands the whole) ever saw the Affinity between the two Examples; the different situations of the Lines makes it appear a very different Operation; and so it really is, though built on the same foundation.

Respecting Plane Objects, only, Perspective is soon acquired; knowing how to delineate Figures in horizontal Planes, it is the same in any other; having found their Vanishing Lines, with their Centers and Distances; which I shall first shew, in Practice (as in Theory) how to find, in all useful Cases whatever. But, without the embellishments of Mouldings and other architectural Ornaments, in Buildings, &c. all the rest would be to little purpose. That Treatise comprehends only Perspective in Plano; but I shall, in this Book, shew how to apply it in every necessary Case that can be devised; and doubt not, to render it, by that means, the most useful work, of the kind, yet published in any Language, that we know of.

Brook Taylor has, indeed, to his immortal Fame (in Perspective) furnished us with new and extensive Principles; but his Work, at best, is imperfect, and greatly deficient. His Theory is too concise, and is not regularly digested. It may perhaps, by some, be objected, that I have made too much of the Theory. I must freely own, that the length of it has exceeded my own design and expectation; and yet, I have greatly abridged my first settled Plan. I could say much more on it, but know not where to curtail it, except in the Examples, given for Illustration rather than Demonstration; and which, I am persuaded, will not be found unnecessary, to some, or trifling to any. One Example, to some Persons, would be sufficient; to others there can scarce be too many, so they are various, and not a repetition of the former. I shall be guilty of the same fault (if it be a fault) in Practice; I had rather say too much than not say enough, yet I would not be tedious; because, all that can be said, to some, will be too little, or rather too much, seeing it will be all to no purpose. To stear the middle course is a difficulty not easily obviated; but, it is my fixed design to aim at it; others must determine how I have succeeded.

It is the opinion of many Artists, that the whole of useful Perspective may be comprised in a little Compass; that nothing is a greater discouragement to the study of it, than to see a voluminous Work on the Subject. 'Tis certain, that the Principles, on which the Theory of it is founded, are contained in a very small Compass; and, I would recommend Brook Taylor's Epitome for that very reason, which contains *multum in parvo*. Yet, notwithstanding that valuable Treatise has been so long published, it is, at this time, but little known, and less understood; which is a sufficient reason, with me, to suppose that he has not said enough on the Subject. The elaborate Work of Mr. Hamilton is spun out to an immoderate length; yet to as little use as the other. 'Tis my Design to comprise the whole of useful Perspective in this Book. Neither of these Authors, I am persuaded, had either taught or practised the Art of delineating; and consequently, they were not qualified for treating it in an easy and familiar manner; one great requisite in a Work of this kind. I have had experience in both, and am well convinced, that, to make it useful, it cannot be comprised in so little compass as many imagine; that it requires frequent repetitions of the same Lessons, somewhat diversified, to familiarize the Rules, in various Cases; without which, not one in twenty will ever be a Proficient. Dr. Taylor truly says, it is much better for the Student to devise Examples, himself, in particular Cases, than to go through those of others; but how few are capable of doing so? nay, I find, many are not able to comprehend them at any rate, nor by any means; and therefore, to make such a Work really useful, variety of Cases and Examples must be devised, for practice and experience.

In Practice, our Author has given some Problems, containing the most elegant and general Rules that can be, notwithstanding they are but seldom practiced; because he has not shewn, properly, how to apply them: that shall be my care to do, where they can be applied usefully. His Diagrams are, in general, very imperfect, and badly devised; 'tis evident he was no practitioner or delineator, himself, even

in

in Plane Objects (for he has given us no other). But, he has departed from his own Principles, in Example V. Fig. 19. Part II. having projected the Dodecahedron, in Perspective, by means of the Ichnography and Orthography, as by the old Authors, instead of Vanishing Lines and Vanishing Points; which is much more masterly, elegant, and perfect; and is what the difference chiefly consists in. Such Subjects are indeed of little use, except to familiarize us to find Vanishing Lines in all Positions of Planes to the Picture, and Situations of the Object, or Picture.

Some Persons are so bigoted to the old Authors, that they cannot be reconciled to the new Terms, by Brook Taylor; nor indeed to his new Principles, till they find their Excellence by experience.

It is not indeed to be wondered at. 'Tis not easy to divest any Person of old habits and methods of practice, though ever so absurd; because it is impossible that they can see the difference, at first, and consequently cannot judge of it; but it is surprizing that they are not to be prevailed on to try; and if they do, it is with seeming reluctance, and with a fixed resolution to prefer, and persevere in their old Prejudices. I am as much against capricious innovations in Science as any Person; but, if there be an appearance of any Acquisition to it, we ought, candidly and unprejudiced, to make a fair trial of their merits; without which we cannot judge of their Excellence.

In respect of the new Terms given us by Dr. Brook Taylor, such as Intersection of the Picture, Center of the Picture, Vanishing Lines and Points, &c. (together with Directing Plane and Line, which are most essential, in Theory, though but of little use, in Practice) I am of Opinion that no better Terms could possibly be devised; nor any other so expressive of what is meant by them. How narrow, how limited is the Base Line and Horizontal Line (the only Vanishing Line known to the old writers on Perspective) when compared with them. What difference is there, either in Theory or Practice, between the Horizontal Vanishing Line and any other, of Planes perpendicular to the Picture? None at all, seeing, they have the same Center and Distance (Th. 6.) nor indeed in any other, except in finding them; the Practice, in all, is the same, in every respect.

It was impossible for him to make the Principles of Perspective general, but by general Terms; which does not regard any Position either of the Picture or of the Original Plane, since all Planes (simply as Planes) are the same. The Intersection of the Picture includes every Intersection whatever, as well as the Base Line, and they are all of the same use. Vanishing Line is not only general, but is, at the same time, so simple and expressive, that it conveys its utility at once to the Mind.

In Theorem 2. Book I. of Direct Vision, it is proved, that parallel Right Lines, however situated, appear to approach towards each other; and, consequently, if they are produced, infinitely, they will appear to meet, and vanish in a Point at an infinite Distance. So likewise, parallel Planes appear to meet each other, and to vanish in a Right Line, (supposed to be infinite) or, properly, in a Point.

Now, if this Theorem be well considered (together with the foregoing) it will be found to be the very foundation of the new Principles of Perspective. For if a Plane be supposed to pass through the Eye, parallel to any Plane, whatever, or any number of parallel Planes, and being produced, or continued till they are lost to sight, they will all appear to unite, and to meet the Plane passing through the Eye at an infinite Distance.

But if the Eye be in the continuation of a Plane, the whole of that Plane is lost to sight, and appears but a Right Line (Art. 3. of a Plane, P. 43.)

And, the Intersection of two Planes is a Right Line, (Ax. 3.) - 1. 7. El. Wherefore, if a Plane (which may be considered as the Picture) be situated any how; the Line, in which this imaginary Plane would cut the Picture, is that in which the parallel Planes unite, and vanish; consequently it represents an infinite Distance; and consequently, the Line, so produced, is their Vanishing Line; for they cannot, if continued infinitely, go beyond it.

E e Hence,

Hence, those Planes are, very aptly, said to vanish, they being lost to sight. Therefore, all Parallel Planes have the same Vanishing Line. (Theo. 5.)

Also, if a Right Line be supposed to pass through the Eye parallel to any number of Lines (not parallel to the Picture) they will appear to converge towards that Line, and to meet it in one Point, at an infinite Distance (Theo. 2. Direct Vis.)

Wherefore, if this Right Line from the Eye cuts a Plane, any how situated, it will cut the Plane in a Point only; which represents a Point at any Distance whatever, in that Direction, and consequently, it represents the Point in which the Lines, parallel to it, converge; which is infinite. Therefore, it is their Vanishing Point; for, they are lost to sight before they reach it, seeing it is infinite.

And, since the Line, producing that Point, passes through the Eye, the whole Line is lost to sight, seeing, one Extreme is at the Eye; and the Extremes of Lines are Points. Therefore, the Point, in which it cuts the Picture, is its whole Representation; and consequently, all Lines, parallel to it, tend to that Point.

Now I must own, that I cannot conceive any Term so fit to express that Line, or Point, in which parallel Planes, or Lines, meet each other, as Vanishing Line, and Vanishing Point; because they are truly said to vanish in them. For the same reason, perhaps, Mr. Noble, the last writer on Perspective, has made use of the Terms Entring Line, and Entring Point; seeing that, the Plane, or Line, begins at the one and vanishes in the other. Had this Author been the inventor of those Principles, I should not have found fault with the Names he had given them; but, since there were Names already given, by the Author, which are more significant, I must blame even an attempt to alter them; because, a multiplicity of Names, for the same thing, occasions a confusion of Ideas, in the Mind, of their significance and use; and cannot possibly be of advantage to the Science.

In the process of this Book, after some necessary observations, on the Proportion of the Picture, the Height and Distance of the Eye, &c. in the third Section, I shall, first, shew how to determine on the Position of the Picture, in respect of the Object and the Eye, the Station being previously determined; then, how to prepare the Picture, for Practice, according to the Principles contained in this Theory.

The Picture being prepared, the Situation and Distance of the Object, and the Position of the Picture being determined, the following Problems, in this Section, shew how to find the Intersections and Vanishing Lines of Planes, in all Positions to the Picture, if they are not parallel to it (for all such have no Vanishing Line. Theo. 2.) with their Centers and Distances. Then, to fix the Vanishing Points of certain Lines in those Planes, and determine their Distances; by which, the Lines, vanishing in them, are proportioned.

The Position, Situation, and Distance of the Picture (in respect of the Object and the Eye) being determined, the Original Lines, in the Objects, are produced (if necessary, and not parallel to the Picture) to their Intersecting Points; which are always found in the Intersection of the Plane they are in (Theo. 11.) or being much inclined to the Picture they do not, perhaps, fall within the compass of the Picture; then, other expedients are used, to find the Representation of some principal Point in the Line, from which Point, the indefinite Representation is drawn (Theo. 12.) And lastly, the finite Parts, which represent certain portions of Lines in the Original Object, are determined (by Theorem 13th) by which means, the Object is compleated; proceeding from one Plane, or Face of the Object, to another; drawing all the Figures, in each, by means of their respective Vanishing Lines. Each two adjoining Faces, having one Line common to both, the Vanishing Point, of that Line, is in both Vanishing Lines (Theo. 11.) consequently, it is in their common Intersection (Cor. 2. Th. 8.) by the help of which, the Vanishing Lines of contiguous Faces are determined.

In

In Section 4th, I shall briefly illustrate all the remaining, practical Problems in Brook Taylor's Essay, respecting the proportioning of Right Lines, perspectively, and shew their great and extensive utility; each of which, founded on the most solid and permanent Principles, is of immense value. For, without knowing the whole of Perspective, or practicing its Rules, rigidly, an Artist, who is accustomed to sketch, by sight, whatever he sees before him, with seeming accuracy, may, by these Problems, rectify any errors, in right lined, or circular, Plane Objects, from the known proportion of one part to another; the affinity of the Planes and Lines to, each other, being known, and the Ratio of one Part to another; which, may frequently be obtained, when the true measures of those Parts can not, by reason of sundry impediments.

These Problems contain all the Rules necessary for Practice. They may be compared to the five fundamental Rules in Arithmetic, by which all others are worked; and, a Person might, with as much propriety, imagine that he had given Arithmetic enough, for every Occasion, in the five Rules, as Brook Taylor had of Perspective, in his first Essay; whereas, the Rules, he has there given, are no more than the Elements of practical Perspective.

This Section contains, also, various Expedients; viz. for finding or determining Vanishing Points, when they fall beyond the limits of the Picture, geometrically and arithmetically, i. e. to determine their Distance from the Center of the Vanishing Line and from the Eye, both which are necessary to be known; how to draw Lines to a Vanishing Point which is not on the Picture, with several others.

In Section 5th, those Rules are applied to real use, in delineating all kinds of Plane Figures; and, on Planes in various Positions. First, by means of the Figure being geometrically drawn, in the Original Plane. Secondly, without it, by their known Proportions, their position to the Picture, and the properties of the Figure, being regular.

In the 6th Section, I shall, from Plane Figures, proceed to Solids, composed of Planes, of various Figures; and in various Positions.

In the 7th, I shall apply them to streight Mouldings, composed of Planes and cylindrical Surfaces, in Cornices, Entablatures, &c.

Section the 8th treats of curve lined Objects, in general.

The 9th shews how to apply the whole to compound Objects, in regular pieces of Architecture, and Buildings of various kinds.

The 10th is for internal Views, and horizontal Pictures, Ceiling Pieces, &c.

The 11th is adapted for the particular Professions of Cabinet-makers, Coach-makers, &c. and for Machines, in general.

The 12th, and last, is on inclined Pictures and Planes, in general; and, applied to Fortification, or military Architecture.

N. B. A Scale of equal Parts is always adapted, or determined on, of the Proportion we intend to delineate the Objects, on the Picture.

The methods of making Scales, for various purposes, are given in the Appendix to my Treatise of Geometry.

SECTION

SECTION II.

A preparatory and elementary INTRODUCTION, to the PRACTICE of PERSPECTIVE.

As this Book is intended to be a compleat practical Treatise, I shall treat it in such a manner (in this Section) as if no Theory, or Elements, had been given. For which reason, I shall define a few more Terms, which are suited to Practice only; as there are several Terms in the theoretic list which may be omitted here; and, there are also, in Practice, several which are not useful, in Theory. I shall deduce from the Definitions such useful Lessons, which, if carefully attended to, will contain all the necessary Theory for a Practitioner.

PERSPECTIVE, is the Art of delineating the true Representations of Objects, on a Plane Surface, by geometrical Rules; according to the Position, and Distance of the Objects, in respect of the Picture and of the Eye. (See the Apparatus.)

The PERSPECTIVE REPRESENTATION of an OBJECT, is the Section of the Pyramid of Rays, AEI, by a Plane, in any Position; which, will be the Subject of the following Sheets of this Third Book. For, the Picture of every Object, truly delineated in Perspective, is supposed to be so situated, in respect of the Object and the Eye, that, if Visual Rays, or Threads, i. e. if Right Lines were drawn from each Angle, or other Point in the Object, to the Eye, they would pass through the corresponding Points in the Representation, on the Picture.

As EA, EB, EF, &c. cut the Pictures in *a*, *b*, *f*; a, b, f, &c. which are, therefore, the Perspective Representations, of the Points, A, B, F, &c. on each Picture.

It is evident, that the Perspective Representation of an Object is nothing more than the Figure projected on a Plane, by its Intersection with the Visual Rays from the Eye to the Object; wherefore, the whole business, of practical Perspective, is to find the true Figure of the Section of the Rays, in all Positions, whatever, of the Picture (which is considered as the intersecting Plane) and in every situation of the Object, or of the Eye.

In order to which, it is necessary to reconsider, well, the construction of the elementary Planes; as in Plate VI. or, to make them more familiar, I have given another, which is better suited for Practice; they are the foundation of the whole, and the origin of all the Lines and Points used in Practice. Also the general Introduction to this Work should be attentively perused, and tolerably well understood, by every one who would be a Proficient in the Practice of Perspective.

Fig. 15. Let BFIL be an Original Object, representing a Building, and, ES a Spectator, viewing that Object, from the Point of View E (the Eye.)

It is manifest, that whilst the Eye remains fixed, in that Point, the Object cannot vary in its Appearance. But, it is also obvious and demonstrable, that every different Section of the Pyramid of Rays, by a Plane in different Positions, will exhibit a different Picture; so very different, indeed, they may be, that they can scarcely be supposed to be Representations of the same Object, much less from the same Point of View and Position of the Object, when viewed direct.

Compare the Pictures, MNOP and *OP*; how different is the Image of the same Object, to each other. The one, on MNO*P*, has the Representation, *bgfdc*, of the end of the Building, BFC, similar to the Original, in every respect; the Representation, *ahdc*, of the Front, AHDC, is distorted, and drag'd out to a preposterous length, when the Eye is opposite to it; whilst the other Picture exhibits a pleasing and natural Appearance, in almost any Point of View, but most so in the true one. Yet, both Pictures affect the Eye alike, in the true Point of View, and appear the same as the Original Object, itself; each Line, Plane, or

or Figure (it is evident) being seen under the same Optic Angle as in the Object. Consequently, the Picture being a true Perspective Representation of the Original, if it had likewise the same degree of Colour, Light and Shade, it would not be possible for the Eye, at E, to distinguish whether the Image (*aifc*, or aifc) delineated on a Plane, or the Object itself (AIFC) was presented to view.

Having, thus clearly, explained what Perspective is, I shall next define all the practical Terms, by means of which, the whole process is performed, and the Representations effected.

Let the Plane AIKB (which may be supposed the Picture) be raised up into a vertical Position, i. e. perpendicular to the Plane it is fixed to; which is the most natural, most general and convenient Position. Pl. VIII. Fig. 37.

Also, raise up the Plane GKIH, parallel to the Picture. This Plane is called, the Directing Plane (Def. 4.) and is very useful, in Theory. In it, the Eye is always supposed to be, as at E; and if the Plane N*IK*L be turned down, parallel to ABGH, the Line *IK* meeting IK, in the Directing Plane; then, EC is the Distance of the Eye from the Picture (equal FD); and EF, (equal CD) is its height above the Plane ABGH, on which, the Spectator (EF) is supposed to stand. Therefore, at whatever Distance the Eye is from the Picture (as EC) a Plane GKIH is supposed to pass through the Eye, parallel to the Picture.

The Plane V being turned up, perpendicular to the Picture, is considered as a part of the prime or chief Vertical Plane; and, N*IK*L is called the Horizontal Plane; both which are supposed to pass through the Eye, at E.

N. B. This Construction of the elementary Planes, so essential in Theory, is, also, very necessary to be well considered by every Practitioner, who may not have an inclination to go through the Theory; he will, most certainly, find his account in it.

DEFINITIONS.

DEF. A. The GEOMETRICAL or GROUND PLANE, is that Plane on which an Original Object, intended to be delineated, is seated.

As GHZZ; on which the Plans, ACDF and XYZ (supposed to be the Seats of some Objects) are geometrically drawn, which are to be represented on the Picture, AIKB.

On this Plane is always drawn, or supposed to be drawn, the Figures of the Seats or Plans of Objects situated on it, in their true geometrical Proportion, according as they are situated, in respect of each other and of the Picture; and they are (in common Perspective) always understood to be beyond the Picture.

In the Apparatus, the Plane ZSZ is the Ground Plane, representing the real Ground, on which the Object, BFIL, stands. The Rectangle ABCL is its Plan, or Seat on the Ground Plane.

Note. If the Object stands on a Floor or Table, &c. it is considered as the Ground Plane.

DEF. B. The GROUND LINE, or BASE LINE, is a Right Line on the Picture, drawn parallel to its Top and Bottom, i. e. parallel to the Horizon.

As AB, Fig. 37. No. 1 and 2. (See Intersection. Def. 7.)

The Plane AIKB, being erected, represents the Picture, and ABZZ, being horizontal, is considered as the Ground Plane. Therefore, the Line AB is the Intersection of the Picture with the Ground Plane. And, since all Objects which are to be represented, on the Picture, are supposed to be beyond it, they must necessarily appear above the Line of Intersection; for which reason, it is, very properly, called the Ground Line.

Note. On the Ground Line is applied all the measures, geometrically, of all Figures (as Plans, &c.) of Objects on the Ground Plane, to be delineated; and frequently, for other horizontal Planes, whose Intersection with the Picture is not given or drawn.

N. B. Of the same use is the Intersection of any other Original Plane with the Picture, viz. for proportioning Lines and Figures drawn on that Plane.

DEF. C. The HORIZONTAL LINE, is a Right Line drawn parallel to and above the Ground Line; as ML. (See Vanishing Line Def. 8.)

Pl. VIII. The Horizontal Line is called so by way of eminence, or for distinction from all other horizontal Lines, being the first, or principal Vanishing Line; which, from its fixed and determined Position and Distance, determines and governs all other Vanishing Lines, whatever, except the Vertical; which is determinable without it.

N. B. Its Distance from the Ground Line is always equal to the height of the Eye from the Ground, being considered as a Plane, on which the Object, to be delineated, is seated. So that, whether we be siting, standing, or elevated, the height of the Eye, from the Ground Plane, being determined, and *AE*, or DC, being made equal to it (by the Scale of Proportion) NL drawn through that Point, parallel to AB, is the Horizontal Line; or Vanishing Line of horizontal Planes. Fig. 37.

Note. It is supposed (in Theory) to be produced by a horizontal Plane (N*IK*L) passing through the Eye and cuting the Picture, in NL, their mutual Intersection.

DEF. D. The VERTICAL LINE, is a Right Line drawn at right angles with the Horizontal and Ground Line; cuting the Picture into two equal Parts, perpendicularly. As ED. (See Defin. 11. Th.)

DEF. E. CENTER of the PICTURE, or POINT of VIEW, is the Point C, in which the Horizontal and Vertical Lines cut each other. (See Def. 16 and 17.)

N. B. The real Point of View is the place of the Eye, where it ought to be fixed when viewing a perspective Picture. The Point C, on the Picture, opposite to it, where a Perpendicular from the Eye would cut the Picture, is, therefore, its Center; and is, generally, understood to be the Point of Sight; i. e. to which the Eye must be opposite.

Hence, if the Center of the Picture, C, be first determined (as it frequently is) then a Right Line drawn through C, parallel to the Horizon, as LM, is the Horizontal Line; and, another Line, drawn through C, perpendicular to it, as ED, is the Vertical Line.

DEF. F. PARALLEL of the EYE.

If EC be taken, in the Vertical Line, equal to the Distance of the Picture; and, through the Point E (which is considered as the Eye) if a Right Line be drawn, parallel to the Horizontal Line, as IK, it is called the Parallel of the Eye, of horizontal Planes. (See Def. 9. Th.)

DEF. G. VANISHING LINE.

The Vanishing Line, of any Original Plane, is a Right Line on the Picture, supposed to be produced by an imaginary Plane passing through the Eye, or Point of View, parallel to the Original Plane; the Line, in which such a Plane would cut the Picture, is the Vanishing Line of that Original Plane.

COR. Hence, the Horizontal Line is the Vanishing Line of the Ground Plane, and all other horizontal Planes.

For, it is produced by a horizontal Plane (N*IK*L) passing through the Eye, and cuting the Picture.

COR. 2. And hence, all Planes, which are parallel between themselves, have the same Vanishing Line†.

† Theo. 5.

For, there can be but one Plane passing through the Eye parallel to them all; and since it can produce but one Line on the Picture, by its intersection with it, that Line is, consequently, the Vanishing Line of them all.

N. B. The Vanishing Line, and the Parallel of the Eye, of any Original Plane, are parallel to the Intersection of that Plane with the Picture‡.

‡ Theo. 3.

Therefore, LM, the Horizontal Line, and IK, the Parallel of the Eye, of horizontal Planes, are both parallel to AB, the Ground Line; which is the Intersection of the Ground Plane, with the Picture.

N. B. 2. Original Planes which are parallel to the Picture have no Vanishing Line, nor Intersection §.

§ Theo. 2.

DEF. H. VISUAL RAY, is a Right Line drawn from the Eye to any Point in an Original Object.

Let the Picture, AIKB, be turned up into a vertical Position; and, let the Plane W be turned over, till *C* coincides with the Center of the Picture; then will *E* coincide with the Eye, or Point of View, E, if the Directing Plane, GEH† be turned up parallel to the Picture.

† Def. 4.

Let

Let the Plane AFDC, on the other Side of the Picture, be also turned over, till it falls into the same Plane with W. Then, the Right Lines EA, EB, EC, are Visual Rays from the Eye, E, to the Original Points, A, B, and C, on the other side of the Picture; which, by their Intersections with the Picture, gives the Representations a, b, and c, of the Original Points, A, B, and C, on the Picture. Fig. 37.

See the Apparatus; in which, the Threads EA, EB, EF, EG, &c. are Visual Rays; producing, by their Intersections with the Picture, the Representations, a, b, g; and *a*, *b*, *g*, &c. of the Original Points A, B, G, &c. in the Original Object.
PM and *P*M are the Ground Lines of those Pictures; VY, and CX are the Horizontal Lines; and, CL, and *OP* are the Vertical Lines. C, their Intersection, is the Center, or Point of View of each Picture; for, the Eye, considered as a Point, is opposite to either.

DEF. I. DISTANCE of the PICTURE; it is the length of the Perpendicular, or the shortest Line that can be drawn from the Eye of a Spectator, in the true Point of View, to the Picture; or the Distance between the Eye and the Center of the Picture. As EC. (See Def. 15 and 18.)

DEF. K. INTERSECTING POINT, of an Original Line, is that, in which any Original Line (being produced) would cut the Picture.

B, P, and S are the intersecting Points of XY, ZY, and ZX of the Three Sides of the Triangle XYZ.

In the Apparatus, *F*, *G*, and *H*, are the Intersecting Points of the Original Lines, IF, HG, and FG.

N. B. The Intersecting Point, of every Line, is in the Intersection of the Plane the Line is in, with the Picture (Theo. 11.) Therefore, *FGH*, drawn through the Intersecting Points *F*, *G*, and *H*, is the Intersection of the Plane FGHI. Also, *BG*, drawn through the intersecting Points, *B* and *G*, of the Lines AB and HG, is the Intersection of the Plane ABGH.

DEF. L. VANISHING POINT, of an Original Line, is that Point, on the Picture, in which, a Right Line, from the Eye or Point of View, parallel to any Original Line, would cut the Picture.

N, O, and L are the Vanishing Points of the three sides of the Triangle XYZ. C is the Vanishing Point of the Lines ABC and FD; and, M and L of the Diagonals AE, BF, &c.

In the Apparatus, V is the Vanishing Point of all the Lines AB, GH, and FI; Y is the Vanishing Point of BC, and GD, &c. and W, of the Lines GF and HI; on the Picture MNOP. Fig. 15.
The Vanishing Point of FD is out of the Picture, below the Ground Line.

In the Picture MNOP, C, its Center, is the only Vanishing Point; viz. of the Lines AB, GH, &c. to which EC is parallel. For they are perpendicular to that Picture.
All the other Lines in the Original Object, in the Plane BFC, are parallel to that Picture; and therefore, they have no Vanishing Point †.
Also, BG, AH, &c. being parallel to both Pictures, have no Vanishing Point, on either.

† Cor. to Theo. 2.

N. B. The Vanishing Point, of every Line, is in the Vanishing Line of the Plane it is in. (Th. 11.)

Therefore, a Right Line drawn through V and W, the Vanishing Points of IF and HG, HI and GF, is the Vanishing Line of the Plane FGHI.
Also, RW, drawn through the Vanishing Points, Y and W, is the Vanishing Line of the Plane BFC; in which, the Lines BC, or GD, and GF are situated.

COR. All Lines which are parallel amongst themselves have the same Vanishing Point ‡; except they are parallel to the Picture; in which Case they have no Vanishing Point (Cor. to Theo. 2.)

‡ Theo. 5. Cor. 1.

For, EV, producing the Vanishing Points, C and V, on both Pictures, being parallel to AB, is consequently parallel to GH, FI, &c §. and since it can produce but one Point on each Picture, that Point is, consequently, the Vanishing Point of them all.
Also, EW, being parallel to FG and HI, produces their Vanishing Point, W.

§ 4. 7. El.

COR. 2. Hence, the Center of the Picture is the Vanishing Point of all Lines which are perpendicular to the Picture.

For EC, the Direct Radial, produces the Center of the Picture, (Def. 17.) and, because it is perpendicular to the Picture, it is parallel to all Lines that are perpendicular to the Picture, and consequently it produces their Vanishing Point; by this Definition.

DEF.

DEF. M. STATION POINT is at the foot of the Spectator; or, it is that Point, in which a perpendicular from the Eye would cut the Ground Plane.

Fig. 37. If the Directing Plane, GHIK, be turned up, on GH the Directing Line, perpendicular to the Ground Plane; E being the Eye, EF perpendicular to GH, cuts it in F, the Station Point; and EF is the height of the Eye above the Ground Plane.

DEF. N. STATION LINE, is a Right Line drawn from the Station Point, in the Ground Plane, perpendicular to the Ground Line; or, it is the Intersection of the Vertical Plane†, with the Ground Plane. As FD, produced.

† Def. 5.

In the Apparatus, S is the Station Point, and SB the Station Line, of the Picture MNOP; and S*P* is the Station Line of the Picture MN*OP*.

SECTION III.

Containing some preliminary observations, concerning the Proportion and Position of the Picture; of the Height and Distance of the Eye, &c. 2ndly, How to prepare the Picture, for Practice. 3dly, How to find Vanishing Lines, in all common Cases; and to fix the Vanishing Points, of Lines in all Positions, in respect of each other and of the Picture.

IT may appear somewhat strange, and unaccountable to many, of what use are all the imaginary and elementary Planes in delineating Objects; or how so many Planes, cuting each other (at right angles, or otherwise) can possibly be applied, in Practice; seeing, that the Plane or Picture, on which the Object is delineated, is the only real Plane made use of. It may also be a matter of wonder, how a Visual Ray can be drawn from an Object, which is beyond the Picture, to the Eye, on this Side; by means of which, it is very obvious (from the Apparatus) the Representations of the several Angles, &c. in the Object, are projected on the Picture; but how this can be effected in delineating, must appear strange to a Novice in Perspective. All which, will be accounted for, very satisfactorily, and shewn, how to manage all the Planes with great facility.

The Art of drawing in Perspective has this advantage of all other. It is not a random sketch, depending on the Hand and Eye; but, every Line (Right Line at least) may be drawn to the utmost exactness; and the Points, where the Visual Rays would pass through the Picture, are determined, mathematically; yet may be done by a Person entirely unacquainted with Mathematics, who shall adhere to the Rules contained in the following Sections.

It is evident, seeing that Vision is conveyed (from the Object to the Eye) in Right Lines, that every Section of the Pyramid of Rays will appear the same, to an Eye in the Vertex. Consequently, whether the Section be made nearer to the Eye or to the Object, whether by a Plane or other Surface, whether it be direct or an oblique Section, the effect, to the Eye, is the same; but it is obvious, that it can be so only in that Point of View; for, every different Section has a different Representation; but, all parallel Sections are similar Representations. So that, whether the Picture be drawn by a larger or a smaller Scale of Proportion, the Representation is the very same, except in Dimensions; and supposes the Section to be made further from, or nearer to, the Vertex of the Pyramid of Rays, from the Object to the Eye.

It

It may also be observed, that, if the Visual Rays were produced or continued, beyond the Model, they would diverge, so, as to take in a real Building, situated exactly as the Model, of any Dimensions. Which being premised, it is manifest, that a Picture, delineated, truly, by a scale of equal Parts, of any Proportion, from a real or imaginary Model, of a Building, &c. will as truly represent the Original, as if the full measures of it were applied; which could not possibly be done.

This consideration may account for the Measures applied in Practice; which must always be in the same Ratio or Proportion, to each other, as the real Measures of the corresponding Parts in the Original Object.

PRELIMINARY OBSERVATIONS.

IN respect of the Shape and dimensions of the Picture, no Rules can be prescribed, it is always at the discretion of the Artist; unless it be proportioned to some particular Place which determines its Figure and Dimensions. The oblong Rectangle is, in general, a more agreeable and convenient shape than a Square; about the Proportion of 3 to 2, i. e. if the length be three feet, the width may be two, or thereabout; as conveniency, for taking in the Objects, may require. Some Objects requiring it upright; others, and more generally, length-ways.

2. Neither can a certain and invariable Rule be given for fixing the height of the Eye, and, consequently, of the Horizontal Line. To fix it to half or a third part of the height, absolutely, would be ridiculous; it must ever be at Discretion, in proportion to the Scale of the Drawing; a Landscape View, from an Eminence, may raise the Horizon to the middle of the Picture, or higher, yet may be very natural. In general, five feet, or five feet six inches, the natural height of the Eye, is the most agreeable, being most accustomed to see Objects at that height; altho' it may not be, perhaps, above one fifth or sixth part of the height of the Picture. Too low, in a general View, is not agreeable, because the recedings of the parts of Objects, on the Ground Plane, are not so distinguishable, as, they approach nearly to Right Lines.

3. Respecting the Distance, something may be ascertained. The Distance of the Picture is a material Circumstance which ought to be well attended to; otherwise the whole performance may be a disagreeable Distortion, instead of a pleasing and natural Representation. The Distance ought always to be considerably more than the height of the Eye; although Brook Taylor and some others, in their Diagrams, have made it much less, which produces a very bad effect; the Representations of the receding parts of the Object, on the Ground Plane are, by that means, drag'd out to an immoderate length. To illustrate it.

EX. Let AFEB be supposed the prime Vertical Plane, in which the Eye is situated, at E. Let EC, be its Distance from the Picture, of which CD may be supposed a Section. Fig. 37. No. 2.

This, I presume, is easy to conceive; for, suppose two Planes cuting each other at right angles, one of which is considered as the Picture; the other, a Plane passing through the Eye, perpendicular to it. The Right Line CD being considered as their common Section; and the Planes turned around on it, as an Axis; (see Fig. 8. Pl. 3. in the general Introduction) in which revolution, either Plane will become Direct before the Eye, and the other perpendicular to it, at CD, their mutual Section.

Now, E is supposed the Eye of a Spectator, and, C the Center of the Picture; EC is, therefore, its Distance. Let an Object be supposed beyond the Picture, whose Seat on the Ground Line is AD.

If a Right Line, EA, be drawn, cuting CD, the common Section of the Picture and the Vertical Plane, in a; it is obvious, that to the Eye at E, the representation of the Point A will be at a, on the Picture; and the length of Ground, between the Picture and the Point A (equal DA) is represented by Da. For, EA is a Visual Ray, from the Eye to the Point A; in which Direction, that Point is seen; and consequently, a is the Point in which it would pass through the Picture.

As it may be observed in the Apparatus, by placing the Eye in a continuation of either Picture and the Ground Plane; i. e. in the Line of their common Section, PM.

Pl. VIII. Fig. 37. No. 2. Then (considering the Eye of the Spectator as the Point E; and, A, or any other Point, on the Ground Plane, answering to A, in the Figure) EA represents the Visual Ray, and a, the Point where it passes through the Picture; which is represented by the Right Line CD.

Now Da represents the length of Ground between the Picture and the Point A, the Eye being at E; but, being moved to *E*, the Appearance of it is at *a*; and, the Point a represents, at that Station, the length DG only, which is preposterous.

For, since DC is the whole indefinite Representation of DA, produced (Theo. 12.) and Da, *a* represents the finite, small Portions DG, GA, only; consequently, the remainder, *a*C, of the indefinite Representation, DC, represent the whole of DA, produced infinitely beyond A.

It is evident, that the Points, a or *a*, from either Station, is mathematically determined; agreeable to Theorem 13th. For, DC being the whole indefinite Representation of DA (Th. 12.) the representation of any Point, A, is found, by making Da (or D*a*) to DC, as AD to AB (or AD added to CE) i. e. Da : DC :: AD : AB, as it is demonstrated in that Theorem.

The Triangles AaD, AEB, and CEa being similar; Da : ac :: AD : CE; consequently, Da : Da + aC (i. e. DC, equal BE) :: AD : AD+CE, equal DB (i. e. AB.)

N. B. While the Eye moves in the Direction EC, the Indefinite Representation DC remains the same, and the finite Parts (Da or D*a*) of AD is continually varied †.

Also, whilst it moves in the Direction EA the finite part Da remains the same, and the indefinite Representation is varied. As, at e, the whole indefinite Representation is D*a*, and the finite part is still *Da*.

† Cor. 5. Th. 12.

Thus, it is evident, that the Height of the Eye is productive of as great Distortion as the Distance; but, to determine, absolutely, in what proportion one shall be to the other, is not possible, as various circumstances may render all such Rules exceptionable. In general, the Distance ought not to be less than twice the Height of the Eye (as at E) or, at the least, as three to two (as at E^2) but there may be a necessity, in some Cases, to make it equal, or perhaps higher, for the conveniency of shewing some particular parts of the Object.

Some Persons make it a general Rule to make the Distance equal to half the Diagonal of the Picture, which is, certainly (if the Center of the Picture be in the middle) a method, if rightly understood and applied, will never produce great Distortion, in the Representations of Objects thereon.

No. 3. For, if S be the Center, or Point of View, of the Picture ANOB, the Distance SN, or SO, may be sufficient, if there be no Objects near the Extremes, at A or B; but if there are, they will be distorted; because, it is evident that the Optic Angle, under which they are seen, is a Right Angle, or 90 Degrees.

But, this Rule, for fixing the Distance, is sometimes injudiciously adhered to; when the Center, or Point of View, is near either Extreme, as at M or L; in which Case, especially if the View be internal, or have Objects situated near the other Extremes A or B, they will be greatly distorted.

Let AS or SN be taken for Radius, and, on M, describe an Ark of a Circle, from E to F. It is manifest that all the part beyond that Circle, towards L, being seen under an obtuse Angle (which the Eye cannot possibly take in) will be preposterous.

Now, this is so very obvious, to a Person of any discernment, or a small share of knowledge in Geometry, that 'tis needless to expatiate longer on it. Yet, this is one principal reason, why many Artists quarrel with Perspective, and pronounce it deficient. For, not being acquainted with the Theory, they would have, on every part of the Picture, the Objects represented as they appear; which cannot possibly be, if they are remote from the Center; for the reason given above.

Wherefore, if the Center of the Picture be judiciously fixed (as at C) in the middle of the Picture (respecting its length, only, not height) as it ought always, except in particular Cases, to be, and CN or CO be taken for the Distance, there can no great inconvenience accrue, especially in external Views; as there will, very probably, be nothing but Clouds represented, at N, or O.

The reason of all this is evident. For, wherever the Center of the Picture is fixed, every part of the Picture, ought to come within a Circle, whose Radius, or semi-diameter, is its Distance.

I shall however give one general Rule; which is, to make the Distance, at least, equal to the length of the Picture, *EF*, the Center, C, being in the middle; but, if it be on either Side, as at M, or L, it must be equal twice M*F*, or *E*L, inclusive;

five; seeing that, the Eye is always supposed to take into the Optic Angle, as much on one side of the Center as the other.

4. The Position of the Picture, in respect of the Object and the Eye, is another essential Point to be well considered, and determined on. Without due regard to that, the other Preliminaries are to little purpose; as all the imagined caution, in the Height and Distance, may be rendered abortive, by that means.

The Distance of the Picture ought always to be regulated and governed by the real Distance of the Object, if it be a single one; or, if there are a multiplicity of Objects, it must be calculated from the nearest, intended to be represented.

Let ABC, be the Seat or Plan of a Building, which is intended to be delineated from the Station E; the Distance from the nearest part is EB. Fig. 38.

Let DF be the Intersection of the Picture with the Ground Plane, or Horizontal Plane; and E the Station Point, or the Eye; then is ES the Distance of the Picture, applied close to the nearest part of the Object, AC, which is seen under the Optic Angle AEC.

The Distance ES (equal DF) is sufficient for that Picture; i. e. for the Object AC. But, if two, or more Objects, X and Z, are extended to the full length of the Picture, it is little enough; they being seen under the Angle DEF, which the whole length of the Picture subtends; viz 55 Degrees, on the Ark *df*, and is the least Distance I would ever use, in such Case, when the Objects extend the full width of the Picture.

ES is therefore the Distance of the Picture DF, and also of the Objects, or of a Plane passing through the nearest Planes of X and Z; in which Case, it is evident that the Picture must be as large as the Objects themselves, being applied close to them.

If they are supposed real Buildings, and the Scale of proportion be determined on, viz. a 10th or 12th part, or any other; take Ef (one third part of ES) for the Distance, and draw df parallel to DF; then is Ef equal df; for Ef : df :: ES : DF; i. e. they are equal, and consequently, the Objects x, y, and z being reduced to the same Proportion will all subtend the same Angles, respectively, as the Originals. By which means the Picture may be delineated of any Proportion.

Now, if the Station, E, be determined, from which the Object, AC is to be drawn, the Position of the Picture is also determined. For, ES, the Station Line, bisecting the Optic Angle, ABC or DEF, ought always to be perpendicular to the Picture; consequently, the Position of the Picture, *DF* or d f, is determined; i. e. it must be perpendicular to ES.

For, when the Station, or Point of View, E, is fixed, from which we are determined to delineate the Object, is it not most rational to suppose the Picture, on which the Object is to be represented, placed direct before the Eye? (as MNOP in the Apparatus) or is it more eligible to place it parallel to either Plane, ABGH or BCDG (as MN*OP*) i. e. to AB or BC, as D*C* or BG, whose Center, or Point of View, is at *C* or G. The very supposition of it is absurd to the last degree; and yet this absurdity is committed by every Artist who places the Point of View at either Extreme, or perhaps entirely off the Picture, as it is very frequently done.

The difference must be obvious to every Person who considers it. For, the Picture being placed direct, according to DF, the Optic Angle is no more than AEC, or *a*E*c*, about 23 or 4 Degrees; but, being placed in the Position D*C*, the Optic Angle is, *a*E*g*, 102 or 3 Degrees; for, E*C*, perpendicular to D*C*, bisects the Optic Angle, on that Picture; as ES bisects the Angle AEC, on the Picture DF; notwithstanding, the Object, ABC, does not occupy one fourth part of the Angle *a*E*g*.

In the Apparatus, S*P*, on the Ground Plane, bisects the Optic Angle, pSt, under which the Object is seen, according to the Position of the Picture MN*OP*; for, the Angle, at the Station Point, S, is the same as in the Horizontal Plane, at the Eye.

How to prepare the PICTURE for PRACTICE.

Let AIKB (when raised perpendicular to the Ground Plane) be supposed the Picture; also, let the Directing Plane, GHIK, and Horizontal Plane, *IK*LM, be placed parallel to the Picture, and the Ground Plane. Fig. 37.

Suppose the Objects AD, and XYZ, on the other Side, are to be represented on the Picture; and the Points a, b, c, &c. where the Visual Rays EA, EB, &c. would pass through the Picture, to be geometrically determined thereon.

In order to which, several preparatory Lines are necessary to be drawn on the Picture; the Center and Distance must always be known or determined, together with the position or situation of the Object, in respect of the Picture and of the Eye, and also its Height, above the Ground Plane.

Now

Pl. VIII. Fig. 37. Now E is the place of the Eye, EC is the Diſtance of the Picture; and EF is the height of the Eye; the Point C, where the Perpendicular EC cuts the Picture, is its Center. ML, the Line in which the Horizontal Plane cuts the Picture, is the Horizontal Line (Def. C) or the Vaniſhing Line of horizontal Planes; and AB in which the Picture cuts the Ground Plane is the Ground Line (Def. B.) conſequently parallel to the Horizontal Line. Theſe two are the firſt and moſt uſeful Lines; the Center of the Picture (C) in this Caſe, is in the Horizontal Line, the Picture being vertical.

The next Line to be conſidered, and of uſe, is the Vertical Line, ED (Def. D.) If the Plane V be turned up, perpendicular to the Picture, it will paſs through the Eye, at E, and conſequently through EC, cuting the Picture in EC, the Vertical Line; which, being produced will paſs through D.

This Line always paſſes through the Center of the Picture, and is of the ſame uſe for all vertical Planes, which are perpendicular to the Picture, as the Horizontal Line for horizontal Planes; i. e. it is their Vaniſhing Line, which has, conſequently, the ſame Center, C, and Diſtance, EC.

Let the Conſtruction of theſe five Planes be well conſidered, and the Lines they generate, by their Interſections; three of which, viz. the Ground Line, the Horizontal Line, and the Vertical Line, are already produced on the Picture, and the Parallel of the Eye, IK (whoſe real place is out of the Picture) is tranſpoſed to the Picture, by turning up the Horizontal Plane on ML, its Interſection, till it coincides with, or falls into the Picture; and with it the Direct Radial, or Diſtance, EC, together with the Eye, E, which falls into the Vertical Line, at E.

When this Conſtruction has been conſidered attentively, let them be puſhed either from or towards you, keeping the Directing and Horizontal Planes, joined in IK; the Vertical Plane (V) may ſtill be ſuppoſed to cut them all at Right Angles, and generates ſtill the ſame Line on each. In all Poſitions, i. e. let the Angles they make with each other be what it may, the Planes are ſtill parallel to each other reſpectively; and their Interſections are ſtill parallel amongſt themſelves. (Th. 3.)

Now, let the Picture be turned down, on AB, its Interſection with the Ground Plane, till they coincide; and let the Horizontal Plane, with the Eye, E, and the Parallel of the Eye, *IK*, be alſo turned down into the Picture (as it is repreſented on the Picture) alſo, let the Vertical Plane, V, be turned down on either Side, into the Picture, and with it the Direct Radial, EC, i. e. the Diſtance of the Picture, falling into the Horizontal Line, with the Eye at *E*. And, laſtly, let that part of the Ground Plane, which lies beyond the Picture (on which the Seats of Objects are geometrically drawn out) be ſuppoſed to be turned, on AB (its Interſection with the Picture) quite over to the other Side; and imagine it, ſimply, a Plane, without thickneſs; ſo that, all the Figures, deſcribed on it, are ſeen on the other Side, inverted; as *ABEF*, and XYZ; in which Caſe, it is obvious, that they have the ſame Poſition, or Situation to the Picture, as before; and the Lines (which are not parallel to the Picture) being produced, cut the Picture, in the Interſection (AB) in the ſame Points, as before.

Thus, is the Picture prepared for Practice, in common Perſpective; and all the elementary Planes are reduced to one Plane, viz. the Picture *.

All the ſame Lines and Points, may be ſeen in No. 3. which is diveſted of that apparent intricacy of Planes on Planes, conſequently it is more ſimple and intelligible; having only the preparatory Lines, anſwering to the Interſections of the elementary Planes with the Picture, on it. The Figures below AB, the Ground Line, are Plans of Objects, on the Ground Plane, intended to be delineated.

* The Directing Plane (GHIK) not being of uſe in common Perſpective is ſuppoſed to be turned down, or removed out of the way.

To

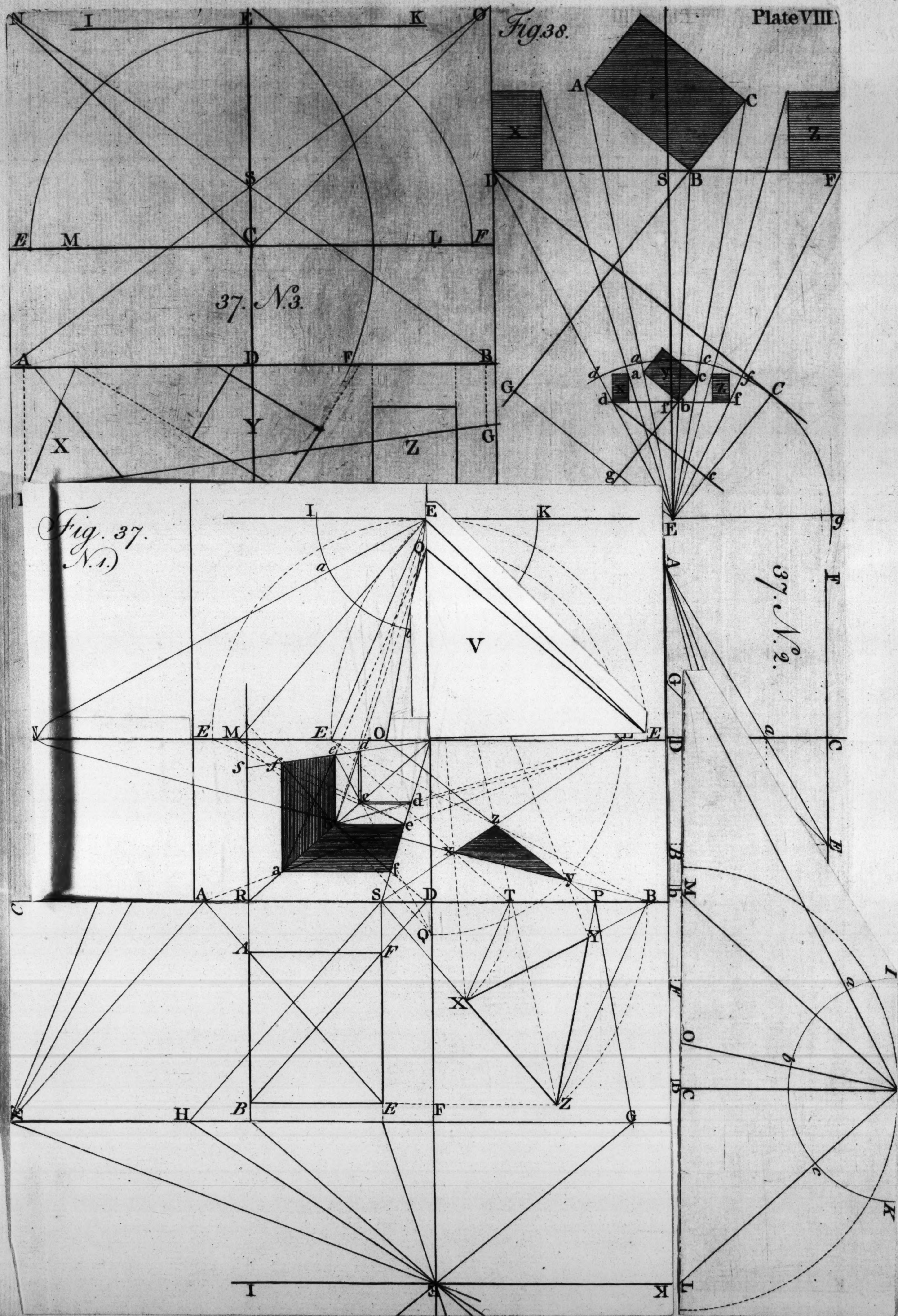
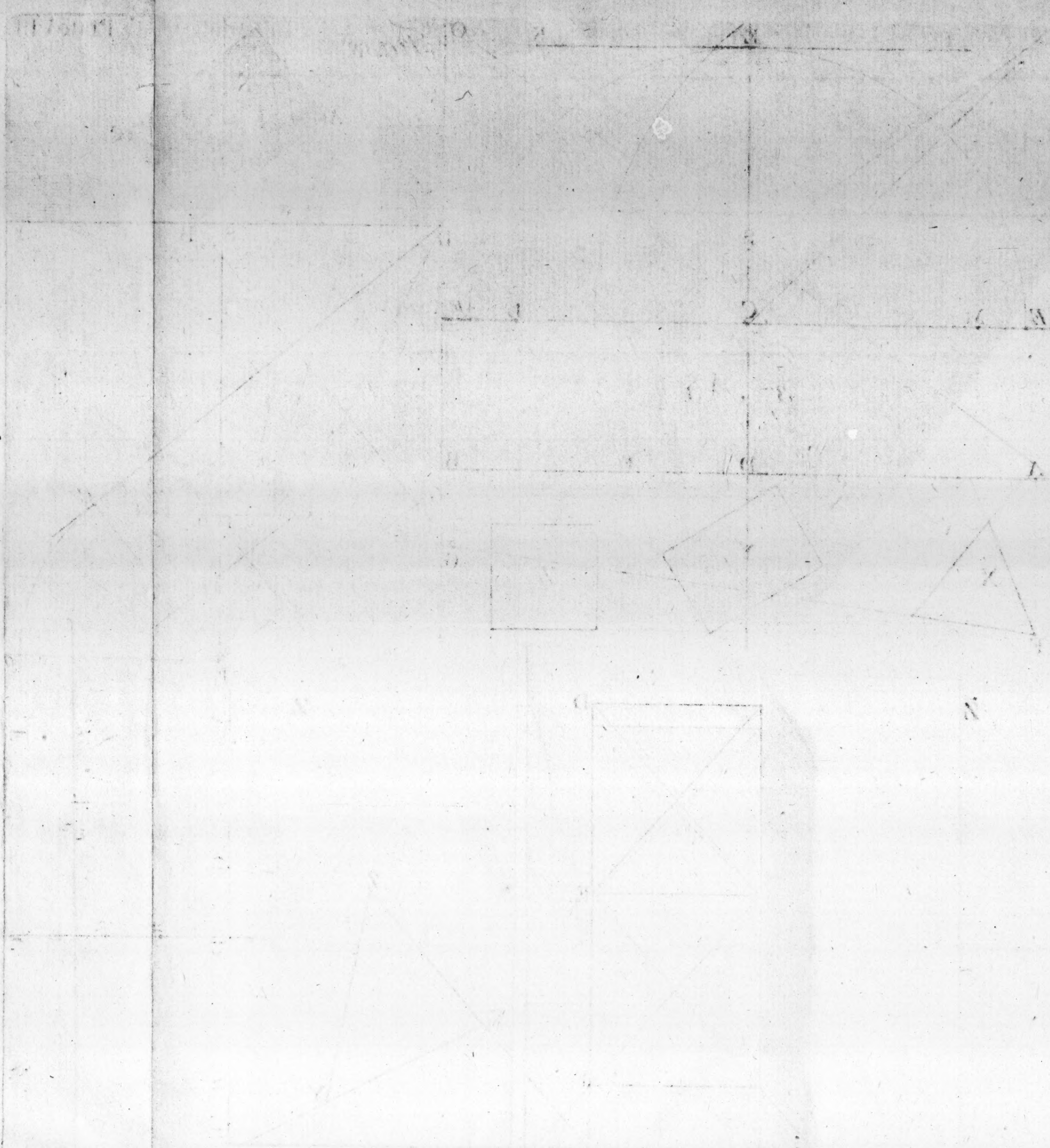


Plate VIII.
Fig. 38.
37. N°. 3.
Fig. 37. N. 1.)
37. N°. 2.



To prepare which, let the Ground Line, AB, be first drawn, at such a convenient Distance from the bottom of the Picture, and parallel to it, as to allow room, below it, for drawing the Plans of Objects (as X, Y, Z, intended to be delineated) in their true geometrical Proportion, Place, and determined Position to the Picture, if necessary. All which space, below AB (as A F B) is not considered as a part of the Picture, but of the Ground Plane, whose real place is beyond the Picture. Fig. 37. No. 3.

Through the middle of the Picture, draw ED, perpendicular to the Ground Line, dividing the Picture into two equal Parts. This Line is the Prime vertical Vanishing Line, in which, the Center of the Picture, or Point of View, is always (though not always in the Horizontal Vanishing Line; but when the Picture is vertical, as it is now supposed to be.) Therefore, make CD equal to the determined height of the Eye, C is the Center; through which, draw ML, the Horizontal Line, parallel to AB, the Ground Line.

Then, with the Radius EC, describe a Semicircle, cuting the Horizontal and Vertical Lines in *E*, E, and *F*; i. e. make C*E*, &c. equal to the Distance of the Picture, which are all considered as the Eye, transposed to the Picture* (generally understood by the Points of Distance) and, through E, draw IK, the Parallel of the Eye, parallel to the Horizontal Line; then is the Picture prepared, having all the fixed Lines and Points determined thereon. It only remains, now, to find other Intersections, Vanishing Lines, and Vanishing Points, necessary for delineating the Objects intended.

The Ground Line, or Intersection of the Picture with the Ground Plane, is the first and principal Intersection, and the only one in general made use of. But, the Intersections of other Planes are often wanted, and although they are frequently made use of, yet very few consider them as such; and consequently, they do not see the generality of the Principles and Rules by which the Work is performed; as they look on the operation in Planes which are vertical or inclined, in a different light from such as are horizontal; whereas, if it be well considered, they will find it is the very same; for, wherever the geometrical measures are applied, in proportioning any Line, it is considered as an Intersection of some Plane, in which that Line is or may be situated. And, since the Intersection of any Plane determines the Position of its Vanishing Line, I shall shew how to find the Intersections of Original Planes, in all necessary and generally useful Cases.

Inclined Planes, in general, I have reserved for the last Section; but, as various Planes, which are perpendicular to the Picture, are inclined to the Horizon, and vertical Planes are frequently inclined to the Picture, I shall not consider them, in either Case, as inclined Planes, but such only as are inclined to both.

As the Horizontal and Vertical Lines both pass through the Center of the Picture, they are, therefore, the Vanishing Lines of Planes perpendicular to the Picture †; which from their fixed and invariable Position, determine all other Vanishing Lines whatever. † Theo. 6.

Therefore, in the following Problems, I shall, in the first place, shew how, by them, to determine the Vanishing Lines of all Planes, that are perpendicular to the Picture, either with or without the Intersection; which are subject to one general Rule.

Secondly, how to find the Vanishing Lines of vertical Planes, in all Positions to the Picture (except parallel) and also, of certain inclined Planes, which are subject to the same invariable Rule, as vertical Planes.

The fifth Problem shews how to find the Vanishing Lines of Planes any how inclined, both to the Horizon and to the Picture (being vertical) from their known inclination to the Horizon, &c. as specified in the Problem. All which are frequently necessary in common Practice.

* The Eye, or Point of Distance, may be any where in the Circumference of a Circle, whose Radius is EC; as occasion requires; or, according to the Position of Original Planes, which are perpendicular to the Picture, and their Vanishing Lines; all which, have the same Center and Distance (Theo. 6.)

Plate IX.

PROBLEM I.

The Intersecting Point of any Line, in a Plane which is perpendicular to the Picture, being given, together with the Angle of its Inclination to the Horizon, to determine its Intersection and Vanishing Line; the Center of the Picture being given.

Fig. 39. Let I be the Intersecting Point of some Line given, C is the Center of the Picture; and X is the Angle of Inclination of the Plane to the Horizon.

Through C, draw AB, the Horizontal Line, parallel to the Bottom of the Picture, and through I, draw ID parallel to AB.

Make the Angle DIF equal to the given Angle X, and IF is the Intersection required.

Through C, the Center, draw GH parallel to IF; GH is the Vanishing Line.

Note. The Vanishing Line may be determined, by making the Angle ACG or BCH equal to the Inclination known; without the Intersection.

DEM. Let ID be supposed the Ground Line, and I the Intersecting Point of the common Intersection of the Ground Plane and the other Plane; which, seeing both Planes are perpendicular to the Picture, is also perpendicular to the Picture †.

† Cor. to 9. 11. El.

‡ Th. 11.

‖ 4. 1. El.

And, since I is the Intersecting Point, of a Line in the Plane, the Intersection of the Plane must necessarily pass through that Point ‡; and consequently, it will make the same Angle with the Ground Line and Horizontal Line, as the Planes make with each other (DIF, equal BLF ‖) equal to the given Angle X. (See Article 4th, of Planes and their Positions. Page 44.)

Therefore, IF is the Intersecting Line of that Plane.

But, the Vanishing Lines of all Planes, which are perpendicular to the Picture, pass through the Center of the Picture §. And, the Vanishing Line of every Plane is parallel to its Intersection. (Theo. 3.)

§ Theo. 6.

Therefore, GH is the Vanishing Line of a Plane perpendicular to the Picture, and inclined to the Horizon in the Angle HCB equal FID, equal X.

N. B. If F be the Intersecting Point of any other Line in that Plane, and EF be drawn parallel to the Horizontal Line; the Angle EFI equal FID (equal X) produces the same Intersection.

Also, if K be the Intersecting Point of any Line in another Plane, parallel to the former; then, AK, parallel to IF, is its Intersection.

For, all parallel Planes have parallel Intersections. But they have the same Vanishing Line (GH).

If *E*C, or EC, be equal to the Distance of the Picture; AK, or EF, may be considered as the Parallel of the Eye; and *E*C, or EC, (perpendicular to GH, or AB) is the Vertical Line.

Note. If the Angle of Inclination be greater or less, the process is the same. But, if the Angle be increased or enlarged to a Right one, the Plane is no longer inclined, but vertical; and, if the Intersecting Point, J, be in a Line, passing through C, perpendicular to AB, the Intersection and Vanishing Line are the same, viz. the Vertical Line EJ; which is the Vanishing Line of all vertical Planes that are perpendicular to the Picture. (See this Problem illustrated by moveable Planes; Fig. 15. No. 2.)

Fig. 15. No. 2. *IKLM* is the Original Plane, TU is its Intersection with the Ground Plane, or other horizontal Plane; *I* is its Intersecting Point, and *IN* is the Intersection of that Plane with the Picture; making the Angle *NIB* equal to the Angle PQR, of the Original Plane with the Ground Plane.

And, if IKLM (the Vanishing Plane of all Planes which are perpendicular to the Picture) be placed parallel to the Original Plane, *IKLM*, it will cut the Picture in ON, the Vanishing Line of *IKLM*, in that Position.

For, it passes through the Eye (at E) and the Direct Radial EC; consequently it is perpendicular to the Picture †, and passes through C its Center.

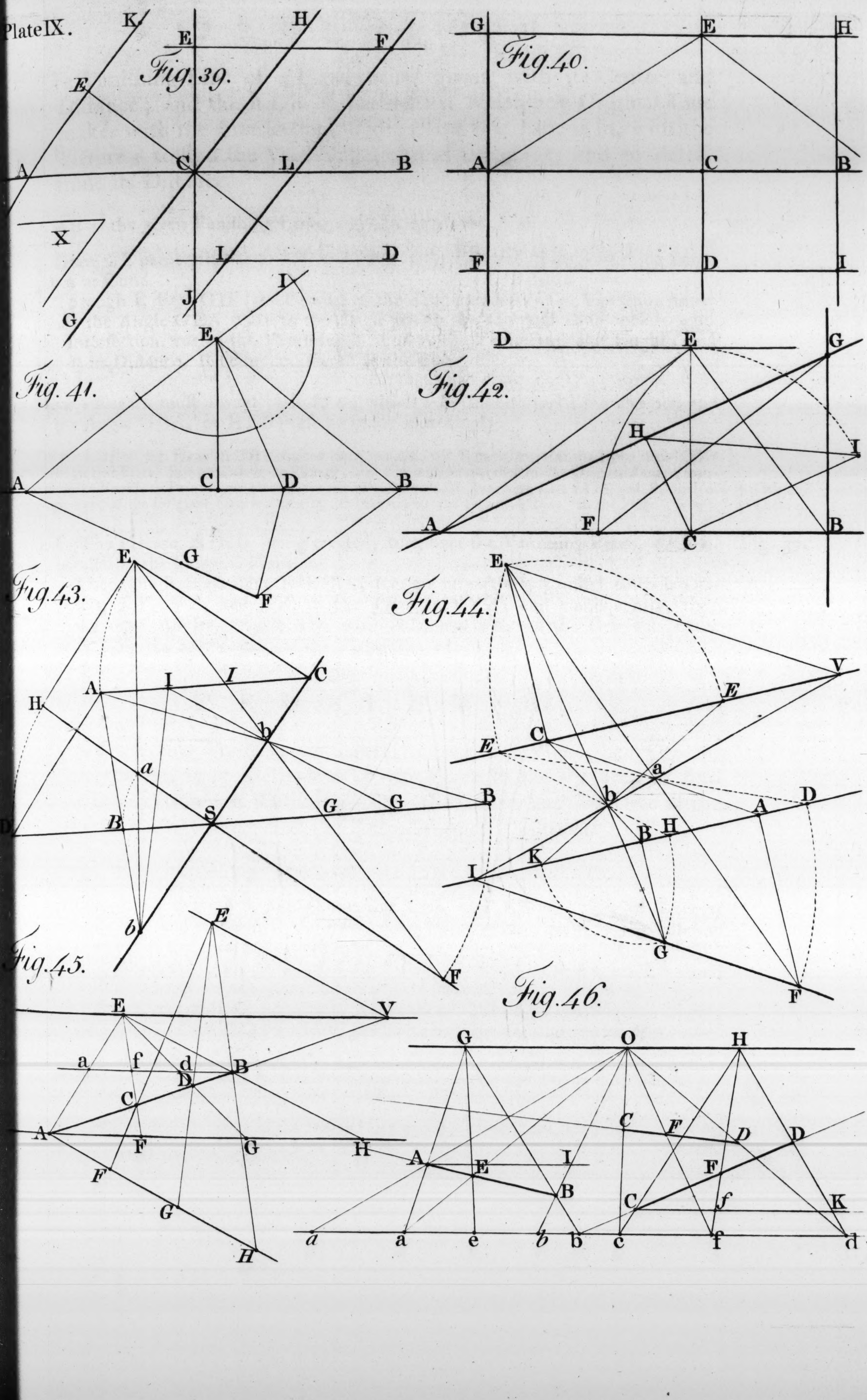
† 9. 11. El.

Hence; all Vanishing Lines, which pass through the Center of the Picture, have the same Center and Distance, viz. the Center and Distance of the Picture.

Note. If the Plane incline on the other Side of its Intersection TU, the Angle of its Inclination is made on the other side of the Vertical Line; to which, particular regard must always be had.

PROBLEM

Plate IX.



PROBLEM II.

The Vanishing Line of a Plane being given, with its Center and Distance; and the Angle of inclination which any Original Line makes with the Intersection, of the Plane that Line is in, with the Picture; to find the Vanishing Point of that Line, and to determine its Distance.

AB is the given Vanishing Line, and C is its Center. Fig. 40.

Draw CE perpendicular to AB, and equal to the Distance of the Vanishing Line, given or found.

Through E draw GH (the Parallel of the Eye) parallel to the Vanishing Line. Make the Angle GEA equal to the Angle which the Original Line makes with the Intersection, cuting the Vanishing Line in A the Vanishing Point sought; and EA is its Distance, E being considered as the Eye.

Note. Regard must always be had, on which side the Original Line inclines, and the Angle GEA, or HEB must be made accordingly.

DEM. Imagine the Plane AGHB turned over, on AB, till E coincides with the Eye; then is GH (in its true Place) still parallel to the Picture; and EA will be parallel to the Original Line, producing its Vanishing Point † and making the same Angle with the Vanishing Line (AB) and Parallel of the Eye, as the Original Line makes with the Intersection and Directing Line, of the Plane it is in ‖. † Def. 22. ‖ Theo. 9.

EX. The Picture, AIKB, being erected, turn over the Vanishing Plane, *IK*NL, parallel to the Original Plane. Fig. 37.

Then, the Radials EN, EO, EL, &c. are respectively parallel to their Originals, XY, YZ, and XZ, in the Triangle XYZ; and consequently, they make equal Angles, respectively, with NL, and *IK*, as the Originals make with AB, the Intersection of the Plane they are in.

PROBLEM III.

The Intersecting Point of any Line, in a vertical Plane, being given, and the Angle of Inclination of the Plane to the Picture, to find its Intersection and Vanishing Line; the Center and Distance of the Picture being given, and the Picture supposed vertical.

Let I be the intersecting Point given, and C the Center of the Picture. Fig. 40.

Through C, draw AB, the Horizontal Line, and ED the Vertical Line at right angles; and through I, the Intersecting Point given, draw IH, parallel to ED; which is the Intersection required.

Make CE equal to the Distance of the Picture; and make the Angle CEA equal to the Complement of the given Angle of Inclination; cuting AB in A.

Or, having drawn EG parallel to AB, make GEA equal to the given Angle; and through A, draw FG parallel to IH, which is the Vanishing Line required.

DEM. Because the Picture and the Original Plane are both Vertical, their Intersection is perpendicular to the Horizon ‡; and consequently, since I is the Intersecting Point of some Line in that Plane, IH perpendicular to FI, the Ground Line, is the Intersection; by Theorem 11th. ‡ Cor. to 9. 7. El.

And, because EAC (equal AEG) is equal to the Angle of Inclination, EA is the Radial, or Parallel of the common Intersection of the vertical Plane with horizontal Planes, producing its Vanishing Point. (By Prob. 2.) Therefore, FG, drawn through A, parallel to IH, or ED (i. e. perpendicular to FI) is the Vanishing Line.

For,

Plate IX. For, imagine the Picture FGHI turned up, vertical, and the Triangle AEC vertical to it, i. e. horizontal; representing a part of the Horizontal Plane; E is the Eye.

Then, a vertical Plane, passing through the Eye and the Line EA, will cut the Picture in FG, the Vanishing Line, which is parallel to IH, the Intersection; by Theorem 3rd.

For, they are the Sections of parallel Planes with the Picture.

N. B. If the Intersecting Point of any other Line, in the Plane, had been given, as B or H, the Intersection IH would be the same.

And consequently, if the Vanishing Point of that Line, or any other, as F, or G, in any vertical Plane, be given or found, the Vanishing Line is determined.

For, it passes through that Point (Th. 11.) perpendicular to the Ground Line, from its Position.

EXAMPLE, by the Apparatus.

Fig. 15. If the Planes ABGH and BFC were produced, they would cut the Picture MNOP, in *BG*, the Intersections of those Planes; which are parallel between themselves, because their common Section BG, is parallel to the Picture †; and, because BG is vertical, they are, also consequently, vertical, being parallel to BG respectively ‡.

† Theo. 7.

‡ 4. 7. El.

And, if a Plane RSTU be supposed to pass through E (the Eye of the Spectator) parallel to BFC, cuting the Picture MNOP, produced, in RU, it is the Vanishing Line of that Plane, parallel to the Intersection, *BG*.

Also, a Plane (STOP) parallel to ABGH, cuts both Pictures, in OP and *OP*, the Vanishing Lines of that Plane, on both; also parallel to RU or W, i. e. to *BG*.

When the Original Plane is inclined both to the Horizon and the Picture, having its common Section with horizontal Planes, parallel to the Picture; it is a similar Case to this; and by turning the Picture sideways, on IH, (Fig. 40.) 'tis the very same in every respect; as it is fully illustrated in Fig. 15. No. 3.

No. 3. EX. *ADFB* is the Original Plane; *AB*, its Intersection with the Ground Plane, is parallel to the Picture; i. e. to AB. Let it be raised up, making the Angle PQR, with the Horizon. AONB is the Picture, which being raised up vertical, the Plane *IKON*, passing through the Eye, at E, will be parallel to the original Plane, *ADFB*; and cuts the Picture in ON, its Vanishing Line, parallel to the Horizontal Line, LM.

And, if the Original Plane was produced, it would almost cut the Picture in a Line parallel to LM, or AB (i. e. to ON) below AB, the Ground Line.

But, if the Original Plane inclined towards the Picture, on the other Side of *AB*; then, its Vanishing Line would fall below the Horizontal Line, and the Intersection above it.

COR. Hence it is manifest, that if the Intersection of an Original Plane, in any Position whatever, be given, and the Inclination of that Plane to the Picture known, its Vanishing Line is determined as by this Problem; seeing that, the Vanishing Line of a Plane is always parallel to its Intersection with the Picture. Consequently, if the Intersection, be parallel either to the Horizontal or Vertical Line, the Vanishing Line sought is also parallel to them.

But, when the Intersection of the Original Plane is not given; and which, by reason of the great Distance of the Plane, from the Picture, or Inclination to it, cannot be had, nor its Position ascertained; then other Expedients are used, to find the Vanishing Lines of such Planes; viz. by finding the Vanishing Points of two Lines in the Original Plane. (See Prob. 5.)

N. B. The Center of every vertical Vanishing Line is the Point in which it is cut by the Horizontal Line; and the Center of every Vanishing Line, which is horizontal, is the Point in which it is cut by the Vertical Line (by the 4th Theorem.)

Fig. 40. The Distance of every Vanishing Line which does not pass through the Center of the Picture, is the Hypothenuse of a Right angled Triangle (as AEC) whose Base and Perpendicular are the Distance of the Picture, and the Distance, AC or EC, between the Center of the Picture and the Vanishing Line.

For, in respect of the Vanishing Line, FG, of vertical Planes, if CE be the Distance of the Picture, EA is the Distance of the Vanishing Line, and A is its Center.

But, if GH be a Vanishing Line of a Plane inclined to the Horizon, in the Angle GEA, and to the Picture, in AEC; having the same Distance AE, E is its Center, and AC is the Distance of the Picture.

PRO-

PROBLEM IV.

The Vanishing Line of a Plane, its Center and Distance, being given, and the Vanishing Point of some Line in that Plane; to find the Vanishing Point of other Lines, making a given Angle with that Line, whose Vanishing Point is given.

AB is the Vanishing Line, C is its Center, and A the given Vanishing Point. Fig. 41.

Draw CE perpendicular to AB, and equal to the Distance of the Vanishing Line, given or found.

Join AE; and make AED, or AEB, equal to the Angle, which the Original Lines make with each other, cuting the Vanishing Line in D or B, the Vanishing Point sought.

DEM. Imagine the Triangle AEB turned up, on AB, perpendicular to the Picture (if AB be considered as the Vanishing Line of a Plane perpendicular to the Picture) or, making the same Angle with the Picture, as a Plane passing through the Original Lines.

Then is AEB the Parallel of whatever Plane the Original Lines are in, producing its Vanishing Line AB‡; E coincides with the Eye, and EA, EB, &c. are respectively parallel to the Original Lines; seeing they pass through the Eye and the Vanishing Points of those Lines (Def. 22.) ‡ Def. 8.

But, the Radials of two Lines producing their Vanishing Points, make the same Angle, at the Eye, as the Original Lines make with each other †. Therefore, D, or B is the Vanishing Point required. † Theo. 9.

N. B. If the Angle AEB be obtuse, it is not the Angle of Inclination of the Lines. In which Case, regard ought particularly to be had to the Position of the Original Lines, in respect of each other, and of the Picture.

For, suppose FG to be the Representation of a Line, whose Vanishing Point is A; then, if FB be drawn, AFB represents an Angle equal to AEB; consequently, the obtuse Angle is towards the Picture. In which Case, let the Angle of Inclination be made on the other Side; that is, produce AE to I, and make IEB equal to the given Angle; for if the Angle AEB (i. e. AED) was made equal to the Angle of Inclination, the Point D (in that Case) would not be the Vanishing Point required.

PROBLEM V.

The Angle of the Inclination of a Plane to the Horizon, together with the Angle which its Intersection, with any horizontal Plane, makes with the Picture, being given, to find its Vanishing Line; the Intersection of the Plane not being given, nor its Position known. The Center and Distance of the Picture is given.

Let AB be the Horizontal Line, and C the Center of the Picture. Fig. 42.

Draw CE perpendicular to AB, and equal to the Distance of the Picture; and, through E, draw DE parallel to AB.

Draw EA, making the Angle DEA equal to the Angle which the Intersection of the inclined Plane (with horizontal Planes) makes with the Picture, cuting the Horizontal Line in A, the Vanishing Point of the common Intersection.

Draw EB, perpendicular to AE, cuting AB in B; and, through B, draw BG perpendicular to AB, indefinite.

Make BF equal BE; and make the Angle BFG equal to the Inclination of the Original Plane to the Horizon, cuting BG in G, and draw AG, the Vanishing Line sought.

This Problem, for finding the Vanishing Lines of Planes, casually inclined to the Horizon and to the Picture, is universal, and applicable in all Cases, when the Angles are determinable; as in the Roofs of Buildings, Pediments, &c. which, being frequently necessary in common Subjects, could not be dispensed

 with

Plate IX. Fig. 42. with here; otherwise, I should have omitted it till the last Section, which will treat more fully on such Subjects. I shall therefore reserve the Demonstration of it till then; where, every Case and Circumstance, respecting inclined Planes, will be fully demonstrated, and exemplified by moveable Planes.

N. B. The Center of the Vanishing Line A G is determined by drawing CH perpendicular to the Vanishing Line, cuting it in H, its Center †, nor does it differ, in that respect, from any other, except in Position; for, CH is the Vertical Line of the Original Plane.

† Th. 4.

And, if CI be drawn, parallel to the Vanishing Line, and equal to the Distance of the Picture, the Line IH, to the Center, is the Distance of the Vanishing Line AG; which, Distance, is as applicable to that Vanishing Line, as the Distance of the Picture to all Vanishing Lines which pass through its Center, i. e. of Planes perpendicular to the Picture.

Fig. 15. EX. In the Apparatus, VW, the Vanishing Line of the Plane of the Roof, HIFG, is determined by this Problem; V being the Vanishing Point of the common Section, GH, with a Horizontal Plane, DGHK, and EW, i. e. the Radial or Right Line from the Eye, parallel to GF determines W, in RU produced, the Vanishing Point of GF and HI, on the Picture MNOP.

For, *E* in the Horizontal Line, represents the Eye transposed to the Picture; *E*Y being equal to the Distance of the Eye from the Point Y; and Y*E*W is equal to the Angle D G F of the Inclination of the Roof to horizontal Planes.

Wherefore, since V is the Vanishing Point of one Line (GH or IF) in the inclined Plane, and W is the Vanishing Point of another Line (GF or HI) in the same Plane; consequently, VW (a Right Line drawn through those Points) is the Vanishing Line of the Plane HIFG (Th. 11. and Cor. 1.)

For, a Plane passing through the Eye, and those Vanishing Points, would be parallel to HIFG, and would cut the Picture in the Line VW; which is, therefore, the Vanishing Line of that Plane. Def. G.

SECTION IV.

The ELEMENTS of PERSPECTIVE.

HAVING, in the foregoing Section, shewn how to find and determine the Intersections and Vanishing Lines, of Planes, in all common Cases; and also the Vanishing Points of Lines, in any Plane whose Vanishing Line is given or found; by means of which Vanishing Points, all original Right Lines (not parallel to the Picture) have their indefinite Representations, on the Picture, truly and accurately described. In this Section I shall shew how to cut off certain portions, from the indefinite Representation, which are the perspective Representations of certain portions, or segments, of Lines in the Original Object.

I would advise the young Student to read over again, with strict attention, the conclusion of the second Section, Book II. P. 53 and 54.

Having well considered, that most regular Objects are bounded by Planes, and the bounds of Planes are Lines; it is evident, that to find the Representations of Lines, in all Positions, is to find the Representation of the Figure, or Object, bounded or circumscribed by those Lines. And, since the extremes of Lines are Points, it follows, that, if the two Extremes, of a Right Line, be found, the whole Line is determined; and, by finding sundry Points in curved Lines, the Representation of the Curve is determinable. Wherefore, the whole, of practical Perspective, consists in finding the Representation of a Point, any how situated.

But, since Points are the intersections of Lines, and, to find the Representation of a Point, in Perspective, it must, necessarily, be suppoſed in some Line; hence it follows, that, to determine Lines, in all Cases, is the whole sum and substance of Practical Perspective.

Now,

Now, Right Lines can have but three Positions, in respect of themselves and of the Picture, viz. they must be either parallel, perpendicular, or inclined; and, having learnt how to manage Lines in all these Cases, by the following Problems, there remains little more to be done; for, by constructing a number of Lines together, properly, an Object is formed.

By Theo. 12. the Indefinite Representation of a Right Line, not parallel to the Picture, is a Line drawn through its Intersecting and Vanishing Points. But, since the Intersecting Point is not always wanted (nor is it always attainable) if any Point in the indefinite Representation be determined, a Line drawn through that Point, to the Vanishing Point, is the same; for, it would, if produced, pass through the Intersecting Point of the Original Line.

In this Section, which contains the whole Substance of practical, rectilinear Perspective, I shall shew how the indefinite Representations of Lines, in all Positions, are determined, and then how to proportion them, in any given or known ratio to the Original, and afterwards, how to manage them, when the Vanishing Point is not within the limits of the Picture, by various Expedients.

Let the Reader take particular notice, that I shall, always (to save repetition) in the following Problems, suppose the Center of the Picture to be given, and its Distance known; except, in particular Cases, when it is otherwise expressed.

The Distance of the Picture (being determined) is applied, in Practice, by the same Scale of Proportion to which the Picture is delineated.

Let it also be observed, that I shall always (in the Diagrams) make use of the initial Letters of the following Terms, viz. C for the Center of the Picture, S for the Seat of a Point, &c. E for the Eye, in its first or principal place on the Picture, and *E* for its first transposed place, in any Vanishing Line, &c. and E^2 for the next transposition, &c. and V L for any particular Vanishing Line. But, seeing that the Ground Line, the Horizontal, and Vertical Lines never vary their Places, and are always stronger drawn than the operative Lines, I think it needless to particularize them otherwise.

PROBLEM VI.

How to find the Representation of a Point whose Seat on the Picture is given, and its Distance from the Picture known.

Let C be the Center of the Picture, and S the Seat of the Original Point. Fig. 43.

Draw a Right Line CS, through the Center of the Picture, and the given Seat, indefinitely beyond S. Draw CA, at pleasure; and SB parallel to CA.

Make AC equal to the distance of the Picture, and SB equal to the distance of the Original Point from its Seat. Draw AB, which will cut CS in b, the Point sought.

Or, if the Original Point be between the Eye and the Picture, make S*B* equal to its distance as before, and draw A*B*, which produce to the Picture, cuting it in *b*. Then is *b* the projected Representation, of the Point *B*, on the Picture.

Compleat the Parallelogram ACSD.

DEM. AC is equal to the Distance of the Picture, SB to the Distance of the Original Point from its Seat, and AC is parallel to SB (Con.) Consequently, the Triangles ACb, and bBS are similar. Wherefore, Sb : bC :: SB : AC; and, consequently, Sb : Sb + bC (i. e. SC) :: SB : SB + CA, equal SD (i. e. BD.) that is, Sb : SC :: SB : BD. Also S*b* : SC :: S*B* : *B*D. - Theo. 13.

Now, because SB is the Distance of the Original Point, from the Picture; and AC is equal to the Distance of the Picture; draw CE and SF, both perpendicular to CS, consequently parallel; make CE equal CA, and SF to SB, and draw EF; which will cut SC in the same Point, b.

For, the Triangles CEb, bFS are similar. Wherefore, Sb : bC :: SF : EC; i. e. as SB : AC.

Hence

Plate IX. Fig. 43. Hence it is evident, that the projection of the Point b does not in the least, depend on the situation of the Lines CA and SB in respect of CS, but on their parallelism and proportion to each other; wherefore, if the true Distances are not known, but only their Ratio, the Point b, will be projected the same. e. g.

Take CI at pleasure; and make SG to CI, as SB to AC, and draw IG; the Point b will be projected the same, on the Picture.

For, since SG : CI :: SB : CA, and Sb : bC :: SB : CA; consequently, Sb : bC :: SG : CI, and consequently, the Point b is the same.

Hence, may be seen the universality of the 13th Theorem. For, conceiving E to be the Eye, and EC the Direct Radial, i. e. the Distance of the Picture, and SF the Distance of the Original Point from its Seat, i. e. from the Picture, imagine the Triangle ECb to be turned up, on bC, till EC is perpendicular to the Picture, and suppose bFS turned back, on Sb, till SF is also perpendicular to the Picture, on the other Side; then is E in the true place of the Eye, and F is in the Place of the Original Point, and consequently, EF is a Visual Ray from the Eye to the Point; which it is evident will pass through the Picture, in the Point b; and, since Vision is conveyed in Right Lines to the Eye, the Point F will appear, on the Picture, at b; which is, therefore, the perspective Representation of the Point F. (See App.)

Again, because EC is parallel to SF and cuts the Picture in C, its Center, C is the Vanishing Point of SF; (Cor. 2. Def. L.) for, the Line SF is perpendicular to the Picture; and S is its intersecting Point; (Def. K.) wherefore, SC is its indefinite Representation †; and EF is a Visual Ray from the Eye to the Original Point; (Def. H.)

And, because EC is parallel to SF, and EF cuts them both, they are, therefore, all in the same Plane ‡, consequently, the Visual Ray, EF, will cut the Picture somewhere in SC, the Intersection of that Radial Plane with the Picture, and consequently in b; making Sb to bC, as SF (or SB) to EC (or AC) or, Sb to SC, as SF or SB, added to EC, or AC; i. e. to BD, or FH§; for, D is the Directing Point of BS, and H of FS.

† Theo. 12. Def. 25.

‡ Ax. 7.

§ Theo. 13.

Thus, the whole business of practical Perspective will be found (when well understood) to consist in finding the representation or projection of a Point on the Picture, any how situated; i. e. to determine that Point, in which a Visual Ray, from the Original Point to the Eye, would cut and pass through the Picture.

For, if the two Extremes of a Right Line are found, the whole Line is determined; and curved Lines can only be represented by finding the representations of several Points in the Original Curve, and joining them carefully, by hand. I would therefore advise the young Student to bestow the utmost attention on this and the following Problems, as they really contain the whole essence of Practical Perspective.

PROBLEM VII.

The Seats, on the Picture, of any two Points in an Original Line being given, and the Distance of the Points from the Picture; to find the Inclination of the Line to the Picture, its Intersecting and Vanishing Points; to draw its indefinite Representation, and to find the Representation of each Point.

Fig. 44. A and B are the Seats of the Points given, and C is the Center of the Picture.

Draw AB indefinite; draw AF and BG perpendicular to AB, and equal to the Distance of the Original Points from their Seats, respectively; and draw FG meeting AB in the Point I.

Then is I the Intersecting Point of the Line, in which the Original Points are situated; and AIF is the Angle of its Inclination to the Picture.

Through C, draw CV parallel to AI, indefinite; draw CE perpendicular to CV, and equal to the Distance of the Picture; and, draw EV parallel to IF, cuting CV in V, the Vanishing Point.

Draw IV, the indefinite Representation of the Original Line; and lastly, draw EF and EG, or AC and BC, cuting IV in a and b, the Representations of the Original Points, F and G.

DEM.

DEM. Because A and B are the Seats of the Original Points, AB is the Seat of the Line they are in, and consequently, it is the Intersection of a Plane passing through the Line perpendicular to the Picture. (See N. B. Art. 7. General Introduction, Page 47.)

Now, since AF and BG are perpendicular to AB, i. e. to the Picture, and measure the Distance of each Point, respectively, from its Seat, if the Triangle AFI was turned back, behind the Picture and perpendicular to it, F and G would be in the true places of the Original Points, in respect of the Picture, and FG would represent the Original Line; which being produced to the Picture would cut it in I, in the Line AB, produced.

For, AF and BG are parallel Lines, and consequently, all other Right Lines which cut them both are in the same Plane †.

Therefore, I is the Intersecting Point of the Line FG, and AIF is the Angle of its Inclination to the Picture, i. e. to its Seat, AB. † Ax. 7.

Again. Suppose the Triangle CEV turned up perpendicular to the Picture, it will be parallel to AFI; for CV is parallel to IA, and, AFI and CEV are both perpendicular to the same Plane.

Now CE is perpendicular to the Picture, and, since it is equal to the Distance, E is the true place of the Eye; and because EV is parallel to IF, and cuts the Picture in V, V is the Vanishing Point of IF; for they make equal Angles with the Picture; (EVC equal AIF; by Theorem 9th.)

But, I is the Intersecting Point of FG, and V is its Vanishing Point; wherefore, IV is its Indefinite Representation; by Theorem 12th.

And, EF, EG, are Visual Rays, from the Eye, E, on this Side, to the Points F and G, on the other side of the Picture, which would cut and pass through the Picture at a and b.

For, EV is parallel to IF, and they are both cut by IV and EF; wherefore, they are all in the same Plane ‡; and IV is the Section of that Plane with the Picture, because, the Points I and V are both in the Picture; the Line FG cuting it in I, and EV in V, (Def. K and L.) ‡ Ax. 7 & 8.

Now, since EF and EG are also in the same Plane, they must cut the Picture, somewhere, in the Intersection, IV, of the Plane they are in, consequently in a and b.

For, the Triangles VEa and aIF, VEb and bIG are similar; and therefore, Ia : aV :: IF : EV :: Fa : aE; and Ib : bV :: IG : EV; as by Theorem 13th.

2nd. AC and BC also determine the same Points, a and b.

For, AF and BG are Lines perpendicular to the Picture; therefore, the Center of the Picture is their Vanishing Point §; EC being parallel to them ‖; also, A and B are their Intersecting Points; therefore, AC and BC are the indefinite Representations of AF and BG; by Theorem 12th. § Cor. to Theo. 6.

But, the Representation of the common Section of two Lines, is the Point in which the Indefinite Representations cut each other. (Cor. 5. Theo. 11th.) ‖ Def. 22.

Therefore, a, the Point in which IV and AC cut each other, is the Representation of F; and b of G, in which the Lines BC and IV cut each other.

Also, because CE is parallel to AF, the Triangles CEa, aFA are similar; wherefore, Aa : aC :: Fa : aE :: AF : EC, i. e. :: Ia or IF : aV, or EV. Q. E. D.

Or, if the Distances of the Points F and G, from the Intersecting Point, are transfered to D and H; and the Distance of the Vanishing Point, EV, be set off, from V to *E*; *E*D and *E*H, being drawn, will give the same Points; that is, they will cut IV the same, in a and b.

For the Ratio of Ia or Ib, to aV or bV, is still as IF or IG, to EV; i. e. as ID or IH, to *E*V.

Also; if AC, or BC be considered as the indefinite Representation of a Line (BG) in which the Point G is situated; make BK equal BG, and C*E* (on the contrary side) equal CE; K*E* will determine the same Point b, &c.

N. B. The Seat of a Point, on any Plane, is the Point where a Perpendicular from the Original Point cuts the Plane, and consequently measures its Distance. See Ichnog. and Orthog. Sect. 2. Page 49.

The Distance of any Point, in an Original Object, from the Picture, may be obtained when the Intersecting Point, of the Line it is in, and its Distance from the Intersecting Point, cannot, for various reasons; and, since it is demonstrated (Theo. 13.) that the Distances of several Points in a Line respectively, from the Picture, are in the same Ratio as their Distances from the Intersecting Point of that Line; consequently, having their Seats on the Picture, and Distance from their Seats, respectively, their Representations, on the Picture, may be determined without the Intersecting Point.

If the Original Points are in a Plane which is perpendicular to the Picture the Seats of those Points are in the Intersection of that Plane with the Picture.

Wherefore, if F and G are two Points in a Line on the Ground Plane, the Seats of those Points are in the Ground Line (AB) where the Perpendiculars FA and GB cut it; A and B are the Seats; and their Distances AF to BG, are as the Distance of the Point F is to the Distance of G, from the Intersecting Point of the Line FG, where it would meet AB produced. Fig. 37. No. 3.

Pl. IX. Fig. 44. COR. AI being the Intersection of the Plane in which the Line FG is situated, VC is its Vanishing Line (Theo. 3. and 11.) and because it passes through C, the Center of the Picture, it is, consequently, the Vanishing Line of a Plane perpendicular to the Picture (Theo. 6.) whose Distance is EC.

COR. 2. If the Intersection, (IA) and Vanishing Line (CV) of any Plane, with its Center, (C) and Distance, (EC) be given, and any Line (FG) in the Original Plane; its Intersecting Point (I) is found by producing the Line; and its Vanishing Point, (V) by drawing EV parallel to FG; or, by making the Angle EVC equal to AIF.

PROBLEM VIII.

The Representation of a Line and its Vanishing Point being given or found, to divide the Line in any known Proportion.

AB is the Line given, and V is its Vanishing Point.

Draw AH, at pleasure, and EV parallel to AH.

Fig. 45. Make AF, FG, and GH in the Ratio, or known Proportion, of the Original.

Draw EH, through B, cuting VE, in E; and draw EF, and EG, cuting AB in C and D.

Then is AB divided, in C and D, perspectively, as AH is divided; in F and G.

DEM. If VE be considered as the Vanishing Line of a Plane in which the Original of AB is situated, AH may be considered as its Intersection; and EF, EG and EH represent parallel Lines †; for they are all in the same Plane, whose Vanishing Line is EV, and E is their Vanishing Point.

Wherefore, since ABH is a Triangle, and CF and DG are supposed parallel to BH; consequently, AB and AH are divided in the same Ratio, in the Points C, D, and F, G ‡.

† Cor. to Theo. 5.

‡ 2. 6. El.

N. B. If aB had been drawn through the other Extreme, B, instead of AH, and aB divided, in the same Ratio as AH, that is, in the given Proportion; Aa being drawn cuting EV in E, and Ef, Ed, produced, will cut AB in the same Points C and D.

Any other Line, *AH*, being divided in the same Ratio as AH, and *E*V being drawn parallel to *AH*; *EH* drawn, through B, and *EF*, *EG* also drawn, will give the same Points C and D, which may be demonstrated in the same manner.

By this Problem, any Right Line in any Object sketched by hand and depending on the Eye, may be divided, with certainty, in the Proportion of the Original, being known, no regard being had to what Plane the Line is in, nor its Vanishing Line.

By the same means, a Line drawn in Perspective may be bisected, as AD, or CB, by making AF, FG, or FG, GH equal; consequently, if the Parts AF, FG, and GH, are all equal, AB is trisected, perspectively.

COR. If AV be the indefinite Representation of a Line, and AC represents some certain portion of the Original; and it is required to cut off another Portion, as CD or CB, in a known Ratio to that represented by AC.

Draw AH, and VE parallel to it, at discretion; assume any Point, E, in EV, and draw EF, through C, cuting AH in F.

Make FG, or FH, to AF in the known proportion of the Original, and draw EG, or EH; which will cut off CD, or CB, in the proportion required.

COR. 2. If CA represents a certain portion, and it is required to cut off, from any other Point, as D, a part, DB, which shall be to AC in a certain Ratio.

Proceed as before, and, from any Point E in EV, draw EF, EG through C and D. Make GH to AF in the known Proportion, and draw EH cuting AV in B, the Point sought.

PRO-

PROBLEM IX.

The Vaniſhing Line of a Plane being drawn, and the Repreſentations of two Lines in that Plane; if one of the Lines be divided any how, how to divide the other in the ſame Ratio.

Let AB and CD be the given Repreſentations, of which, let AB be divided, any how, in E; it is required to divide CD in the ſame Proportion. Fig. 46.
GH is the Vaniſhing Line of the Plane they are in.

At any Diſtance, at diſcretion, draw ad parallel to GH; and, from any Point, G, in the Vaniſhing Line, draw GA, GE, and GB, and produce them till they cut ad, in a, e, and b.

From the ſame Point, G, or any other, in the Vaniſhing Line, as H, draw Hc, Hd, through C and D.

Divide cd, in f, as ab is divided, in e; i. e. make cf to cd, as ae to ab †. † Prob. 32. Geom.

Draw Hf, which will divide CD, in F, in the ſame Ratio as AB is divided

This Problem may be performed as well by one Point, O, as by two; if the Line, *CD*, be ſo ſituated, as to come within compaſs. From which conſtruction it is evident, that AB and *CD* repreſent equal Lines, as well as being equally divided, *ab* and cd being equal, and the two Lines having the ſame Vaniſhing Point.

For, they repreſent parallel Lines, and they are both cut by parallel Lines, *a*O, *b*O, cO, &c. at equal Diſtances, *ab* equal cd, &c.

Or, it may be done by drawing two Lines, AI and CK, which are parallel to GH, through either Extremes, A and C, of the given Lines; by which conſtruction, the Demonſtration evidently appears from the laſt; ſeeing that, AI, and ab, or *ab*; CK and cd are all divided in the ſame Ratio.

After the ſame manner, CD may be divided, perſpectively, into as many parts as AB is divided into, and in the ſame ratio to each other.

The utility of this Problem is much of the ſame nature as the other; the former being, how to divide a Line, perſpectively, in the proportion of the Original; and this ſhews how, to find the Proportion of the Original from the perſpective Repreſentation; by which means, any other Line may be divided the ſame.

In the former, the Vaniſhing Line was not neceſſary, only the Vaniſhing Point of the Line, which may be in any Plane whoſe Vaniſhing Line would paſs through V, its Vaniſhing Point, as VE or V*E*. But, in this Caſe (the Lines having different Vaniſhing Points) the Vaniſhing Line, of the Plane they are in, muſt be had; and, it muſt alſo be obſerved, that both Lines are in the ſame Plane, or the Operation cannot be performed by one Vaniſhing Line.

PROBLEM X.

The Vaniſhing Line of a Plane being given, and the Repreſentations of two Lines in that Plane, having different Vaniſhing Points, to cut off, from any Point in one of them, a portion equal to that repreſented by the other, or in any known Ratio.

The different poſitions or ſituations of the Lines, to each other, may make this Operation appear very different; for which reaſon I ſhall give it variouſly, of which, the firſt is according to Brook Taylor.

Let AB be a given Repreſentation of a Line, and FG the indefinite Repreſentation of another Line. NM is the given Vaniſhing Line, and C its Centre. Fig. 47.

It is required to cut off (from the Point F) a portion, which ſhall be to that repreſented by AB in a known Ratio.

Pro-

Plate X. Fig. 47. Produce AB to its Vanishing Point, D; and, through F, draw AH, to the Vanishing Line.

Draw CE perpendicular to the Vanishing Line and equal to its Distance, and join ED and EG.

Make EK and EL in the Ratio required; i. e. make EK to EL as the Original Line, represented by AB, is to the other; and draw EM, or EN, parallel to KL.

Draw BH and FD, intersecting in I; draw IM, or NI, cuting FG, in O or P.

Then, FO, or FP, represents a Line having that Proportion, to the Line which AB represents, as EL to EK.

DEM. If NM be considered as the Vanishing Line of a Plane perpendicular to the Picture; imagine the Triangle NEM turned up perpendicular to the Picture; or, if the Plane in which the Original Lines are situated, be inclined to the Picture, let NEM be supposed parallel to it; then, E is in the true place of the Eye.

Now, ED and EG are the Radials of AD and FG, producing their Vanishing Points, consequently they are parallel to them, respectively;† and make the same Angle DEG, at the Eye, as the Original Lines make with each other. (Cor. 1. Theo. 8.)

† Def. 22.

And, because EM is parallel to KL, D, G, and M are the Vanishing Points of the three Sides of a Triangle, EKL.

But, the three Lines, FI, FO, and IO, vanish in those Points, respectively; consequently, FIO represents a Triangle similar to EKL; the Angle DFG represents the Angle DEG (Prob. 4.) and consequently, IM represents a Line parallel to EM; i. e. to KL.

Therefore, the other two sides, FI and FO, or FP, have that Proportion to each other, perspectively, as EL to EK.

But, because AB and FI, AF and BI have the same Vanishing Points, D and H, respectively, ABIF represents a Parallelogram; consequently FI represents a Line equal to the Original of AB;‡ and therefore, FO, or FP, represents a Line which has that Proportion to the Original of AB, as EK to EL.

‡ 15. 1. El.

N. B. If the Situation of the Line AB was such, that a Right Line, joining A and F, was parallel to the Vanishing Line NM, then BI must be drawn parallel also.

But, if ab, the given Line, be so situated, that the Line aF is so much inclined to the Vanishing Line as not to reach it within the compass of the Picture, take any Point, J, in the Vanishing Line, and draw Ja, Jb, indefinite; draw AD, at discretion, cuting them in A and B, and proceed as before.

For, AB and ab having the same Vanishing Point, D, also, Aa and Bb have the same Vanishing Point J, abBA represents a Parallelogram; and consequently, AB, ab, represent equal Lines.

Fig. 48. Case 2nd. Let the given Lines, AB and FP, be so situated as to cross each other; and, it is required to cut off, from the Point I, of their Intersection (or any other) a part IF or IP equal, or in any Ratio to the Original of AB.

Draw Ac parallel to the Vanishing Line, MD, indefinite; and, from the Point H, or any other, at discretion, in the Vanishing Line, draw HI and HB, cuting Ac in c and b; then, Ab is to bc in the Ratio of the Original of AB to BI.

Make ac equal to Ab and draw aH, cuting AB in d; dI represents the same measure as AB (Prob. 8.) viz. ac equal to Ab.

Produce AB to its Vanishing Point, D, and FI, to H, if it be necessary.

C being the Center of the Vanishing Line, draw CE perpendicular to HD, and equal to its Distance; and draw ED, EH.

Make EK, EL equal, or in the Ratio required, and join KL; to which, draw EG parallel.

Draw Gd, cuting IF in F; then, if EK be equal EL, IF represents a Line equal to the Original of dI, that is, of AB, as it was required; or, whatever Ratio EK has to EL, FI represents the same to AB.

Again. Draw mn, through I, parallel to Ac, cuting aH in m.

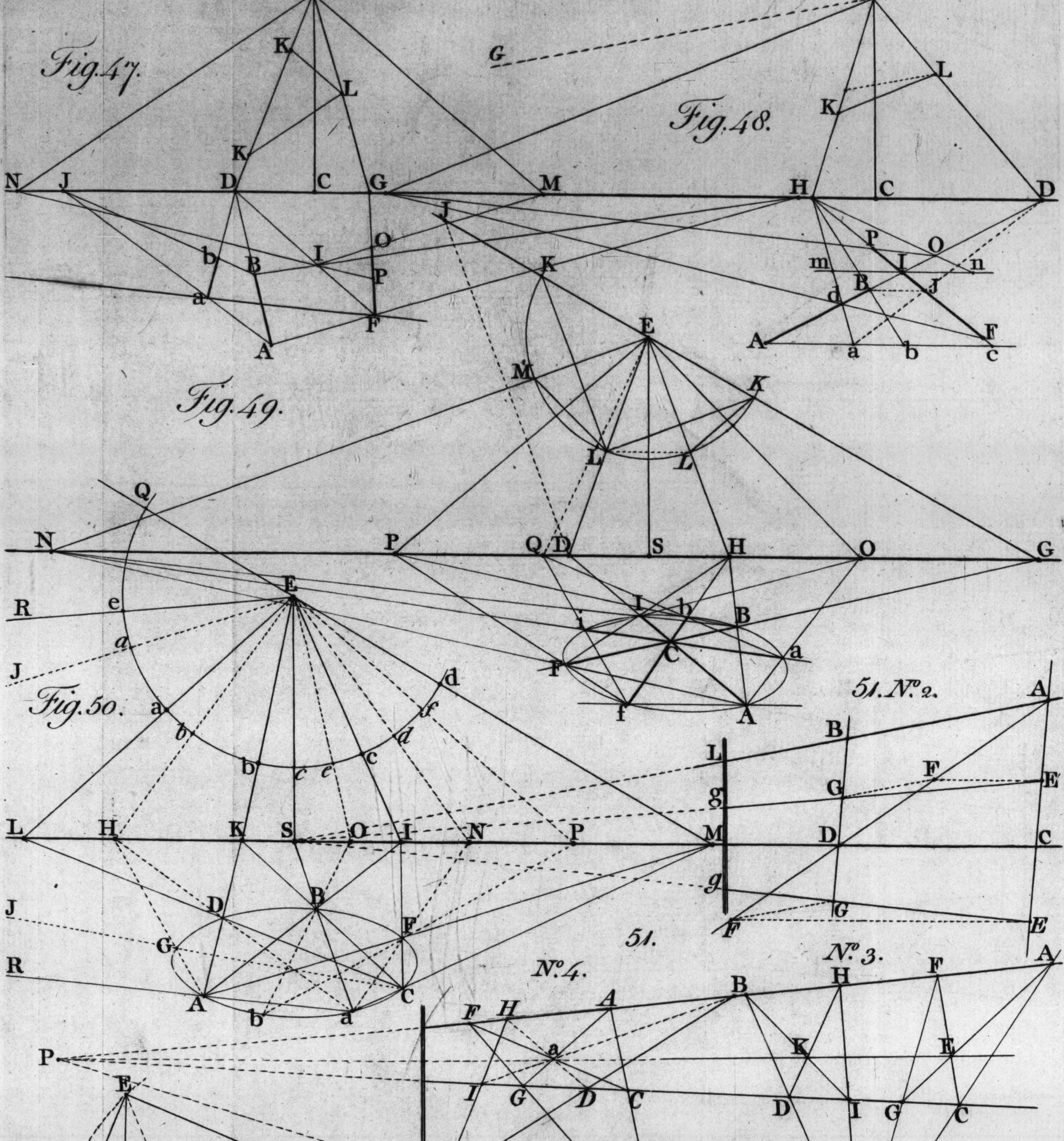
Make In equal Im; draw nH cuting AD in O; and draw OG, cuting FH in P.

IP represents a Line equal to IF, equal AB; i. e. to their Originals; or in the Ratio of EK to EL, whether equal or otherwise.

For, because am and FI have the same Vanishing Point, H, they represent parallel Lines; and, because mIn is parallel to ac, and to the Vanishing Line HD, amIc represents a Parallelogram; consequently, the Originals of mI and Ic are equal.

But

Plate X.



But, I n was made equal I m, and n H represents a Line parallel to d H; consequently, I O and I d represent equal Lines. (See Problem 8th.)

And, since OP and F d have the same Vanishing Point, G, they also represent parallel Lines; and consequently, IOP, I d F represent equal Angles †; wherefore, they are similar Triangles, for the Angles PIO, d I F, are equal‡. † 4. 1. El. ‡ 2. 1. El.

But, they also represent congruous Triangles, for IP : IF :: IO : Id :: PO : dF; and IO represents a Line equal to I d; consequently, IP represents a Line equal to IF, and PO to d F.

If it had been required to cut off, from the Point J, in FH, a portion, in a certain Ratio to AB; draw BJ.

If BJ be parallel to HD, draw DJ cuting AB in a; then J a represents an equal Line as AB.

But, if BJ be not parallel to the Vanishing Line, produce it to its Vanishing Point, and proceed as by Fig. 47, using J a for AB, as I d for AB, before.

N. B. If KL was parallel to the Vanishing Line, then PO and d F would also be parallel; for EG would be parallel, and consequently, could not produce a Vanishing Point.

Suppose it was required to cut off, from the Point a (Fig. 47.) in the Line aF, a part, equal or in any Ratio to the Original of AB.

Draw A a till it cuts the Vanishing Line in J, and draw JB; also draw aD to the Vanishing Point of AB. Then a b represents an equal measure as AB, and aF may be cut in any Ratio to a b, or equal to it, as in the next Case; which is the 3d Example, Prop: 16, of Brook Taylor, mentioned in the Preface to this Book.

Case 3d, Let AC or CB be the given Line. It is required to cut off from the Point C, a part, which represents an equal measure as the other. Fig. 49.

D and G are the Vanishing Points of the two Lines, and E is the Eye.

Draw ED and EG, and bisect the Angle DEG, by the Line EH.

Draw AH, which will cut CG in the Point B, as required.

This Case, or this position of the Lines, is but another Example of this Problem, which Brook Taylor calls finding the Representation of a Circle from the representation of one Radius given. It is certain that the whole Circle may be compleated, from a Radius given, though not by this Case only, but by both, as I shall exemplify.

Let AD and FG be drawn indefinite; C, their Intersecting Point, is the representation of the Center, and CB is the Radius given, in the Position by Brook Taylor.

He says, " make CA to represent a Line equal to that represented by CB (by the 15th)" viz. by the first Case of this; " i. e. bisect the Angle EOD;" and, to make it still less intelligible, in his Diagram, the Angle DEG (i. e. EOD, in his Treatise) is not bisected, nor trisected, but is cut very nearly in the Ratio of 2 to 5, which negligence, in a Person of his sagacity, is, to me surprizing.

It is demonstrated in the 3d of the 6th of El. that, if any Side of a Triangle be divided in the Ratio of the other two Sides, a Right Line joining the opposite Angle and the point of Section will bisect that Angle. But, there may, very justly, be an exception to this; for, it is necessary that the greater Segment be contiguous to the greater Side, and consequently the lesser Segment to the least Side.

In the Triangle DEG, if DG be divided, in H, in the Ratio of DE to EG, then, EH bisects the Angle DEG; because GH, the greater Segment, is contiguous to the greater Side, EG; which, it is obvious, could not be otherwise.

In forming the Construction for proof of the Assertion, we are told to produce either Side, as GE, aud make EJ equal to ED: and join JD; which is proved to be parallel to EH.

For, because ED = EJ the Angle EJD = EDJ (9. 1.) and, because DEG is equal to them both (10. 1.) and DEH = HEG (Con.) JDE = DEH (Ax. 2.) therefore, EH is parallel to JD (4. 1.)

But, JE is the Radial of CB, produced, and ED of AC; and EJ is made equal to ED; also, EH is a Right Line from the Eye, parallel to JD (i. e. to KL, as in the former Case) producing the Vanishing Point H, of the Line AB (i. e. of IO and Fd, in the two former Cases) which cuts off CA and CB, representing equal portions of those Lines.

Hence may be seen the affinity between the two Cases, in bisecting the Angle DEG.

Plate X. Fig. 49. For EJ=ED, i. e. EK ~~to~~ EL, and EH is parallel to KL; consequently, ACB represents a Triangle similar to KEL, or JED; and consequently, D, G, and H are the Vanishing Points of the three Sides AC, CB and AB; which is Isosceles, by Construction.

And, since DCG represents an Angle equal to DEG (Prob. 4.) consequently, ACG represents an equal Angle to DEJ†; therefore, the Sides, containing the external Angle DEJ, are, in this Case, made in the ratio (EK to EL) of the Originals, which AC and CB represent; and since they are equal, consequently, AC and CB represent equal Lines.

† 1. 1. El.

Now, CA and CB represent equal Lines, from the same Point C; wherefore, if C be considered as the Center of a Circle, in Perspective, CA and CB are Radii of the Circle; and this is all that Brook Taylor has done towards finding the whole Representation, which is far from being sufficient.

In BC produced, make CF represent a part equal to what CB represents, and CI to CA, or CB; then, BF and AI represent Diameters of the Circle.

Proceed as in Case 1st. by making EK equal to EL, and draw EN parallel to KL; draw AN and BN, cuting BF and AD in F and I, the Points sought.

Or, having drawn AN, only, cuting BF in F; draw FH, cuting AD in I.

The four Points, A, B, I, and F, are all in the Circumference of a Circle, whose Center is C; for, CA, CB, &c. represent equal Lines.

But these four Points are not sufficient for compleating a Circle in Perspective.

Draw HC and NC indefinite, and cut off, from the Center (C) Ca, Cb, Ci, and Cf, representing also equal measures, to those represented by CA, &c.

Make EM equal to EL; join ML, and draw EO parallel to ML.

Also, make EL equal EK; and draw EP parallel to KL.

Draw AO, OI, BP, and PF, cuting the indefinite Lines in a, i, b, and f.

The Angle NEH being bisected, by EQ, shews the Affinity to the first Case.

Through the eight Points, A, a, B, b, I, i, F, and f, an Ellipsis may be described, which will be the representation of a Circle in Perspective; from the given representation of one Radius, AC or CB.

PROBLEM XI.

From three Points given, in the Circumference of a Circle, to find the Representation of the Circle; having the Vanishing Line of the Plane the Circle is in, and the place of the Eye.

Fig. 50. A, B, and C are the three given Points, LM is the Vanishing Line, and E is the place of the Eye, ES is the Distance of the Vanishing Line.

Draw AB and CB, cuting the Vanishing Line in I and K; draw EI and EK.

Make the Angles KEL and IEM each equal IEK, producing the Points L and M, in the Vanishing Line.

Draw AK and AM, CL and CI, intersecting in D and F; the Points D and F are in the Circumference, which passes through A, B, and C.

DEM. Because the Angles KEL, KEI, and IEM are equal (Con.) and E is the Eye, the Angles LDK, KBI, and IFM represent equal Angles (Prob. 4.) and consequently ADC, ABC, and AFC also represent equal Angles‡.

‡ 2. 1. El.

Therefore, those Angles touch the Circumference §; for they are in the same Segment, or stand on the same Ark, AbC.

§ 10. 3. El.

Brook Taylor has made a very lame affair of this elegant Problem; notwithstanding his Principles are the same. By reason of the short Distance he has taken for the Eye, and the prodigious Dimensions of the Circle, it is the most distorted and preposterous Diagram in the whole Book. He finds no more than one Point and leaves his Readers to find out the rest. One thing I am much surprized at; he says, "make the Angle dOe equal to DOE; or, having made an Instrument, containing

the Angle DOE, turn it round the Center till it comes into the Position dOe," &c. which, is so ungeometrical, that I could scarce conceive it to be the expression of so great a Man.

Now, here are five Points, A, D, B, F, and C, in the Circumference; but they are not sufficient for ascertaining the true Curve of the Ellipsis.

Therefore, draw AH at pleasure, cuting the Vanishing Line, in H.

Make the Angle HEJ equal IEK, and draw CJ, cuting AH in G, which is another Point in the Circumference of a Circle.

Thus may as many Points be found in the Circumference as are necessary to describe the whole Curve; which, passing through all the Points, A, G, D, B, &c. will be an Ellipsis; for, it is the Representation of a Circle in Perspective, seeing that those Points are all in its Circumference.

If there be too much Space between any two Points, another Point may readily be found; as a, or b, between A and C.

Draw aB, or bB, at pleasure, cuting the Vanishing Line in S, or O. Join SE, or OE, and make the Angles SEH and SEN, or OEP, equal to IEK, cuting the Vanishing Line in H, N, or P, respectively.

Draw HD, or NF, cuting aB in a; or PF, cuting bB in b; which Points, a and b, are also in the Circumference.

For, because AD and CB have the same Vanishing Point, K; also, AB and CF having the same Vanishing Point I, the Originals of the Arks, DB and DF, are each equal AC †; for AD and CB represent parallel Lines‡; consequently, BaD or F, and BAD or F, or BbF, represent equal Angles§, each being equal to the Original of ABC; i. e. IEK.

† Cor. to 10. 3. El.
‡ Cor. 1. Theo. 5.
§ Cor. 2. 9. 3. El.

Otherwise; draw MC, indefinite; produce ME, and make the Angle QER equal to IEK; ER, being produced, will cut the Vanishing Line, somewhere, if it be not parallel to it.

If ER be parallel to the Vanishing Line, draw Aa also parallel; or to the Point R, in which ER would cut the Vanishing Line, cuting MC in a.

For, the Angle QER + REM = two Right Angles†. And, the Angle MaR (i. e. AaC) represents the obtuse Angle MER; and, ABC represents an Angle equal to QER.
But, the opposite Angles of every Quadrilateral, inscribed in a Circle, are equal to two Right Angles ‡. Consequently, AaC represents an Angle in the opposite Segment; for, AaC and ABC are opposite Angles, in the Quadrilateral ABCa.

† 1. 1. El.
‡ 11. 3.

There is not, perhaps, in the whole Book, a more elegant Problem than this, which induced me to give it a place, and to perfect it. Its utility is not so great; yet it may frequently be applied, by those Artists who do not care to be confined strictly to the Rules of Perspective.

For, having obtained three or more Points, the whole Circle may readily and accurately be determined; if they know the Distance of the Picture, and the Vanishing Line of the Plane of the Circle.

Having had occasion, in this Problem, for a Vanishing Point (R) which was not within the compass of the Picture, the next Problem shews how to determine the Distance of such Vanishing Points from the Center of the Picture, or Vanishing Line, and also from the Eye.

PROBLEM XII.

The Vanishing Line of a Plane being given, and the place of the Eye, with a Line, from the Eye, inclining to the Vanishing Line; to find the Center of the Vanishing Line, and to determine the Distance of that Point, in which the inclined Line would cut the Vanishing Line, from the Center, and from the Eye.

AB is the Vanishing Line given, and ED is the inclined Line from the Eye, at E. Fig. 51.

Db is considered as the bounds, or limits of the Picture, and B is the Point in which ED, produced, would cut the Vanishing Line, AF, produced.

Draw

Plate X. Fig. 51. Draw EC perpendicular to the Vanishing Line, cuting it in C, its Center.

Take CG any part of CE, a half, a third, a fourth, as ED is less or more inclined to AB, and draw GF parallel to ED.

† 2. and 4. of 6. El. Then, as CG : CE :: CF : CB†; that is, if CG be a third part of CE then CF is a third part of CB, and GF of EB.

Or, draw EA perpendicular to ED, cuting the Vanishing Line in A.

Then, find a third Proportional to AC and CE (Prob. 31. Geo.) it will be CB.

For, AEB is a Right Angle (by Construction) and EC is perpendicular to AB;

‡ 7. 6. El. therefore, as AC : CE :: CE : CB‡;

Also as AC : AE :: CE : EB; i. e. EB is a fourth Proportional to AC, CE, & AE.

In Numbers, they are thus determined, by a Scale of equal Parts.

First. Take AC and CE by the Scale; square CE, and divide the Product by

§ 9. 6. El. AC, which, will give CB§; for AC, CE, and CB are three Proportionals.

Or, if the Triangle CFG be used, multiply CF by CE, and divide the Product

† same. by CG; the Quotient will be CB†. For, CG, CF, CE, and CB are four Proportionals.

2nd. Since AC : CE :: AE : EB, consequently they are four Proportionals. Wherefore, multiply CE into AE, and divide the Product by AC, the Quotient will be EB, the Distance of the Vanishing Point, B, from the Eye.

If the Distance of the Picture, CE, be known, and the Angle of the inclination to the Picture, of one Side of a right angled Object, be determined; the Vanishing Points, and their Distances, are determinable.

Let AB be the Horizontal vanishing Line, and C the Center of the Picture.

Draw CE perpendicular to AB, and equal to the Distance given.

Make the Angle CEA equal to the Angle given (or to its Complement) cuting

‡ Def. 22. the Vanishing Line in A, the Vanishing Point of one Side‡; from which all the rest are determinable.

Or, if the Distance, CE, be so great, that it cannot be laid down on the Picture (as is frequently the Case) take CG half, a third, a fourth, or any portion of CE, and proceed as before; making the Angle CGH equal to CEA, the given Angle. Then will CH be half, a third, &c. of AC, or whatever portion CG

§ 4. 6. El. was taken of CE§; by which means, the distance of the Vanishing Point A, from the Center, C, is ascertained.

Make HGF a Right Angle, i. e. draw GF perpendicular to GH, cuting the Vanishing Line in F; then will CF be also half, a third, &c. of CB, the distance of the other Vanishing Point, B, from the Center C. Also, GF will be the same portion of EB.

Thus may the real Distances, be found, and the place of the Eye transposed to the Picture, as it will be exemplified in Practice, when their real places cannot be had thereon; all which may be found arithmetically, as follows.

Let the Distance of the Picture, CE, be given, equal 6,5 (Feet, or what you please) and let the Vanishing Point A be determined as above, or at discretion, on the Picture; viz. AC equal 2,6, from which all the rest may be determined.

First; to find the Distance of the Vanishing Point, A, from the Eye.

Square AC and CE, which being added together, the square Root of that Pro-

† Prob. 3. 7. 6. El. duct will be AE†; for AE square is equal to AC added to CE square‡.

‡ 20. 1. Secondly; to find the Distance of the other Vanishing Point B, from the Center.

§ Prob. 3. 7. 6. El. Square CE, and divide that Product by AC, the Quotient will be CB§.

For, AC : CE :: CE : CB. Also, AC : AE :: AE : AB.

Thirdly; to find the Distance of the Point B from the Eye.

Having obtained AE, by the first, multiply CE by AE, and divide that Product by AC, the Quotient will be EB; for AC : CE :: AE : EB.

Th

The Distance of each Vanishing Point, A and B, from the Eye E, are as necessary to be had as their places on the Picture, which I shall exemplify and explain in its proper place.

First. AC and CE being given, to find AE; the Distance of A.

Square AC, viz. 2,6;	also, square CE, viz. 6,5	AE=7
multiplied by 2,6	multiplied by 6,5	7
15,6	325	squared 49
52	390	
67,6 Square of AC.	42,25	
	square of AC added 6,76	
	sum of both)49,01(7, sq. Root; =AE.	

2ndly. To find CB; AC and CE being given.

Square CE; 6,5
6,5

divide by AC=2,6)42,25(16,25=CB
. . . .

3dly. To find EB.

Multiply CE=6,5
by AE=7,

divide by AC=2,6)45,5(17,5=EB.
. . .

Or; having found CB (by the 2nd) add the two Squares, of CE and CB, into one Sum; the square Root, of which, will be EB. (By 20. 1. El.)

When the two Vanishing Points, A and B, are known, to find the Distance of the Picture, CE; the Center C being given.

Multiply CB by AC, and extract the square Root of that Product, it will give CE the Distance of the Picture; by Prob. 4. 7. 6. El.

To find the Distance, CE, geometrically; A and B being given, the vanishing Points of Lines at right angles with each other, and C, the Center.

Draw CE, perpendicular to AB; and, having bisected AB, on F (the point of bisection) with the Radius AF, describe the Ark AE, cuting CE in E. CE is the Distance required, of the Picture, or of the Vanishing Line, AB.

For, AC : CE :: CE : CB; consequently, AEB is a Right Angle; by 7. 6. El.

PROBLEM XIII.

To draw a Line to a Vanishing Point which is not on the Picture,* its Distance from the Center being given, or some Line tending to the Point; the Vanishing Line of the Plane it is in, being also given.

Let AB be the given Vanishing Line, and C its Center.

Let I be the Intersecting Point of some Line; or the Representation of any other Point in the Line, given or found.

It is required to draw a Right Line, from the Point I, tending to B, which is out of the Picture; by the known Distance of the Vanishing Point B, from C, the Center of the Vanishing Line. D b is the limits of the Picture. Fig. 51. No. 1.

* By the Vanishing Point not being on the Picture is meant, only, that it does not fall within the prescribed Bounds or Limits. But the Picture may be imagined to be produced, as occasion may require, so that the Vanishing Points are always supposed to be on the Plane of the Picture.

Plate X. Fig. 51. No. 1. Take CF equal half, a third, or any other equal part of CB.

Draw CI, and FJ parallel to CI. Make FJ : CI :: FB : CB; i. e. make JF equal half, two thirds, or the same Complement of IC, as FB is of CB.

Draw IJ which will tend to the Point B.

Or; make CK : CI :: CF : CB; i. e. if CF be a third part of CB, make CK one third of CI. Join KF, and draw IJ parallel to KF.

DEM. First. In the Triangle CBI, because JF is parallel to IC, CF : FB :: IJ : JB. - - 2. 6. El.
Consequently, CF : CF+FB (equal CB) :: IJ : IJ+JB (equal IB) i. e. as JF is to IC.
Therefore, IJB is a Right Line; for CBI is a right lined Triangle. - - 2 and 4. 6. El.
2nd. In the Triangle ICB, because CK : KI :: CF : FB (Con.) KF is parallel to IB. Q. E. D.

2ndly; by a Right Line tending to the Point B; the Distance not being known.

Let IJ be a Line tending to the Point B, in the Vanishing Line, AB.
It is required to draw, from the Point a, a Line tending to the same Point.

Draw aC and aI, at pleasure, making an Angle IaC, cuting the Vanishing Line in any Point, C, and the given Line at I.

Join IC; take any Point, K, in IC, and draw KF parallel to IJ, cuting the Vanishing Line in F. Draw KL parallel to Ia, and join LF.

A Right Line, ab, drawn through a, parallel to LF, will tend to the Point B.

Note. The Point is given at a and *a*, both without and between the given Lines; and the process would be the same if the Point was given on the other Side of the Vanishing Line; or any other Line, instead of the Vanishing Line, which tended to the same Vanishing Point.

DEM. In the Triangles IaC, I*a*C, because KL is parallel to Ia, CL : La :: CK : KI. - 2. 6. El.
But, CK : KI :: CF : FB; for KF is parallel to IB, by Construction.
Consequently, CL : La :: CF : FB; therefore, LF is parallel to aB. Q. E. D.

Or, the same thing may be done in another manner.

No. 2. Let AB and CD be the two given Lines, and E is a Point given, through which it is required to draw a Line tending to that Point, in which AB and CD would intersect.

Draw AC, at pleasure (through E) and BD, parallel to AC, at discretion.

Draw either Diagonal, as AD; draw EF parallel to CD, and FG to AB, cuting BD in G; a Right Line, drawn through E and G, will tend to the same Point with AB and CD.

‡ 2. 6. El. DEM. In the Triangle ADC, because EF is parallel to CD, the Sides AC and AD, are cut proportionally, in E and F‡; i. e. AE : EC :: AF : FD; and because FG is par. to AB, AF : FD :: BG : GD; wherefore, BG : GD :: AE : EC; i. e. as AB is to BS, or CD to DS, the Point where the Lines meet.
2ndly. When *E* falls without the two Lines Because *EF* is parallel to CD, AC : C*E* :: AD : D*F*; and because *FG* is parallel to AB, BD : D*G* :: AD : D*F*, i. e. as AC is to C*E*; i. e. as AB to BS.

Hence it appears, that nothing more is required than to find a fourth Proportional. For, having drawn a Right Line, AC, at pleasure, through the given Point, E, cuting any two Lines which tend to the same Vanishing Point; and, at a proper Distance, another, BD, parallel to the former (the farther off the better) BG (or LM) being divided, in G, as AC is divided in E; then, if a Right Line be drawn through the Points E and G (or g) it must necessarily tend to the same Point; for Right Lines, proceeding from the same Point, cut parallel Lines proportionally§. This method is the most eligible because it is the readiest. (§ Cor. to 6. 6. El.)

There are various ways of performing this, of which, those by Brook Taylor are very ingenious, and do not require any parallel Lines to be drawn.

No. 3 & 4. Let AB and CD be two given Lines, tending to one Vanishing Point, P.

It is required to draw a Line, through E, which shall represent a Line parallel to the Originals of AB and CD; and consequently, it must tend to the same Vanishing Point; seeing they are not parallel.

First,

First, when the Point E is between the two Lines.

Through the Point E draw two Lines, at pleasure, cuting the two given Lines, in A, G, C, and F; and, through A, C, and F, G, draw two Lines, meeting at a. No. 3.

Draw a H and a B, at pleasure, cuting the given Lines in I, H, B, and D; join BI and DH, intersecting at K.

A Right Line drawn through E and K will tend to the same Point, with AB & CD.

2nd. When the Point *E* is situated without the two given Lines, AB and CD. No. 4.

Draw *EA* and *EF* at pleasure, cuting the two Lines, B*F* and *CI*, in *C*, *A*, *F*, and *G*; join *AG* and *CF*, intersecting at a.

Draw any other Line, B*K*, indefinite; and, at *D*, where it cuts *CI*, draw *DH*, through a; also, draw B*I*, through a.

Lastly, through *H* and *I*, draw *HK*, cuting B*D* produced, at *K*.

A Right Line, drawn through *E* and *K*, will represent a Line parallel to the Originals of *AF* and *CI*; and consequently it will tend to the same Van. Point, P.

Although Dr. Taylor has given Demonstration of all the Problems, previous to this, he has not favoured us with a Demonstration of it. But a little consideration will make it very obvious, on inspection of the Figure.

In No. 3, because AB and CD have the same Vanishing Point, they represent parallel Lines; and if a be considered as the Vanishing Point of AC, FG, &c. they, consequently, represent parallel Lines.

Wherefore, AFGC and IHBD represent Parallelograms; and consequently, E and K, the Intersections of their Diagonals AG, CF, &c. represent the Centers of those Parallelograms†. † 16. 1. El.

And since the Parallelograms are between the same Parallels, their Centers are, consequently, equally distant from each; therefore, EK represents a Line parallel to AB and CD.

In No. 4; because B*F* and *CI* represent parallel Lines, the Lines *AG*, *CF*, &c. which pass through the Point a are all cut proportionally in that Point; wherefore, the Originals of B*H* and *ID*, of *AF* and *CG* are in the same Ratio respectively, i. e. the Originals of $BH:DI::AF:CG$; consequently, $EA:EC::EF:EG$, i. e. as $KH:KI$, or $KB:KD$. N.B. Cor. 1. 2. 6. El.

Wherefore, since $EA:EC::KH:KI$, consequently, $EC:CA::KI:IH$; or as any other Line drawn from *K*, whose Original is parallel to *EA*, would be cut, by B*F* and *CI*.

But, if *KH* be supposed parallel to *EA*, and *AH* to *CI*, *CAHI* represents a Parallelogram; wherefore *IH* is equal *CA*; consequently, *IK* represents a Line equal to *EC*; and consequently, *EK* represents a Line parallel to *CI*. For, *ECIK* also represents a Parallelogram.

5. If it be required to draw several Lines to a Vanishing Point which is out of the Picture, from various Points, in a given Line.

Let A, B, and C, be three Points in the Line, AC; let DE and FG be two Lines which tend to the same Vanishing Point, P. No. 5.

At any Distance, at discretion, draw HL parallel to AC, and draw AH, DI, BK, and CL parallel to either Line (as FG) cuting HL, in the Points H, I, K, & L.

Assume any Point, G, in the same Line, FG, and draw GI, cuting DE in E; and, through E, draw a c parallel to AC and HL; lastly, draw GH, GK, and GL, cuting a c, in the Points a, b, and c.

Then, if Right Lines A a, B b, and C c are drawn, they will tend to the same Point, P.

For, considering the Originals of AC and HL to be parallel to the Picture, and FG as the Vanishing Line of some Plane; because AH, DI, &c. are parallel to FG, and H a, IE, &c. have the same Vanishing Point, G, they are all in parallel Planes, of which FG is the Vanishing Line; and, because a c is parallel to AC and HL, the Original of a c is parallel to their Originals, and consequently may be in the same Plane with either (Ax. 5.) wherefore, A a, DE, &c. represent parallel Lines, in the same Plane, A a c C, or in parallel Planes AH a, DIE, &c. of which FG is considered as the Vanishing Line; and consequently, they vanish in the same Point in that Line.

In the Preface to this 3rd Book, I proposed giving some other Expedients, in this Section; but, on more mature consideration, I shall omit them, till they occasionally occur in the Work; when they will be more intelligible and better understood; and, being immediately applicable, in Practice, they will, at the same time that we acquire them, shew their use; by which means, they will be deeper rooted in the Mind, and the application of them, in future Examples, will be more familiar.

SECTION V.

Of the PRACTICE of PERSPECTIVE, respecting PLANE FIGURES.

HAVING, in the foregoing Section, exemplified and illustrated the Elements of Practical Perspective, according to Brook Taylor; which, notwithstanding they are so excellent in themselves, and of great utility in Practice, yet they do not lay a foundation whereon to begin; but teach, only, how to proportion one Part by another, either given or found, on the Picture; so that, a Novice, in these matters, cannot possibly apply them to real use. Nor, in my Opinion, would any Person ever be made a Practitioner, from that Treatise, unless he was endowed with an extraordinary Talent, and a very comprehensive Capacity; being quite conversant in Geometry, and particularly acquainted with the Doctrine of Proportion; having a clear Idea of Planes and their Intersections with each other, and of Right Lines cut by Planes.

Before I proceed to Figures, I shall shew, in three Problems, how to find the perspective Representations of Right Lines, in the three Positions, parallel, perpendicular, and inclined to the Picture; having the Intersection and Vanishing Line of the Plane they are in (their Distance from the Picture being known and the Position of the inclined Line; that is, the Angle of its inclination to the Picture, or to the Intersection, and its intersecting Point, or the seat of some Point in it; or the Representation of some Point in the Line.) How to proportion them, that is, to cut off such Portions, or Parts, as are the true perspective Representations of certain Parts in the Originals, (in which the whole foundation of Practical Perspective consists) is contained in the last Section (Prob. 8th, 9th and 10th) and are exemplified in this.

Prob. 6th shews how to find the representation of a Point on the Picture, any how situated; its Seat on the Picture and its Distance from its Seat being given; which, in reality, contains the whole; as it will be found hereafter. I shall, in the next Problem, give a more familiar and introductory Lesson, how to find the representation of a Point, situated on the Ground Plane; which is, undoubtedly, the first Plane to be considered. At the same time, let it be observed, that there is not the least difference, in the Operation, between the Ground Plane and any other, whose Intersection and Vanishing Line is given, and its Distance known; as it will be shewn.

Having, in the preceding, and in the next four, Problems, given the Elements of the whole, and fully demonstrated it; I shall not trouble the Reader with the Demonstration of every Operation, in the following Work; but only in particular Cases, which may not readily be deduced from the foregoing; as it would only swell the Work to an enormous bulk, but would not be of use to the Practitioner. Therefore, where I see occasion, I shall refer to the elementary Problems, or Theorems, for Demonstration, and to shew how each particular Problem is applicable in Practice, in various Cases, in the course of the Work.

Let it, here, again, be observed, that whenever any Point, Line, or Figure, is given in the Original Plane (the Intersection of the Plane being also given) it is supposed to be so situated on the other side of the Intersection, as it would be, if the Original Plane was turned over, on the Intersection, to the other Side of the Picture, making the same Angle with it; and (if it be not perpendicular) inclined towards the same Part.

See Fig. 37; the Triangle XYZ, on the other Side of the Picture, is inverted on this Side, for Practice.

PROBLEM

PROBLEM XIV.

To find the Representation of a Point, situate on the Ground Plane, or other horizontal Plane, its real Place being given thereon; the Intersection of the Plane it is in being given.

There are several ways to find the representation of a Point. Plate XI. Fig. 52. No. 1.

Let A be an Original Point in the Geometrical Plane, of which, BD is the Intersection; C is the Center of the Picture, and CE its Distance; E is the Eye.

Through C, draw *E*F parallel to BD, the Intersection; then is *E*F the Vanishing Line of the Original Plane; which being horizontal, consequently, *E*F is the Horizontal Line, or Vanishing Line of the Ground Plane.

Now, since BD is the Intersection of the Original Plane, and *E*F is its Vanishing Line, if the Original Plane was produced infinitely, beyond the Picture, its whole perspective Representation would be between its Intersection and Vanishing Line; consequently, the Representation of any Point, Line, or Figure, in that Plane (beyond the Picture) must be somewhere between those Lines; to find which observe the following Rules. (See Introductory Preface; Page 109.)

Find S the Seat of the given Point; i. e. draw AS perpendicular to the Ground Line or Intersection, cuting it in S; which is the Seat of the Point A; because it is in a Plane perpendicular to the Picture.

Draw SC, and AE cuting it, in a, the Point sought. (See Problem 6.)

For, EA is a Visual from the Eye, at E, to the Point A, which would pass through the Picture at a, making Sa to aC, as AS to CE†; therefore, a is the Representation of the Point A.

† Theo. 13.

Method 2nd. On C describe a Semicircle, with the Radius CE, cuting the Horizontal Line in *E* and F; which are the transposed places of the Eye E to the Vanishing Line. (See Fig. 37, No. 1, the Vertical Plane, V, being turned down, on either Side, into the Picture.)

Therefore, having made C*E* or CF, on either Side, equal to the Distance of the Picture, make SB, or SD, equal to SA, and draw D*E* or BF, on either Side, it will cut SC in the same Point, a, as before.

For C*E* is parallel to SB or D, and, C*E* is to SB or SD, as CE to SA, viz. as the Distance of the Picture, to the Distance of the Original Point from its Seat, i. e. from the Picture. (See Prob. 6.)

If CE^2, and S*F* be drawn perpendicular to SC, or parallel between themselves, it is still the same. For *F*a is to aE^2, as Sa is to aC, as before.

Note. This Method is the most eligible of all other; for, the Distance of the Picture being known (as it is always supposed to be) and being placed on either Side of the Center, and SB, or SD, equal to the Distance of the Point, on the other Side, *E*D or BF, is a Visual Ray from the Eye to the Original Point, as before.

N. B. If the Original Point be situated in the Station Line, as at *A*, the first method cannot be applied; because its Seat, *D*, being also in the same Line, *D*C and E*A* are one Line, and cannot intersect; in which Case, the Eye being transposed to *E*, on either Side, and *A* to *B*, then is *EB* a Visual Ray, from the Eye, on one side of the Picture, to the Point *A* (i. e. *B*) on the other Side; supposing the Vertical Plane, in which the Visual Ray must be, turned on CD, its Intersection with the Picture, direct before the Eye.

Plate XI. Fig. 52. If ſ be the Seat, on the Picture, of a Point in any other Plane, (ſuppoſe Vertical) then ſG being drawn, parallel to CE, is its Interſection; and, having drawn ſC, make ſG equal to its Diſtance, and draw GE, cuting ſC in g, the Repreſentation of G; i. e. of a Point ſituated beyond the Picture at the Diſtance ſG.

Thus the Repreſentation of any Point may be obtained, knowing its ſituation and Diſtance from the Picture.

No. 2. METHOD 3rd. Through the given Point, A, draw AB and AD, at pleaſure, cuting the Interſection in B and D.

From E, the Eye, draw EF and EG, parallel, reſpectively, to AB and AD, cuting the Vaniſhing Line in F and G, the Vaniſhing Points of AB and AD.

Draw BF and DG, interſecting in a, the Repreſentation of A.

For BF is the indefinite Repreſentation of BA, and DG of DA; conſequently, the Point, in which they interſect, is the Repreſentation of the Point in which the Lines BA and AD interſect. (Cor. 5. Theo. 11.)

N. B. The 2nd Method will be found, on inſpection, to be the ſame as this; for, if E*E* and AD be drawn, they will be parallel, and *E*C is parallel to AS; alſo C*E* is parallel to SD; conſequently, C and *E* are the Vaniſhing Points of AS and AD reſpectively. So that to find the Repreſentation of a Point is to find the Repreſentation of a Line; ſeeing, it muſt be ſuppoſed in ſome Line.

No. 3. METHOD 4th is rather a matter of curioſity than real uſe, being ſeldom, if ever, practiſed; it may be performed without either Interſection or Vaniſhing Line; nothing more being required than the Place of the Eye, and the Original Point; their Diſtances from the Picture being known, or the Ratio of their Diſtances.

On A the given Point, with the Radius AS, its Diſtance from the Interſection, or from the Picture, deſcribe an Ark, BSD; and, on E, with the Radius EC, the Diſtance of the Vaniſhing Line, or of the Picture, deſcribe the Ark FCG.

Draw the Tangents, BF and DG, interſecting in a, the Repreſentation of A.

Or, if the Radius be taken, in the Ratio of the Diſtances, reſpectively, it will be the ſame, i. e. the Tangents will cut each other in the ſame Point, a.

DEM. Draw AB and AD, EF and EG perpendicular to the Tangents, BF and DG.

Then the Trapezium ABaD is ſimilar to aFEG; for aB=aD and aF=aG; (C. 2. 16. 3. El.) alſo AB=AD, and EF=EG; the Angles at B, D, F, and G are Right, and BaD=FAG (2.1.El.) conſequently, BAD=FEG. (See Th. 1. 10 1. El.)

Wherefore, Ba : aF :: AB : EF; i. e. as AS : EC; or, as Aa is to aE, as in the firſt Method (No. 1.) for, AE is a Viſual Ray, in both, and a is the Point in which it cuts the Picture.

PROBLEM XV.

To find the Repreſentation of a Line perpendicular to the Picture; its Place being given in the Original Plane, the Interſection and Vaniſhing Line of the Plane it is in, and the Place of the Eye.

Fig. 53. Let AB be the Original Line; AD is the Interſection of the Plane it is in, and *ECE* is the Vaniſhing Line; E is the place of the Eye.

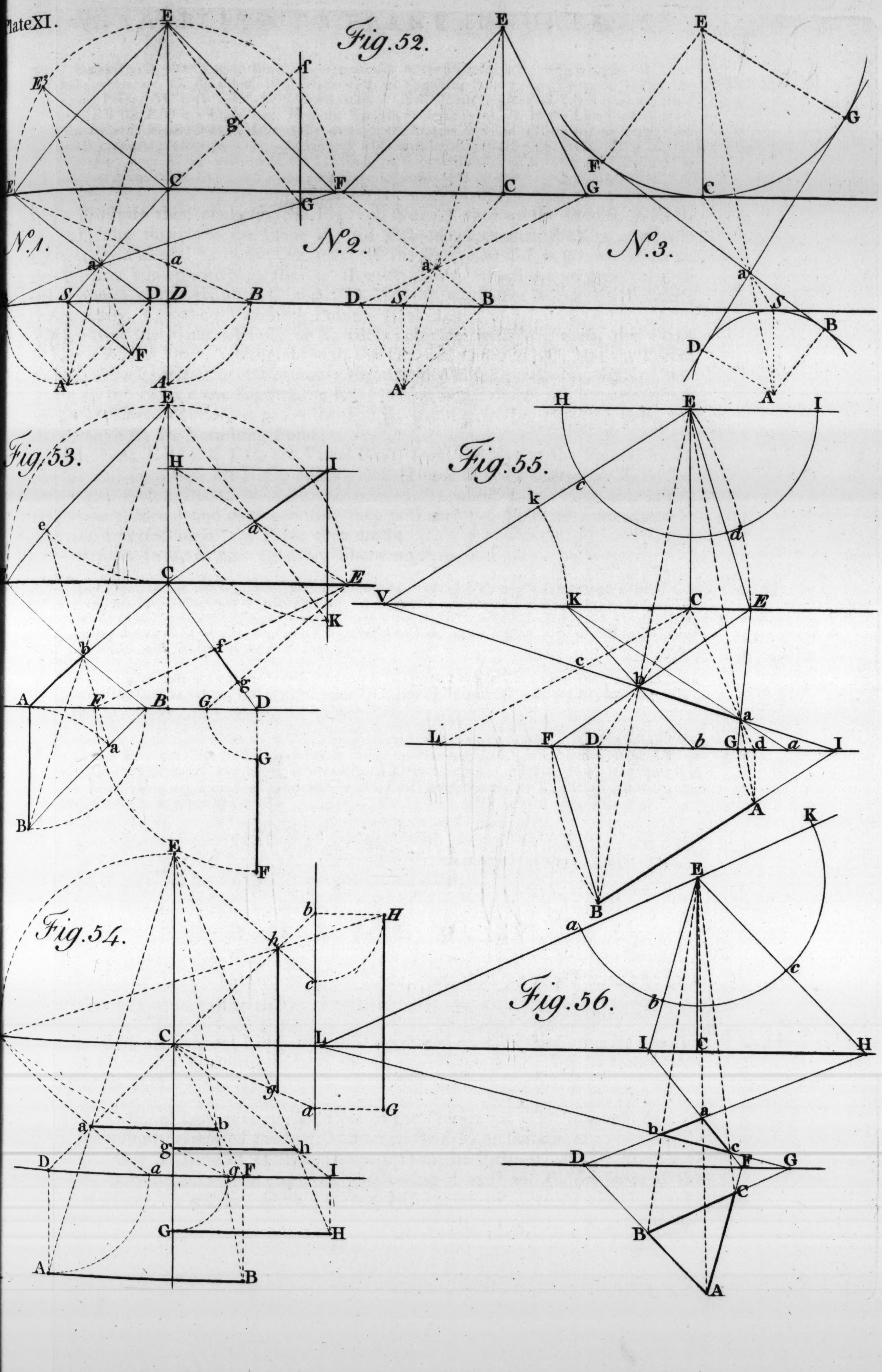
Draw EC perpendicular to the Vaniſhing Line, cuting it in C, the Center; and, becauſe AB cuts the Interſection, A is its Interſecting Point.

Draw AC, and BE cuting it in b; then is Ab the Repreſentation of AB.

Or (having drawn AC) make C*E* equal to the Diſtance of the Picture, and A*B* equal to the Original Line, AB, and draw *BE*, cuting AC, in the ſame Point b. In which proceſs, the meaſures, it is obvious, muſt always be placed on contrary Sides of the indefinite Repreſentation, or the Viſual Ray cannot cut it.

DEM.

Plate XI.



DEM. Because AB, the Original Line, is perpendicular to the Picture, EC, the Direct Radial, is its Radial; wherefore, C, the Center of the Picture, is its Vanishing Point †; and, because A is its Intersecting Point, AC is the indefinite Representation of AB, infinitely produced (by Theorem 12th.) † Def. 22.

But, EB (or *EB*) is a Visual Ray, from the Eye, E, to some Point, B, in the Line; which must cut the Picture somewhere in the Line AC; which is the Section of a Radial Plane passing through the Original Line and the Eye ‡; and consequently it will cut it in b; making Ab : bC :: AB : CE. ‡ Theo. 1.

Therefore, since A, one extreme of the Line AB, is in the Picture, and b is the Representation of B, another Point in the Line, Ab is the perspective Representation of AB. Q. E. D.

To illustrate this, ocularly (See Fig. 37, No. 1.) turn up the Picture AIKB, vertical; also turn over the Plane W, till *EC* coincides with the Center of the Picture, then *E* will be in the true place of the Eye, and *EC* is perpendicular to the Picture, consequently parallel to all other Lines, which are perpendicular to the Picture; therefore, to AC and FD, &c. on the other side of the Picture; consequently, C is their Vanishing Point. (Def. L.)

Turn over the Plane AFDC, or X, till it coincides with W; then, the Plane WX is a Radial Plane, passing through the Original Line, ABC, and the Direct Radial, *EC*, its Parallel, consequently through the Eye; producing, by its Section with the Picture the Right Line RC; which is the indefinite Representation of AC; (Theo. 12) for, it passes through R, the intersecting Point of AB, and also through C, its Vanishing Point.

Now, EA, EB, and EC, are Visual Rays, from the Eye to the Points A, B, and C; and since they are Right Lines (Def. H) and the two Extremes, E, A, &c. are in the Radial Plane, WX, consequently, the whole Line, EA, &c. are in that Plane (Ax. 1.) and consequently, they will cut the Picture, somewhere in RC, the Intersection of the Plane they are in.

Therefore, in a, b, and c; as by Theorem 13th.

N. B. Any Point, B, in the Line AB, may be found, by drawing B*B*, at pleasure, and E*E* parallel to it; cuting the Intersection and Vanishing Line, in *B* and *E*, and drawing *EB*; as in Prob. 14. Fig. 53.

Or, having drawn B*B*, at pleasure, and E*E* parallel to B*B*; make Ba to Ee, as AB to EC, and draw ae; which will cut AC in the same Point, a.

If the given Line does not cut the Intersection, as FG, it must be produced till it does, as at D; and then proceed the same as before; making D*G* equal DG, and D*F* equal DF; draw DC; and *EF*, *EG*, cuting DC in f and g: fg is the Representation of FG.

If I be the Intersecting Point of a Line, perpendicular to the Picture, whose Representation is required; draw IC; and IH, or IK, parallel to either the Horizontal or Vertical Line; make IH, or IK, equal to the Distance of any Point, in the Original Line, from its intersecting Point, and draw H*E* or K*E*, cuting IC in *a*. I*a* is the Representation of a Line, perpendicular to the Picture; from its intersecting Point, I, equal to IH or IK.

It may appear somewhat strange to begin with perpendicular Lines before parallel, but since parallel Lines are found, best, by means of perpendicular ones, I presume it may not be thought improper or irregular.

PROBLEM XVI.

To find the Representation of a Line, parallel to the Picture; in a Plane perpendicular to the Picture.

Let AB be an Original Line, in the Ground Plane; C is the Center of the Picture, *E*CL is the Vanishing Line of the Ground Plane, and DI is its Intersection with the Picture. Fig. 54.

If the Original Line was applied close to the Picture (as DF) it would have its full Dimensions delineated thereon, and be its own Representation.

But, if AB be situated at some Distance from the Picture (as AD) draw AD and BF, perpendicular to the Intersection, cuting it in D and F, the Seats of the extreme Points, A and B. Draw DC and FC.

Draw

Plate. XI. Fig. 54. Draw CE perpendicular to the Vanishing Line, and equal to its Distance; and draw AE and BE, cuting DC and FC in a and b; which are the representations of A and B.

A Right Line, joining a and b, will be parallel to AB, and is its Representation.

Or, having drawn AE only, projecting the Point a, in DC; a b, drawn parallel to the Intersection, or Vanishing Line, till it cuts FC, is the Representation of AB.

DEM. Because AD and BF are Lines perpendicular to the Picture, C, the Center, is their Vanishing Point (Prob. 15) and D and F are their Intersecting Points †; consequently, DC and FC are the indefinite Representations of AD and BF‡.

† Theo. 11.
‡ Theo. 12.

But EA and EB are Visual Rays, from the Eye to the Original Points, A and B; which, it has been proved, would cut the Picture, somewhere, in the Indefinite Representations, DC and FC, making D a : a C :: AD : CE, &c. by Theorem 13.

§ Theo. 10. But, the Representation of a Line, parallel to the Picture, is parallel to its Original §; and consequently to the Vanishing Line of the Plane it is in; and has that proportion to the Original, as the Distance of the Picture, to the Distance of a Plane passing through the Original Line parallel to the Picture; which shall be proved, here.

‖ 15. 1. El. Now, DF=AB ‖; and, in the Triangle DCF, because a b is parallel to DF, the Triangle a C b is similar to DCF; wherefore, as C a : CD :: a b : DF, i. e. as a b is to AB, equal DF. (2 & 4. 6. El.)

But C a : a D :: CE : AD; consequently, C a : CD (i. e. C a + a D) :: CE : CE + AD.

But, CE is equal to the Distance of the Picture, and AD is equal to the Distance of the Original Line, beyond the Picture; wherefore, AD + EC is equal to the Distance of a Plane, passing through the Original Line, parallel to the Picture.

Therefore, a b, the Representation, is to AB, the Original Line, as the Distance of the Picture, CE, is to CE + AD. Q. E. D.

If C*E* be made equal to CE, the Distance of the Picture, and D*a* equal to the Distance of the Point A, from the intersecting Point D; *E* is the place of the Eye, transposed to the Vanishing Line; and *Ea* is a Visual Ray, as before, projecting the same Point, a.

Which Method is the best, on account of its intersecting DC more direct; especially when the Point D falls near the Vertical Line. Besides, it is not always necessary or convenient to place the Eye in the Vertical Line, at E, or to have the Original Line drawn in the Geometrical Plane.

The Demonstration of it is the same as before. For C*E*=CE, and D*a*=DA.

Note. The Representations, a and b, of the Extremes, A and B, of any Line, may be found, by any of the Methods for finding the Representation of a Point, in Problem 14th, which, it would be superfluous to repeat here.

N. B. Any other Line, as GH, being parallel to the Picture, is found after the same manner.

Or, being situated in any other Plane, as *GH*, whose Intersection, *a b*, and Vanishing Line, ECG, is given, or found; whether the Plane it is in be perpendicular to the Picture, or inclined, having the Center and Distance of the Vanishing Line, or of the Picture.

a and *b* are the Seats of *G* and *H*, and *EH* or E*c* are Visual Rays; the rest is obvious.

PROBLEM XVII.

To find the Representation of a Line, any how inclined to the Picture, situated in a Plane perpendicular to the Picture; having the Intersection and Vanishing Line given.

Fig. 55. Let AB be an Original Line in the Ground Plane, whose Intersection is LI, and Vanishing Line, VC; C is the Center of the Picture.

Draw CE, perpendicular to the Vanishing Line, and equal to the Distance of the Picture. Produce BA, to its intersecting Point, I; and draw EV, parallel to AB, cuting the Vanishing Line in V, the Vanishing Point of AB.

Draw IV, the indefinite Representation of AB; and the Visual Rays, AE, BE, being drawn, will cut it in a and b, the Representations of the two Extremes, A and B. a b is the finite perspective Representation of AB.

Or make V*E* equal to VE, IG equal IA, and IF equal to IB.
Draw G*E* and F*E*, cuting IV in the same Points, a and b.

Or, the Seats of the extreme Points being found, by drawing Ad and BD, perpendicular to the Intersection; dC and DC, being drawn, project the same Points a and b, as before.

DEM. Because EV is a Right Line, from the Eye, parallel to the Original Line, and cuts the Picture in V, V is the Vanishing Point of the Line AB†; and I being its intersecting Point, IV is its indefinite Representation‡; a and b are the Intersections of the Visual Rays, with the Picture, as proved in the former Problems. Therefore ab is the Representation of AB. † Def. 22. ‡ Theo. 12.

G*E* and F*E* also project the same Points, a and b.
For V*E* is parallel to FI, and equal to EV, the Distance of the Vanishing Point; also IG is equal to IA, and IF to IB; by Construction.
Wherefore, if E*E* and BF be drawn they will be parallel; for the Triangles, EV*E* and BIF, are similar Isosceles; and since IF, IB are respectively parallel to EV and V*E*, conf. BF is parallel to E*E*.
Therefore, *E* is the Vanishing Point of BF; and conf. Fb : b*E* :: Bb : bE, i. e. as Ib : bV, &c.

N. B. This Method of proportioning inclined Lines will be found the most convenient, in Practice, of all other; and it may also be observed, that if the Representation, a, of any Point, A, in the Original Line, be given or found; and the Distance of that Point, from the Intersecting Point, I, or from the Picture (as Ad) the Representation of any other Point, B, in that Line, may be determined, its Distance from A being known, without the Intersecting Point I.

The Inclination of AB being known (equal BIF) and the Vanishing Point, V, found; by making the Angle HEV, with the Parallel of the Eye, HJ, equal BIF; or CEV equal to its Complement; or, if E*E* be the Radial of any other Line whose Vanishing Point is *E*, make the Angle *E*EV equal to the Angle which the two Lines make with each other; all which, produce the same Point V.

Then, having made V*E* equal VE, the Distance of that Vanishing Point from the Eye; and, if a be the known Representation of some Point A, in the Original Line (AB) and FI the Intersection of the Plane it is in, draw the indefinite Representation, aV, from that Point, and draw *E*a cuting the Intersection, in G; make FG equal to the Distance the Point B lies from A, and draw *E*F, cuting the indefinite Representation, aV, in b, the Representation of B.

By which means any other Point in that Line may be obtained.

If the Vanishing Point, V, be out of the Picture, having found its Distance from the Center, C, and from the Eye, E (by Prob. 12) C*E* will be equal to the Difference; i. e. make C*E* equal VE, less by VC.

Or, when the Vanishing Point is in the Picture, and the Point *E* cannot be had thereon; take any Portion of VE, viz. VK half VE; draw Ka, cuting the Intersection in *a*; make *ab* equal half AB; or the same Portion VK was taken of VE, draw K*b*, which will project the same Point, b, for the Representation of B, as before. This Expedient will be found, frequently, necessary in Practice.

If the Representations, a and b, of any two Points, A and B, in any Line, be found; their Seats, d and D, are ascertained by drawing Ca and Cb, from the Center, to the Intersection, FI, of a Plane perpendicular to the Picture, in which the Original Line is situated, cuting it in d and D.

By which means, the Representation of any other Point, in that Line, may be found; making DL to Dd, as the Distance of the Point from B, is to AB, and draw CL, cuting aV in c, the Point sought. (See Prob. 8.)

After some one or other of these Methods, all Lines inclined to the Picture are drawn on the Picture, and divided perspectively, as the Original is divided; either by the real measures, being applied, or by the known Ratio of the Originals. Various Expedients for performing the same thing will be found necessary, because the same Method cannot always be applied, in all Cases; for which reason, I have given all the Methods which are really useful.

These Methods of proportioning inclined Lines are general and applicable in all Cases whatever; and, on inspection of the Figures, they will be found the very same as for Lines perpendicular to the Picture. For the Center of the Picture being the Vanishing Point of all such Lines, its Distance is laid down, on either Side as is most convenient, on the Vanishing Line, from the Center; and the Lines, which vanish in the Center, are proportioned by the very same means. (See Prob. 15, Fig. 53)

Plate XI. Fig. 55. When the Vanishing Point, V, cannot be had, on the Picture, the transposed place of the Eye, *E*, may be thus found.

The inclination of the Original Line, to the Intersection, being known subtract it from 180 Degrees, or two Right Angles; take half the sum of the remainder, and make the Angle VE*E* equal to it.

It may be found, geometrically, thus.

Through E, draw HEJ, the parallel of the Eye, parallel to the Vanishing Line; make HEk equal to the Angle of Inclination of the Original Line; bisect the remainder, kEJ, by the Line E*E*, cuting the Vanishing Line, at *E*, the transposed place of the Eye, for the Vanishing Point V.

§ 9. 1. El. For, the Triangle EV*E* is Isosceles (EV being equal V*E*) consequently, the Angle VE*E*=V*E*E§ equal *E*EJ, by construction, and by 4. 1. of Elements.

Hence it is evident, that Ib is the perspective Representation of IB.

For, suppose BIF to be the Original of the Triangle bIF; one, Side, IF, being in the Picture, therefore, it has no Vanishing Point†.

† Cor. 1. Theo. 2.

Now, EV is parallel to the Side IB, and E*E* to BF; wherefore, V and *E* are the Vanishing Points of the other two Sides, and IV, F*E* their indefinite Representations; consequently, Ib and IF represent equal Lines; for IF being in the Picture, retains its full Dimensions, and is its own Representation on the Picture; and, Fb represents the other Side, FB; therefore, Ib represents IB.

PROBLEM XVIII.

How to find the perspective Representation of a Triangle, having the Original given, in any Plane, whose Intersection and Vanishing Line is also given, and the Place of the Eye.

Fig. 56. ABC is the Original Triangle, DG is the Intersection, and LH the Vanishing Line; E is the Eye.

Produce the three Sides, AB, AC, and BC, to the Intersection, cuting it in D, F, and G, the intersecting Points of the Sides, respectively.

Draw EH, EI, and EL, respectively parallel to AB, AC, and BC, cuting the Vanishing Line in the Points H, I, and L, their Vanishing Points.

Draw the indefinite Representations, DH, FI, and GL, cuting each other in the Points a, b, and c, and giving the representation of the Triangle by their Intersections.

This is manifest, seeing that the Sides of the Triangle, are in the Lines AD, AF, and BG; consequently, their Representations must be in the indefinite Representations of those Lines, and consequently, between their Intersections (by Theorem 1 and 12.)

N. B. The Angles LEI, IEH, and HEK, at the Eye, are equal, respectively, to the Angles ACB, BAC, and ABC; and they are also equal to two Right Angles†; and so are the three Angles of the Triangle‡.

† Cor. 3. 1. 1. El.
‡ 10. 1. El.

Hence, if any one Side of the Triangle, as ab, be given, on the Picture, the Vanishing Points, I and L, of the other two Sides, are determined by Problem 4th. And, by drawing Ia and Lb, intersecting at c, the Triangle is compleated.

See this illustrated, by moveable Planes, in Fig. 37.

Raise up the Picture AIKB perpendicular to the Ground Plane, and let the Horizontal Plane be placed parallel to the Ground Plane, i. e. perpendicular to the Picture; or if the Picture be inclined to the Ground Plane, the Horizontal Plane making an equal Angle with the Picture, as it makes with the Ground Plane, will be parallel to the Ground Plane. In which Case, the Lines EN, EO, and EL are the Radials of the three Sides of the Triangle XYZ on the other Side of the Picture; being respectively parallel to them §. Consequently, the Points N, O, and L, are their Vanishing Points ‖; B, P, and S are their Intersecting Points, and BN, PO, and SL are their indefinite Representations, producing the perspective Representation, xyz, by their Intersections.

§ Def. 14.
‖ Def. 22.

Let the Picture be turned down, and imagine the Geometrical Plane turned over on its Intersection, AB, to the other Side; in which Case, the Triangle is inverted. Turn down the Horizontal Plane into the Picture; the Radials EN, EO, and EL, are still parallel to the three Sides of the Triangle, and produce

produce the same Vanishing Points, N, O, and L; also, the Visual Rays EX, &c. being drawn on the Picture, it is obvious they would pass through the Picture, to the Original Points X, Y, and Z, cuting it in x, y, and z, their Representations.

N. B. The Angle NEO, which the Radials EN and EO make, at the Eye, is equal to the Angle Y, of the Triangle: for, they are respectively parallel to them. EO and EL, the Radials of YZ and XZ, make the Angle OEL equal to the Angle Z; and if NE or LE was produced beyond the Eye, E, they would make an Angle equal to X; which is consonant to Cor. 1st, Theorem 8th.

PROBLEM XIX.

How to find the Representation of a Square or other Rectangle, given in the Geometrical Plane; one Side being parallel to the Picture.

If the 15th and 16th Problems be well understood, there will be little occasion to explain the Method of proceeding in this; for, if any Rectangle have one Side parallel to the Picture, the other two adjoining Sides must, necessarily, be perpendicular to the Intersection of the Plane it is in, if not to the Picture.

Let ABFD be a Square; one Side, AB, lying close to the Picture, consequently it is in the Intersection; AD and BF are therefore perpendicular to it. C is the Center of the Picture, or of the Vanishing Line, *EE*, of the Plane of the Square, AB, is its Intersection. Plate XII. Fig. 57.

Now, AD and BF are perpendicular to the Intersection, therefore they vanish in the Center. (See Problem 15.)

Draw AC and BC, their indefinite Representations; make C*E*, on either Side, equal to the Distance of the Picture, or Vanishing Line, and draw B*E* (or A*E*) cuting AC in d, the Representation of D, (or BC in f.)

Draw d f parallel to AB. A d f B is the representation of the Square ABFD.

For, B*E* is the indefinite Representation of the Diagonal, BD; *E* being its Vanishing Point; for E*E* is parallel to BD, (C*E* being equal to CE, and AD to AB) and DF being parallel to the Picture, its Representation, d f, is consequently parallel to the Intersection. (Prob. 16.)

2. If another square be required, draw f*E*, cuting AC in g, and draw g h parallel to d f. By which Expedient any length of AC may be obtained.

If the length AG had been required, equal twice AD; AG on the Intersection being made equal twice AB, and *GE* drawn, gives the same Point g; as before.

Or, if the Diagonal of the whole be drawn, BG, draw EK parallel to BG; or make the Angle CEK equal GBH, cuting the Vanishing Line in K, the Vanishing Point of that Diagonal; draw BK, cuting AC in g, as before.

3. If the Square be at some Distance from the Picture, as *ABFD*, produce the two perpendicular Sides to the Intersection, cuting it in a and b; also produce the Diagonal *FA* to I, its intersecting Point; or, make a I equal a A, and proceed as in the former Case. The Figure explains the rest.

COR. Hence, a Pavement of Squares, having their Sides parallel to the Picture, may be delineated, with great facility.

Let AB be the Ground Line, and ECD the Horizontal Line. No. 1.

Take the geometrical measure of a Square, and apply it, on the Intersection, AB, as often as it is required, from A to B; as a, b, &c.

From each Division, draw Right Lines to the Center, AC, a C, &c. and draw a Diagonal, from A or B, to the Eye, at E or D, cuting each indefinite Representation in the Points *a*, *b*, *c*, &c. through which, draw the Lines FG, HI, &c. parallel

Plate XII. parallel to the Interſection, or Vaniſhing Line, and the ſeveral Figures X, Y, Z, &c. are the Repreſentations of Squares on the Picture; which may be repeated, by drawing other Diagonals to any length or width.

Now, if one of theſe Squares (as Y or Z) was ſingle, it would ſcarce have the appearance of a Square, having no other, contiguous to it, to biaſs the judgment; but, by the affinity of the whole, the Eye (being accuſtomed to ſee Objects as they appear, in all ſituations) is not offended, and readily gives the aſſent; altho' it is certainly capable of determining, that the ſeveral Repreſentations of Squares, are not Squares, but have the appearance, only, of Squares, in certain Poſitions and Situations.

PROBLEM XX.

To find the Repreſentation of a Square, or other Rectangle, whoſe Sides are all equally inclined to the Picture.

Firſt, by the Original Figure being geometrically drawn, in its determined Poſition to the Picture; the Interſection of the Plane it is in being given.

Fig. 58. Let ABCD be a Square, the Sides of which are equally inclined to the Picture. C is the Center of the Vaniſhing Line *EE*, or of the Picture; *E*, *E*, are the tranſpoſed places of the Eye, to the Vaniſhing Line, viz. C*E* equal CE.

As one Angle of the Square, A, touches the Interſection, it is, conſequently, the Interſecting Point of the two Sides AB and AD.

Let the other two Sides be produced to the Interſection, at F and G.

Draw A*E*, both ways; alſo, draw F*E* and G*E*, diagonal ways, cuting each other. A b c d is the Repreſentation of ABCD.

For the Point A being in the Picture, is its own Repreſentation, A*E*, A*E* are indefinite Repreſentations of AB and AD; and F*E* and G*E*, of FC and CG (Theo. 12) conſequently, they cut each other in the repreſentations of the ſeveral Angles, B, C, and D. (Cor. 5, Th. 11.)

If the Rectangle HIKL be at ſome Diſtance from the Picture, the Sides being produced to their interſecting Points, *a*, *b*, *c*, *d*, and the indefinite Repreſentations *a E*, *b E*, &c. being drawn (as in the Figure) gives the Repreſentation h i k l of that Rectangle; its Sides being parallel to the Sides of the Square.

EV, parallel to the Diagonal, IL, produces its Vaniſhing Point.

Note. In this Caſe, it may be obſerved, that there is no neceſſity for the Eye, i. e. the Diſtance of the Picture or Vaniſhing Line, being placed above it, but placed equally on either Side of its Center, C, in the Vaniſhing Line, as C*E*. The Squares, or Rectangles (wherever they are ſituated in the Original Plane) having the ſame Poſition to the Picture, have their Sides parallel, and conſequently, they have the ſame Vaniſhing Points.

N. B. The Diagonals of the Square are, in this Caſe, the one, BD, parallel, and the other, AC, perpendicular to the Picture; conſequently, their Repreſentations are either parallel, as b d, or vaniſh in its Center, as A c.

Let it be, here alſo, particularly noticed, that *E*, being the place of the Eye, in the Vaniſhing Line (commonly called the Point of Diſtance) is the Vaniſhing Point of the Diagonal of a Square, whoſe Sides are parallel and perpendicular to the Picture, Conſequently, if the Diagonals are parallel and perpendicular to the Picture, they are the Vaniſhing Points of its Sides.

By which Points, all Lines perpendicular to the Picture are proportioned, perſpectively; as it may be obſerved in the preceding Problems.

2nd. How to find the Repreſentation of a Square, in this Poſition, not having the Figure drawn out geometrically, only its meaſure and place known.

Let A be the Interſecting Point of an Angle of the Square, ſituate on the left ſide of the Station Line; at the Diſtance AJ.

Make AF and AG each equal to the Diagonal, and proceed as before.

Or,

Or, by the measure of its Sides.

Make *EF* equal to the Diagonal of a Square, whose Side is CE; i. e. make *EF* equal to the Distance of the Vanishing Point, *E*, from the Eye, equal *E*E.

Make Af equal to AB and draw f*E*, cuting the indefinite Representation A*E* in b, the Representation of the Angle B.

Draw bd parallel to FG, the Intersection, cuting A*E* in d; and, lastly, draw b*E* and d*E* diagonal ways, cuting each other, which compleats the Figure.

COR. Hence, a Pavement of Squares, diagonal-ways, may be delineated. Fig. 58. No. 1.

Let AB be the Ground Line and ED the Horizontal vanishing Line.

Make Aa, ab, &c. equal to the Diagonal of the Square; make CE, CD equal to the Distance of the Picture (C being the Center.)

Draw AE, aE, &c. and AD, aD, &c. cuting each other in the representations of Squares, placed diagonal-ways.

Through d, where AD and BE intersect, draw ef parallel to AB; and where it cuts the several Lines, drawn from a, b, c, &c. viz. in *a*, *b*, &c. draw D*a*, D*b*, and E*d*, &c. contrary ways, by which means they may be continued at pleasure.

PROBLEM XXI.

To find the Representation of any Rectangle, obliquely situated to the Picture.

First, by having the Original drawn in the Geometrical Plane; the Intersection, Vanishing Line and its Center being given.

ABCD is the Rectangle to be delineated; FI is the Intersection of the Plane it is in; KL is the Vanishing Line, and *C* its Center. Fig. 59.

Make *C*E, equal to the Distance, and perpendicular to KL.

Draw EK and EL, parallel to the Sides of the Rectangle, AB and BC, respectively, producing their Vanishing Points K and L.

The Original being at some Distance from the Picture, produce every Side DA, CB, &c. to the Intersection, cuting it in F, G, H, and I, their Intersecting Points.

Draw the indefinite Representations FL, GL, HK, and IK; which, by their mutual Intersections, give the Representation, abcd, of ABCD.

Method 2. By the Seats of every Angle, on the Picture, and their Distances from their Seats, respectively.

If the Original be in a Plane perpendicular to the Picture, the Seats of all its Angles are in the Intersection of the Plane they are in.

Let *a*, *b*, *c*, and *d*, be the Seats of the Angles A, B, C, and D, respectively.

Draw *aC*, *bC*, &c. to the Center; make *ab* equal to the Distance of the Point A, from its Seat, *be* equal to the Distance of B, *cd* of C, and *df* of the Ang. D.

Make *CE* and *CF* each equal to the Distance of the Picture, and draw *bE*, *eF*, *dF*, and *fE*, cuting *aC*, *bC*, &c. in the Points a, b, c, and d, respectively; which are the representations of the several Angles A, B, C, and D.

Join the Points a and b, b and c, &c. as in the Figure; and the Quadrilateral, abcd, is the representation of the Rectangle, ABCD, situated as in the Figure.

Note. This Method, is used by all the old writers on Perspective; particularly in the Jesuits. In which process it may be observed, that there is no occasion for the Vanishing Points, K and L; for if they made use of Vanishing Points, at all, they were found by producing the Sides ba, bc, &c. to the Vanishing Line; i. e. to the Horizontal Line, for they know no other.

The Difference, which is very material, will be explained in the Appendix.

PlateXII. Fig. 60. METHOD 3. By the known Proportion of the Original; its Position, Situation, and Distance being determined.

The Original Figure, ABCD, is drawn out in the Geometrical Plane, to shew how the measures, &c. are applied, but it is, otherwise, of no use in the operation; as the careful and accurate observer will perceive.

IN, the Intersection; KL, the Vanishing Line; C, its Center; and CE its Distance, are given, the same as before.

Through E, draw HI, parallel to the Vanishing Line; draw EK and EL, making the Angles, HEK, and IEL, equal to the known Inclination of the Sides of the Rectangle (AB and DC) to the Intersection, respectively; i. e. make HEK equal to ABF, and IEL equal to CBG; FG being parallel to the Intersection, the Angles ABF and CBG are equal to their inclination to it †.

† 4. 1. El.

Make *DB* equal to JB; i. e. to the distance of the nearest Angle, B, to the Station Line, and draw *BC*; also, make *Bd* equal to its distance from the Intersection (equal B*B*) and, *CE* being made equal to *C*E, draw *dE*, cuting *BC* in b, the Representation of the Angle B. (Prob. 14.)

Draw the indefinite Representations, b K and b L, from the Point b.

It remains, now to cut off (from b) b a and b c, in the perspective Representations of AB and BC, the Originals.

Make KE^1 equal to KE, and LE^2 equal to LE; which points are used for proportioning all Lines which vanish in K and L respectively. (Prob. 17.)

Draw E^1 b, cuting the Intersection in *b*; make *ab* equal AB, and draw aE^1, cuting b K in a, the Representation of A.

Also draw E^2b cuting the Intersection in b^2 and make b^2c equal BC (the other Side, whose vanishing Point is L) and draw cE^2 cuting b L in c, the Representation of C, another Angle of the Figure.

From the Points a and c, draw a L and c K, intersecting at d, which compleats the Figure; the Originals of those Sides, being parallel to the Originals of a b and b c.

This Method, which is perfectly consonant to the new Principles by Brook Taylor, may appear to some Persons more difficult than either of the other, particularly the first, which is performed, also, entirely on the same Principles; the difference is very obvious, notwithstanding the effect is the same, as it may be seen, by comparing the Figures; or by comparing both, with the 17th Problem, which contains all the various Methods of proportioning inclined Lines, in general.

In the first, there is a necessity for having the Original Figure drawn geometrically, either on the Picture or somewhere apart; for which there is not, frequently, room to spare on the Picture; and we are liable to errors, in transfering the intersecting Points to the Intersection of the Picture; but, without the Original being placed in the very Position we cannot draw the Radials, or Parallels, from the Eye producing the Vanishing Points; as by that Method.

Whereas, by the last Method, the Angles, being known, are made equal to the Originals; in which Case, the Radials would be parallel to the original Lines being in their true places.

Or, if there be not room on the Picture, or interfere with the Objects, the Vanishing Points and their Distances, from the Center and from the Eye, are all determinable, by Problem 12th; the Vanishing Points, K and L, and their Distances KE^1 and LE^2 being ascertained by it.

There is likewise no need for the Original Figure on the Picture, but being any where drawn out, geometrically, or its measures being known, and the Position determined, they are applied to the Picture with the greatest facility, for which, a little Practice will render it quite familiar; regarding always, carefully, to make use of the proper Points for particular Lines, and to distinguish between the Vanishing Points and those which are used, only, for cuting indefinite Lines in the Ratio required.

Note. The Rectangle ABCD, in this Figure, is in the same Situation, but differently inclined to the Picture, as the Rectangle ABCL, which is the Plan, or Seat of the Object, on the Ground Plane of the Apparatus.

PROBLEM XXII.

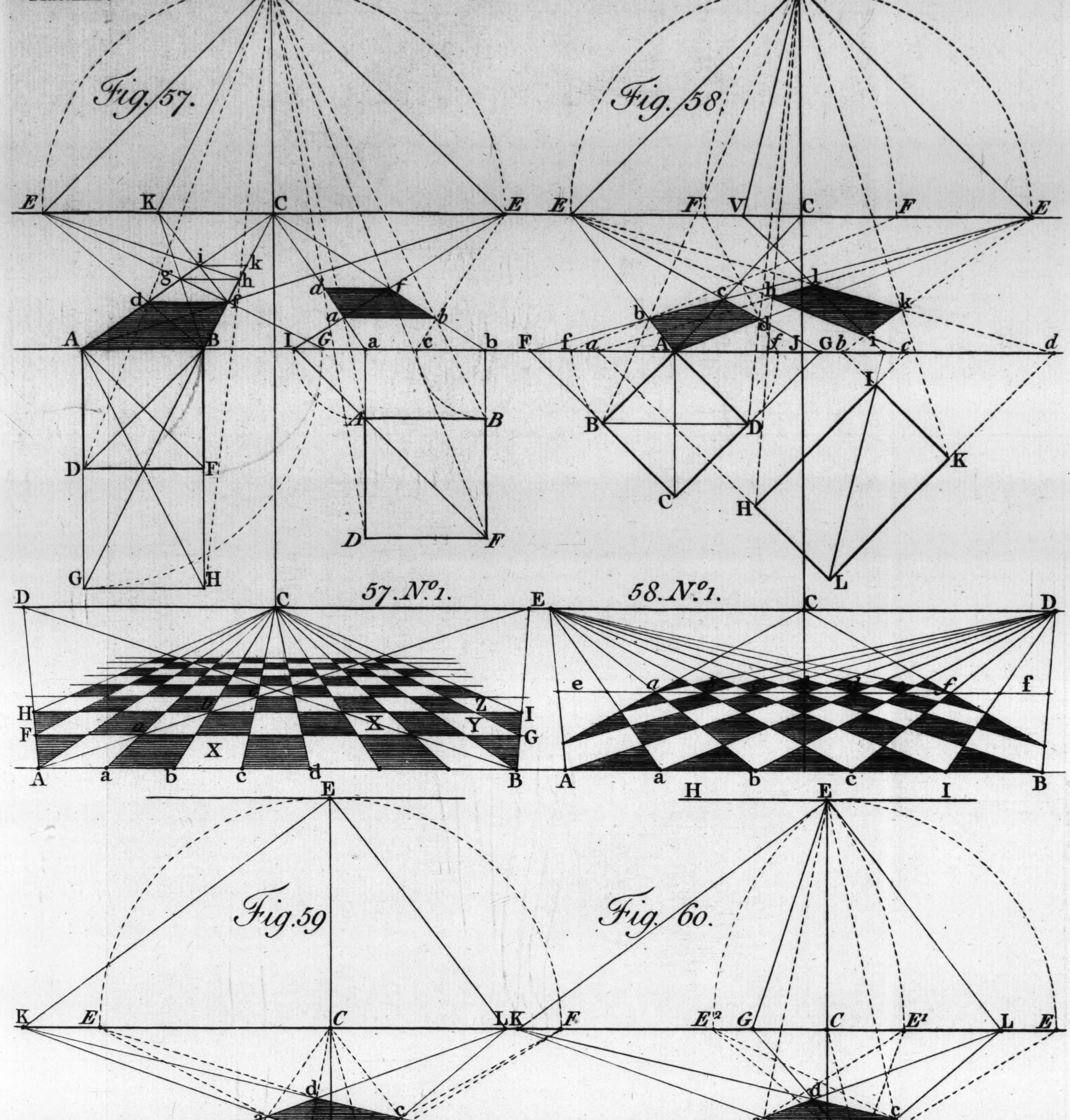
To find the Representation of a regular Pentagon, in any Position to the Picture.

PlateXIII. Fig. 61. Let ABCDF be a regular Pentagon, having one Side, AB, parallel to the Intersection, IK, consequently parallel to the Picture.

LM is the Vanishing Line, and E the Eye.

Produce

Plate XII.



Produce the inclined Sides, FD, CD, AF, and BC, to the Intersection, cuting it in G, H, I, and K.

Draw EL, EM, &c. respectively parallel to them, producing the Vanishing Points, L, M, N, and O, of each Side.

Draw the indefinite Representations GL, HM, IN, and KO; producing, by their intersections, the Figure, fjcd, of the Original FJCD, (No. 1) and *Ifdc K* of *IFDCK* (No. 2) so that, the representation of one Side, AB is wanting, in each; which, on account of its parallelism to the Picture, has neither Intersecting nor Vanishing Point †; and must be found by Problem 16; or, by drawing a Visual Ray, EA, cuting the indefinite Representation, IN, of the Side AF, in a; or, make NP equal NE, and I*a* equal IA, and draw *a*P. (Prob. 17.)

† Cor. to Theo. 2.

Having, by any of these Methods, found a, the Representation of A, draw ab, parallel to the Intersection, cuting KO in b; which compleats the Figure.

2nd. How to determine all the Vanishing Points in this Position, without the Original Figure on the Picture.

Through E, draw RS parallel to the Vanishing Line; and make the Angles REL, LEO, &c. each of 36 Degrees; i. e. divide the Ark of the Semicircle, R23S, into five equal Parts, and produce the Lines E1, E2, &c. to the Vanishing Line, cuting it in L, O, &c. the Vanishing Points of the Sides of a Pentagon, having one Side parallel to the Picture.

The reason of this is obvious; for three of those Angles, viz. REN, is equal to the Angle of a Pentagon; and since one Side is parallel to the Picture, RE is parallel to it; and consequently, EN will be parallel to AF, another Side; the Angle REN being equal BAF.

Also, the Angle which any two Sides, not contiguous (as AF and CB) make with each other, viz. FJC, is equal to one of those Angles; wherefore, EN being parallel to one of those Sides, EO is consequently parallel to the other. (By 6. 1. El.)

See this fully explained, after Problem 11th, in the 4th Book of Elements.

To find the Representation when every Side is inclined to the Picture has nothing particular; for, if every Side be produced to its intersecting Point, and their Vanishing Points are found by drawing parallels to every Side, from the Eye; the indefinite Representations being drawn, produce the Representation of the Original; as in the last Problem, or Prob. 18.

If the Vanishing Point of any Side be found, all the rest are determined as before; by producing that Side and describing a Semicircle; and dividing it into five equal Parts, as in the Figure.

PROBLEM XXIII.

How to find the Representation of a Pentagon, having only one Side given, in its true Place and Position, in the Geometrical Plane.

Let AB be the given Side. Fig. 62.

It is required to find the Representation of a regular Pentagon, whose Side is equal to AB; and inclined to the Picture in the Angle ADF; or to the Intersection FD. IM is the Vanishing Line, C is its Center, and CE its Distance.

Through E, draw EH parallel to the Vanishing Line, and make the Angle HEI equal to BDG, the given Angle, producing the Vanishing Point I.

Produce BA to D; draw the Indefinite Representation DI; and find the finite part ab, the representation of AB (by Prob. 17) i. e. draw the Visual Rays AE and BE; or, make IP equal IE; and, DF, DG, equal to DA, and DB, respectively; draw FP and GP, cuting DI in a and b.

Produce IE, and describe a Semicircle N*ac*O; divide the Ark into five equal Parts, at *a*, *b*, *c*, and *d*; draw E*a*, E*b*, &c. to the Vanishing Line, producing the Vanishing Points, K, L, &c. of the remaining Sides of the Poligon.

Having obtained ab, the Representation of AB, as above; draw aM and bL the indefinite Representations of two Sides, from those Points; for, IaM represents an Angle, IEM, equal to the Angle of a Pentagon; and IbL represents an Angle equal to IEL; which is equal to an external Angle of a Pentagon.

Draw

Plate XIII. Fig. 62. Draw a K, cuting b L in c; b c is another Side of the Pentagon; for, a c represents a Diagonal (which, in a Pentagon, is parallel to the opposite Side) whose Vanishing Point is K.

Draw I f, through c, cuting a M in f; c f represents a Diagonal parallel to the Side a b; consequently, they have the same Vanishing Point I; and consequently, a f represents another Side.

Lastly; draw f K, and b M cuting it in d, and draw c d, which compleats the Figure, a b c d f, required.

The Vanishing Point Q (which is out of the Picture) of the Side c d, and its parallel Diagonal b f, has not been wanted in this process.

N *c* and O *b* are Diagonals, E N, E *c*, or E O and E *b*, being supposed Sides of a Pentagon; and E Q, E K are Parallels to them, from the Eye, producing their Vanishing Points. (See Prob. 10.)

Note. If a b or b c, or any Side in the Representation had been given, the whole Figure may be compleated, as above, without the Intersection.

PROBLEM XXIV.

To find the Representation of a regular Hexagon, situated any how to the Picture.

There is no Figure whatever, except a Square, easier to describe than a Hexagon.

To have the Original Figure drawn in the Geometrical Plane, it is evident, is the same as has already been explained, in the foregoing; I shall, therefore, beg leave to pass over that description.

Fig. 63. Let A B be a Side, given, of a regular Hexagon; the Original of which, is parallel to the Picture. K L is the Vanishing Line.

Make K E L an equilateral Triangle, whose Perpendicular is *C* E.

Through E draw H I parallel to the Vanishing Line, and, on E, describe a Semicircle; which, divide into three equal Arks, at *a* and *b*.

Draw E *a* and E *b*, and produce them to the Vanishing Line, cuting it in K and L; which, are all the Vanishing Points required for this Position.

Draw A K and B L; make A b and B a each equal to A B, in A B produced; and draw a K and b L.

Draw B K and A L, or A *C* and B *C*, cuting them in F and G, and join F G, which is parallel to A B, and compleats the Figure. A B C G F D.

A regular Hexagon having an even number of Sides, the opposite ones are consequently parallel, and the Diagonal, which passes through the Center, is also parallel to them; and because A B is parallel to the Vanishing Line, its Original is parallel to the Picture; and consequently, the opposite Side F G is also parallel to A B; and, because they are equal, A F and B G are parallel, and consequently their Originals were perpendicular to the Intersection; therefore they vanish in the Center of the Vanishing Line.

The two Sides, A D and C G, and the Diagonal, B F, represent parallel Lines; consequently they have the same Vanishing Point, K; also B C, A G, and D F; all which may be seen in the Hexagon A B *C G F D*, which may be supposed the Original Figure, one Side A B, being in the Picture.

By drawing the Diagonals A *G*, B *F*, and *C D*, the Hexagon is divided into six equilateral Triangles, A O B, B O *C*, &c. and K E L has its three Sides respectively parallel to them; consequently, since one Side in each Triangle is parallel to the Picture, and has no Vanishing Point (Theo. 2) E K and E L are parallel to the other two Sides of each Triangle; and consequently, to the Sides A *D*, *C G*, and B *C*, *D F*, of the Hexagon; K and L are, therefore, their Vanishing Points.

COR. Hence, a Pavement of regular Hexagons, or equilateral Triangles, may be easily delineated.

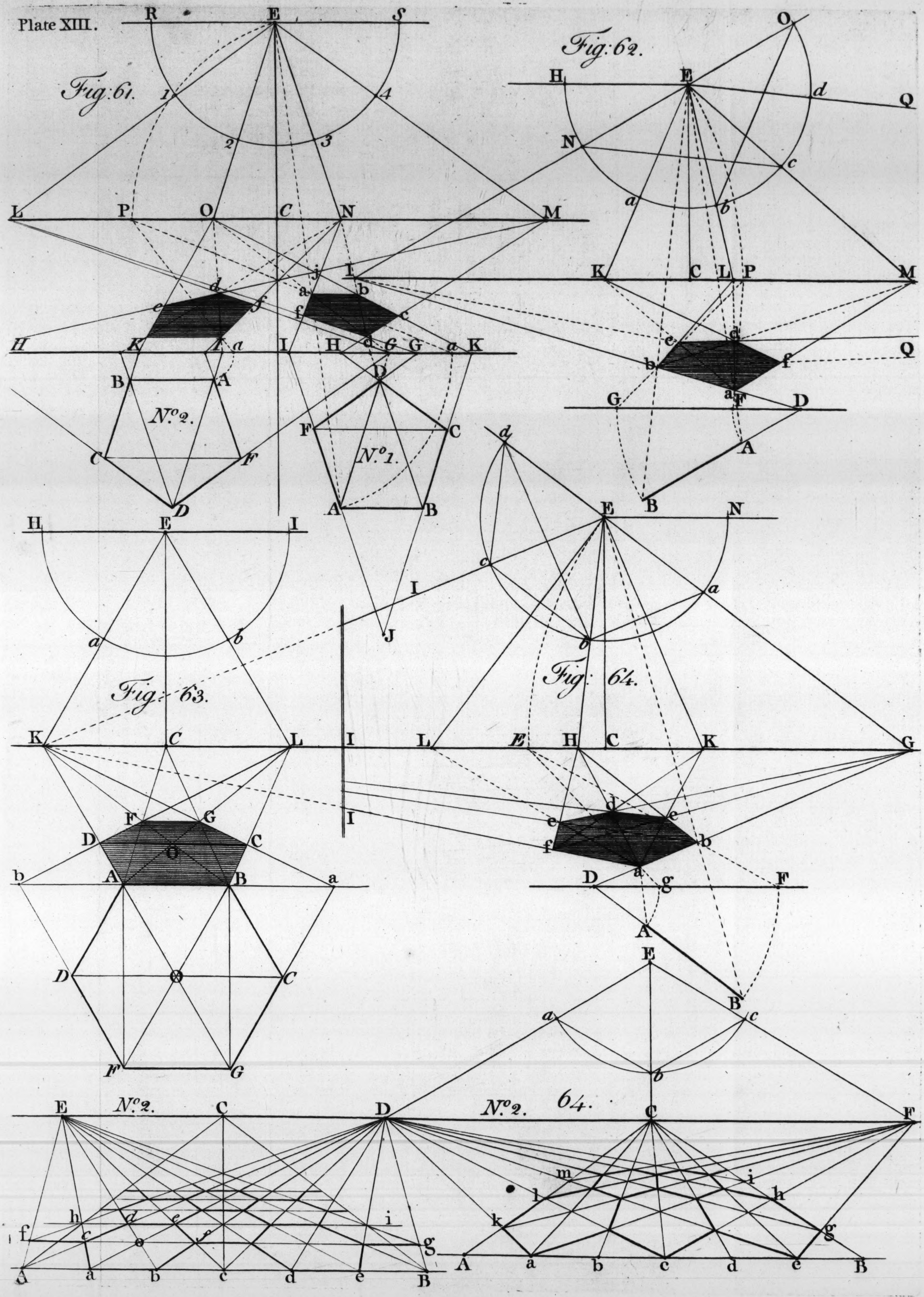
Let A B be the Intersection of the Picture, or the Ground Line of the Pavement; and let A a be the determined measure of one Side of a Hexagon.

Make a b, b c, &c. equal to A a, as often as there is occasion.

If C be the Center of the Picture (its Distance being known) find the Vanishing Points, D and E, as before.

2 Draw

Plate XIII.



Draw AE, AD, and aE, aD, &c. and, where they interfect, draw Lines parallel to the Interfection, as fg, hi, &c. which divides the whole into equilateral Triangles; fix of which, round the fame Point o, as a*cdef*b form a Hexagon; and thus, as many may be defcribed as are required; for which, the infpection of the Figure is fufficient.

Cafe 2. When the given Side is inclined to the Picture.

Let AB be the Side given, FD is the Interfection, and GI the Vanifhing Line of the Plane it is in; E is the Eye. Fig. 64.

Find ab, the reprefentation of AB, by any of the former Problems, its Vanifhing Point is G; draw GE, and produce it; on E defcribe a Semicircle, *abcd.*

Make *ab* and *bc* each equal *a*E, and draw E*b*, E*c*, cuting the Vanifhing Line, in H and I, the Vanifhing Points of the other Sides.

Draw aI and bH, the indefinite Reprefentations of two Sides.

For, baf reprefents the Angle GEI, which is the Angle of a Hexagon; equal *a*E*b*+*b*E*c*, twice 60 Degrees; and abc reprefents the Angle *b*E*d*, equal *a*E*c*.

Make af and bc reprefent Lines equal AB (by Prob. 10, Cafe 3rd) i. e. bifect the Angle GEH, by the Line EK; or, E*d* being equal E*b*, having drawn *bd*, draw EK parallel to *db*.

Or, becaufe EI is the Radial of af; confequently parallel to its Original, make IEK a Right Angle, i. e. draw EK perpendicular to EI; and EK is the Radial of a Diagonal of two contiguous Sides, which is perpendicular to the Original of af, and parallel to *ab* (for *abcd* is half a regular Hexagon in the Pofition of the Original) confeqnently, EK is parallel to the Diagonal whofe Vanifhing Point is K.

Draw aK cuting bH in c, and draw Gc till it cuts aI in f; for, the Original of the Diagonal cf is parallel to the Original of ab. As *ad* is parallel to *bc*.

Draw cI, aH, and fH; and Gd, cuting fH in e, which compleats the Figure.

If the Vanifhing Point H had coincided with C, the Center of the Picture, G and I would be equally diftant from C; and the Original of the Hexagon would be regularly pofited; having two Sides and a Diagonal perpendicular to the Picture; or to the Interfection of the Plane it is in.

By which means, a Pavement of regular Hexagons may be drawn in that Pofition.

Let AB be the Ground Line of the Pavement, C the Center of the Picture, and DF the Horizontal vanifhing Line. No. 2.

On AB, take as many equal divifions Aa, ab, &c. as you pleafe, each equal to half the width of the Hexagon required.

Make CE equal to the Diftance of the Picture, and make the Angles CED, CEF, each of 60 Degrees, i. e. having defcribed an Ark *abc*, on the Center E, make *ba* and *bc* each equal E*b*, and draw E*a* and E*c*, to D and F; which are the Vanifhing Points of all the Sides.

From all the Divifions A, a, b, &c. draw Lines to the Center, C; and from every other Divifion a, c, e, draw aD and aF, &c. to both the other Vanifhing Points D and F, and, where they cut AC and BC, viz. in g, h, i, and k, l, m; draw gD, kF, &c. as in the Figure; by which means they may be continued at pleafure. The reft is obvious, on infpection.

PROBLEM XXV.

To find the Reprefentation of an Octagon and of a Circle infcribed, having only one Side given.

As an Octagon is a Figure which may be infcribed in a Square, it is very readily delineated in Perfpective, feeing that, if one Side be parallel to the Picture, two are neceffarily perpendicular to the Interfection, and four are inclined to it, equally, viz. in an Angle of 45 Degrees or half a Right Angle; confequently, the Vanifhing Points are the Center, and Diftance, of the Vanifhing Line of the Plane it is in.

Plate XIV. To draw an Octagon inclined to the Picture has nothing singular or particular in it; as every Line, which is inclined to the Picture, is projected, on the Picture, by the same general Rule (Prob. 17) which has been so often exemplified, in the preceding Problems, it would be quite superfluous to repeat it.

Fig. 65. Let AB be the given Side of an Octagon, parallel to the Picture; *EF* is the Vanishing Line of the Plane it is in, and C its Center.

Make C*E* and C*F* each equal to the Distance; also make *F*J equal *FE*.
Draw A*E* and B*F*, contrary ways, which are two Sides, indefinite.

For, since the given Side, AB, is parallel to the Picture, the adjoining Sides are consequently inclined in half a Right Angle to the Intersection; therefore, *E* and *F* are their Vanishing Points. (Pr. 19 & 20.)

Produce AB, both ways; make BD equal AB, and draw DJ, cuting B*F* in G; and draw FG parallel to AB, cuting A*E* in *F*.

Draw CF and CG, and produce them, till they cut AB, produced, in H and I; draw I*E* and H*F*, cuting HC and IC, in K and L, and draw KL.

Draw AC and BC, cuting KL in M and N; and draw *E*N and *F*M, and produce them, cuting IC and HC in O and P, which compleats the Figure AFPMNOGB, of an Octagon.

Or, without drawing AC and BC, make KM equal NL, where DJ cuts KL, and draw *E*N and *F*M, as before.

Or, from a and b, where the Diagonals HL and IK cut AF and BG, draw aC and bC, cuting the Diagonals, again, in d and c; and through them, draw *E*c and *F*d.

A Curve described, carefully, by hand, touching every Side of the Octagon, passing through the Points a, b, c, and d, will be an Ellipsis; for it is the representation of a Circle, in Perspective.

Or, for greater exactness, through S, where the Diagonals HL and IK intersect, draw ef parallel to AB; and CS to g, bisecting AB and KL, in g and h; by which means there are eight Points obtained in the Circumference.

Note. The Figure below HD (which may be considered as the Intersection of the Plane of the Original) is the Original, geometrically drawn. The Italics, corresponding with the Roman Letters in the Representation, shew the Original of each Point, or Line; S represents *S* the Center.

Of all the various Methods for finding the Representation of a Circle, in Perspective, there is none preferable to this; which I shall exemplify more at large in the 8th Section. For, if any regular Poligon whatever, be drawn in Perspective, a Circle may be described through all the Angles, or touching every Side. Six Points are not sufficient for describing the Curve with accuracy; besides, an Octagon is easier to describe than a Hexagon, seeing that, the Vanishing Points, required, are only the Center of the Vanishing Line and its Distance. The Octagon being inscribed in a Square, renders it, of all others, the most expeditious for finding the Representation of a Circle.

PROBLEM XXVI.

The Intersection of a Vertical Plane being given, and its Inclination to the Picture, to find the Representation of any Figure in the Plane.

Fig. 66. Let AB be the given Intersection, and C the Center of the Picture.

Through C, draw SD perpendicular to the Intersection, and CE perpendicular to SD, and equal to the Distance of the Picture.

Make CES equal to the Complement of the Angle of Inclination, cuting CS in S; and, through S, draw MN parallel to AB; MN is the Vanishing Line, S is its Center, and ES is its Distance. (See Prob. 3rd.)

If it be required to find the Representation of a Square, whose Side is parallel to the Picture, and equal to AB; at the distance A*d* from the Intersection.

Draw AS & BS; make SE^1 & SE^2 equal SE, and draw dE^1, cuting AS in F.

Draw FE^2, cuting BS in H; and draw FG and HI, parallel to AB.

Then, FGHI represents a Square, in that Plane, at the distance A*d* from the Intersection.

If

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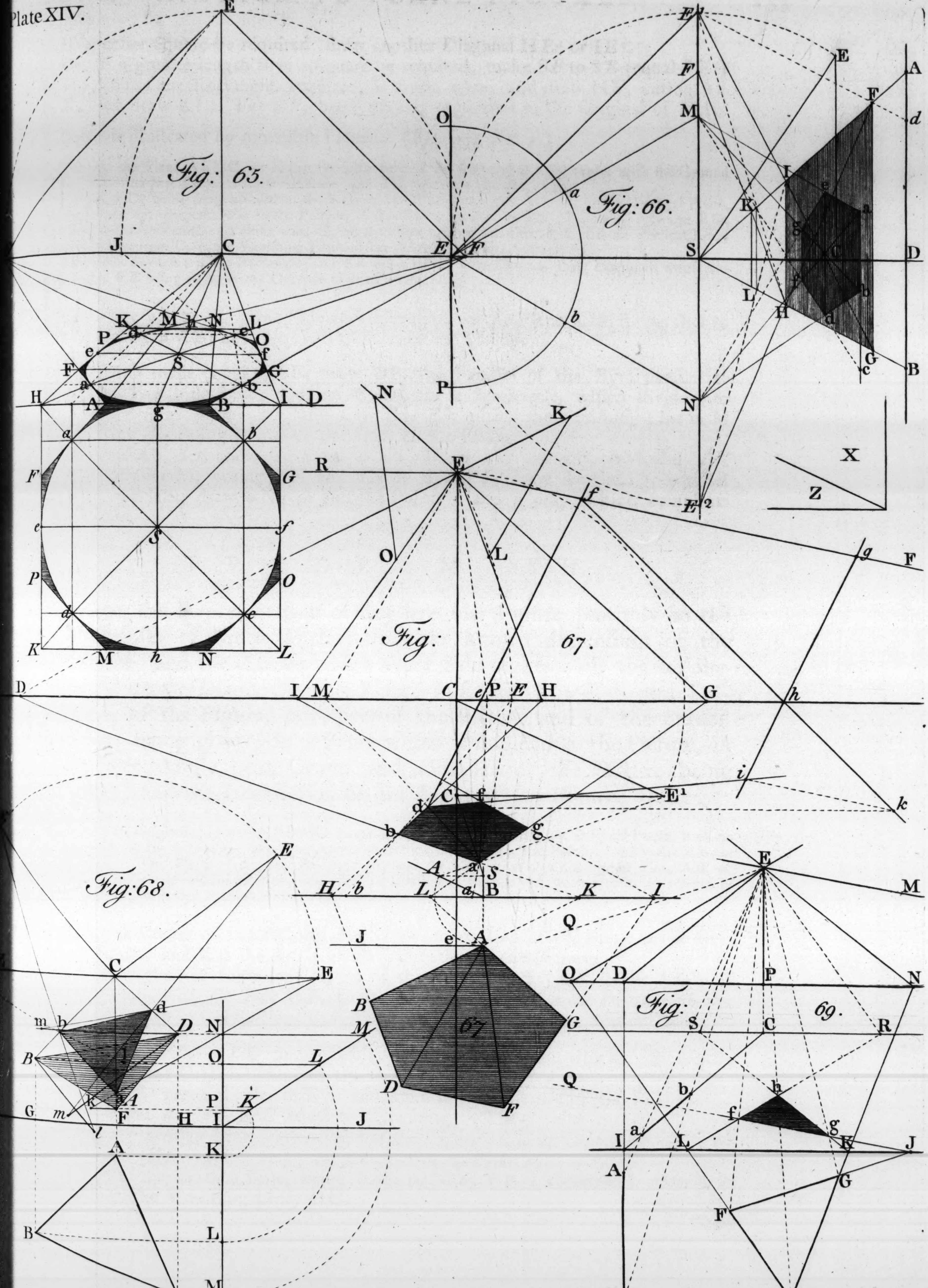
Plate XIV.



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If another Square be required, draw another Diagonal HE^1 or IE^2.

Or, if a greater length than a Square be required, make *SF* to S*E* (equal S*E*¹) as one Side of the Rectangle, required, is to the other, and draw *HF*, cuting AS in K, and draw KL. For *EF*, being drawn, is parallel to the Original of *FH*.

See this illustrated by moveable Planes. (Fig. 37, No. 1.)

Turn up the Plane AFDC, or X (on the other Side of the Picture) at right angles with the Ground Plane, RS is its Intersection with the Picture, and AB, with the Ground Plane.

AB and FD, being perpendicular to the Picture, vanish in its Center, C; and, if the Vertical Plane, V, be turned up, perpendicular to the Picture, *E*Q will be parallel to the Diagonals, AE and BD, and produce their Vanishing Point; and, Q, on the other Side of the Center, C (in the Vertical Line) equally distant from C, is the Vanishing Point of the other Diagonals, FB and EC.

The rest is obvious; if the Rectangles AFEB and BEDC were Squares, their Diagonals would vanish in E; CE being equal to the Distance of the Picture.

2nd. Let a b be the given Side of a Hexagon, in the Plane AIHB, parallel to the Intersection AB; it is required to describe the Hexagon.

S*E* being made equal to SE, draw OP, the Parallel of the Eye, parallel to MN, the Vanishing Line; and, on *E* describe a Semicircle, which divide into three equal Arks, in *a* and *b*, and draw *Ea, Eb*, cuting the Vanishing Line in M and N, the Vanishing Points of the Sides of the Hexagon.

Produce a b, and make b c equal to a b. Draw a M and a N, b M and b N. Also, draw c M, cuting b N and a N in d and f; draw d e and f g, parallel to a b; and lastly draw e N; or join the Points e and g only, which compleats the Figure.

PROBLEM XXVII.

To find the Representation of any irregular Figure (entirely on the Principles of Brook Taylor) from the known dimensions of the Figure, and the Angles which every Side makes with the adjoining Side or Diagonal; the Place, Position, Situation, and Distance of the Figure, in respect of the Picture and of the Station Line, being given; in a Plane which is inclined to the Picture, in a given Angle, the Center and Distance of the Picture being given, and the Intersection of the Plane of the Figure.

Note. The Original Figure (*ABDFG*) being geometrically drawn, in the Original Plane, is of no other use (in the following Operation) than to determine its Situation and Position; otherwise it is not necessary. The Place of the nearest Angle, A, and the Inclination of a contiguous Side, A*B*, or A*G*, in respect of the Intersection, being all that is wanted; the rest are determinable, in respect of themselves; as it will be exemplified. Fig. 67

C is the Center of the Picture, *HI* is the Intersection of the Plane of the Original Figure, and X is the Angle of its Inclination to the Picture.

A is the place of the nearest Angle of the Figure, and J*AB* is the Angle of the inclination of the Side A*B*, to the Intersection; AB is the Distance of the Angle A from the Intersection, and A e is its Distance from the Station Line.

Through C, draw EC perpendicular to the Intersection, which is the Vertical Line of the Original Plane of the Figure. (Def. D.)

Draw CE^1 perpendicular to EC, and equal to the Distance of the Picture.

Make the Angle CE^1C equal to Z, cuting EC in *C*; and through *C*, draw DG, the Vanishing Line of the Plane of the Figure; *C* is its Center, and *C*E¹ its Distance. (Def. 19 and 20.) See Exam. No. 3, Prob. 3.

*C*E, on the Vertical Line, being made equal to *C*E¹; through E, draw ER parallel to the Vanishing Line, DG.

Draw

Plate XIV. Fig. 67. Draw ED; making the Angle RED, equal J*AB* (i. e. to *BKH*, the Inclination of the Side A*B* to the Interſection) cuting the Vaniſhing Line in D; the Vaniſhing Point of A*B*†, and make the Angle DEG equal *B*A*G*, cuting the Vaniſhing Line in G, the Vaniſhing Point of A*G*‡.

† Prob. 2.
‡ Prob. 4.

Alſo, make the Angle DEH, equal to A*BH*, the Complement of A*BD*; and H is the Vaniſhing Point of *BD*; by the ſame.

Make the Angle GEI equal A*GI*, the Complement of A*GF*; cuting the Vaniſhing Line in I, the Vaniſhing Point of *FG*.

Laſtly, make the Angle IEF equal to *DFG*; EF is the Radial of *FD*; which, if produced would cut the Vaniſhing Line, in the Vaniſhing Point of *FD*.

Thus, are the Vaniſhing Points D, G, H, and I, produced, by the known Inclination of one Side to the Interſection, and the Angles which one Side makes with another (by Prob. 4.)

The place of the Angle A being given, in the Geometrical Plane (which is inclined to the Picture, in the Angle E'CC; and to the Horizon, in the Angle *C*E'C) find its Seat on the Picture.

Draw AB, perpendicular to the Interſection, *HI*, and make the Angle *A*B*L*, with the Interſection, equal to *C*E'C, by drawing B*A* parallel to E'C.

Make B*A* equal BA, and draw *A*S parallel to *L*B, i. e. perpendicular to AS, cuting AB produced, in S, S is the Seat of the Point A, on the Picture, and S*A* its Diſtance from its Seat. (See Orthography, Page 49, General Introduction.)

§ Prob. 6. Draw SC, and *A*E' cuting it in a, the Repreſentation of A §

Draw aD and aG the indefinite Repreſentations of A*B* and AG, from the Point a; which, if produced, would paſs through their Interſecting Points, *K* & *L*,

Make D*E* equal DE; and, through a, draw *E a* to the Interſection, cuting it in *a*.

Make *ab* equal A*B*, and draw *bE*, cuting aD in b; ab repreſent a Line equal to A*B* (Prob. 7, or 17.) Draw bH.

Produce DE, and make EK and EL reſpectively equal to A*B* and *BD*; or in the ratio of A*B* to *BD*; and draw EM parallel to KL, cuting the Vaniſhing Line, in M.

Draw aM, cuting bH in d. bd repreſents a Line in the ratio to that which ab repreſents, as EK to EL, i. e. as A*B* to *BD*. (Prob. 10*.)

|| Def. 22. For, EK is parallel to A*B*, and EL to *BD*; wherefore KL, conſequently EM, is parallel to the Diagonal A*D*; therefore, M is its Vaniſhing Point||; and abd repreſents the Triangle A*BD*. (Pr. 18.)

Now, F, the Vaniſhing Point of *FD* (being much inclined) is not in the Picture.

Draw d*f* at pleaſure, cuting the Vaniſhing Line, and EF, in *e* and *f*; and draw *gi* parallel to d*f* (at any diſtance, at diſcretion) and make *hi* to *gh*, as *e*d is to *ef*.

e. g. Draw the Diagonal *fh*; and draw d*k* parallel to the Vaniſhing Line, cuting *fh*, produced, in *k*; draw *ki*, parallel to FE, cuting *gi* in *i*; and draw d*i*; which will tend to the Vaniſhing Point F; (by Prob. 13, No. 2.)

Produce GE, and make EN equal A*G*; and, on EI, take EO equal *GF* (or, make EN and EO in the ratio of AG to *GF*) and draw EP, parallel to NO, cuting the Vaniſhing Line in P; the Vaniſhing Point of the Diagonal A*F*.

Draw aP, cuting d*i* in f; and laſtly, draw If, cuting aG in g, which compleats the Figure.

N. B. If the Sides of the Original Figure be produced to their interſecting Points, *H*, *I*, *K*, and *L*, the truth of the proceſs may be perceived, by its affinity and agreement with the former Methods, viz. when the Original Figure is drawn in the Geometrical Plane, its Vaniſhing Line, Center, and Diſtance being given; which, in this Problem, are given only by Poſition and Diſtance; the Original Figure being uſeleſs in the Operation, otherwiſe than to ſhew the meaning of every ſtep; and that, if the Original Figure be ſuppoſed in its true Place, and Poſition, each Line, from the Eye, producing a Vaniſhing Point, would be parallel to the correſponding Side, or Diagonal of the Figure.

In

* From the Poſition of A*B* and *BD* the meaſure of A*B*, or its Ratio to *BD*, is taken beyond the Eye, in the Radial of A*B*, produced.

I

In this Problem, I have summed up most of the Rules given in the two last Sections; which are called elementary, because, by them, the whole of practical Perspective is performed.

It will, I know, appear, to young practitioners in Perspective, somewhat difficult; yet, if they do but see the Principles on which it is performed, and will be at the trouble to go through the Process a second time, and compare each Operation with the Problem it refers to, that apparent difficulty will vanish; and they will find it of great advantage to them, in applying all the elementary Problems to real use, in delineating Plane Figures.

Now this Process, difficult and operose as it may appear, at first, is infinitely preferable to any other Method of performing it; nay, it is scarce possible to perform it at all, by any Rules given by the old writers on Perspective, or indeed by any; and, considering that I have given the whole procedure, from the begining to the end, it is not a long one. I have, not only, referred to the foregoing Problems for each particular step in the Operation, and (as is customary) left the Reader to apply it; but, I have gone on with him, socially, hand in hand; and, like a trusty Guide, pointed out every step he should take.

I have, also, drawn every Line in the whole Operation, which remains in the Plate; yet there is not the least confusion, the use of each being obvious, on inspection; but, being taken regularly, in the course of the Work, every apparent intricacy is unravelled.

At the same time, let it be observed, that there is no need for half the Lines to remain at once; e. g. after having found the Vanishing Line, DG, its Center and Distance, and the Vanishing Points of all the Sides of a Figure, all the operative Lines may be rubbed out before we begin to draw the Figure, itself; also, having found the Seat of A, all that were preparatory to it are of no further use; but they must all remain in the Diagram; which circumstance frightens many, with the apparent intricacy; which, in reality, is but apparent.

In order to compare the difference, I will shew, how it may be performed otherwise. But, as a Figure of so many Sides would be somewhat confused, I have made choice of a Triangle, in which the whole may be distinctly seen.

Let ABD be the given Figure, in the Geometrical Plane; of which, GI is the Intersection, and JI*M* is its Inclination to the Horizon; let NI be considered as a vertical Section of the Picture. Fig. 68.

Produce NI; draw AK, BL, & DM, parallel to GI, cuting IM in K, L, and M.

Make I*K*, I*L*, and I*M* respectively, equal to IK, IL, and IM; and draw *M*N, *L*O, and *K*P perpendicular to the Picture; NI being perpendicular to JI.

Draw AF, BG, and DH, perpendicular to GI, and produce them, till they cut *K*P, *L*O, and *M*N, produced, in *A*, *B*, and *D*, the Seats, on the Picture, of the Angles of the Triangle.

Having thus found the Seats of the three Points A, B, and D, the Representation of each may be found (by Prob. 6) as a, the Representation of A, in the preceding Figure.

C being the Center of the Picture, draw CE at pleasure, and equal to the Distance of the Picture; and, having drawn *A*C, *B*C, and *D*C, draw *A*k, *B* *l*, and *D*m (or *D* *m*) parallel to CE (or C*E*) and equal to the Distance of each Point from its Seat, respectively; viz. *A*k equal *K*P, *B* *l* equal *L*O, and *D*m or *m* equal *M*N.

Draw k E, cuting *A*C in a; l E (or *l* *E*) cuting *B*C in b, and m E (or *m* *E*) cuting *D*C in d.

The three Points a, b, and d, are the representations of the Angles A, B, and D; which, being joined by Right Lines, compleats the Figure.

In this Process, it is obvious, there is neither Vanishing Line nor Vanishing Point required, the whole being performed by the 6th Problem. If the Intersection, GI, be parallel to the Horizon, ECE may be considered as the Horizontal Line. But, if the Intersection of the Plane of the Figure, be given, it is immaterial what Position it has to the Horizon; the Center of the Picture and Distance, the Place and Position of the Object, and the Inclination of the Plane it is in, being also given.

Although this Method is consonant to the Principles of Brook Taylor, yet it is by no means so masterly or correct, that is, it is more liable to error, than the other, by means of Vanishing Points; which, in complex Objects, having various Lines in various Planes, is preferable to all other. In short, it is truly Perspective, which cannot be said of the old Authors; having few or no elementary Principles, to support or demonstrate the Rules they prescribed.

Plate XIV.

PROBLEM XXVIII.

To find the Representation of any Line, or Plane Figure, by means of the Directing Plane.

Fig. 69. Let AB be a Line perpendicular to the Picture, or to the Intersection, IK, of the Plane it is in. Let E be the Eye, in its true Place as in the foregoing Problems.

At the Distance EP, equal to the Prime Director*, draw NO, the Directing Line, parallel to the Intersection, IK†.

† Theo. 3.

The Space between NO and IK represents all that Space which lies between the Directing Line and the Intersection of the Original Plane with the Picture.

In which Construction, the whole of that Space, together with the Directing Plane, NEO, is supposed to be turned up, on the Intersection IK, till it falls into the Picture; the Original Figures being turned along with it, into the same Position as usual.

Now, IK is the Intersection, and NO is the Directing Line.

Produce AB, cuting the Intersection in I, its Intersecting Point; and the Directing Line in D, its Directing Point. Draw ED, which is the Director of AB‡; to which, draw IC, parallel.

‡ Def. 12.

For, C being the Center of the Picture, and I the Intersecting Point of a Line perpendicular to the Picture, IC is its indefinite Representation; and EC, the Direct Radial, is parallel to AB, and equal to ID, a part of AB, produced; consequently, IC is parallel to ED. (15. 1. El.)

As EC (Fig. 37) is equal to *B*R, a part of AC, intercepted between the Intersecting and Directing Lines; also, EO is equal to GP, and EL to H*S*; and being parallel, respectively, the Lines RC and E*B*, &c. are also equal and parallel.

If the Sides of the Triangle, FGH, be produced, till they cut the Directing Line, in the Points M§, N, and O, their Directing Points, EM, EN, and EO, are their Directors; to which their several Representations are parallel; L, K, and J, are the intersecting Points of those Lines.

Draw Lh parallel to EO, Kh parallel to EN, and Jf parallel to EM; which, by their Intersections, produce the Figure fgh, the Representation of FGH.

Through C, parallel to IK, draw RS, the Vanishing Line of the Plane of the Triangle; and, having produced the Indefinite Representations to their Vanishing Points, R, S, Q, the Lines ER, ES, and EQ, are the Radials of the Sides of the Triangle, and consequently parallel to them, respectively.

The agreement between these two Methods of producing the Representation, fgh, is so very obvious, as not to need any further explanation.

See this further illustrated, in Fig. 37.

The Sides of the Triangle, ZY and ZX, on the other Side of the Picture, being produced, cut the Intersection, AB, in *S* and P; and, the Directing Line, GH, in G and H; EG and EH, in the Directing Plane, are respectively parallel to the Representations of those Lines, PO and *S*L, the Directing Plane being parallel to the Picture. For, they are the Intersections of Radial Planes with the Picture and Directing Plane, which are parallel; by Def. 4.

The Planes W and X being turned over, till *E* coincides with E; the Section with the Picture, RC, is the indefinite Representation of AC (Theo. 11 and 12) and E*B*, its Section with the Directing Plane, is the Director of the Line AC (Def. 12) which is parallel to RC (by Theo. 14) being the sections of parallel Planes, by another Plane.

* In Theory, the Prime Director is a Right Line joining the Eye, or drawn through the Eye, and the Station Point of the Original Plane.

§ Suppose FG produced till it cuts ON, produced, and EM a Line tending to that Point; then EM is the Director of FG.

SECTION

SECTION VI.

THE last Section comprehends the whole Art of practical Perspective, in respect of Plane Figures; illustrated with variety of Cases and Examples; in which, I have studied more to render it easy and familiar, than to embellish the Subject with what is no way useful. Every Figure, which I have given are such, as are common, and applicable to Buildings and other Objects, except the Triangle and Pentagon, which are not so useful in themselves, as for the sake of managing inclined Lines, in all Positions; so that, the Lessons deduced from them are excellent. By varying their Positions and Situations, and diversifying the Cases, as much as is consistent with utility, I am persuaded that no Person, who has a tolerable Capacity, will be at any loss to know how to describe any Plane Figure, and in any Position whatever.

Although some Persons, of keener Talents, may imagine, that the preceding Section contained Instructions sufficient for every occasion (it certainly contains the Elements of the whole) yet, I presume, if no more had been given, it would be found of but little use to many; when, by applying those valuable Lessons to familiar Subjects, what, before, might appear somewhat mysterious will, hereafter, be applied with ease, as occasion may require.

It may with equal propriety be alledged that there is no need for shewing how Plane Solids are formed; seeing they are only composed of Plane Figures, in various Positions, all which have, already, been copiously treated of. Yet, a few Lessons, and familiar Examples, will not be found superfluous or unnecessary. I shall however dwell as little as possible on any thing not really useful, but hasten to apply the whole to useful Objects; for which, all the foregoing is but preparatory.

In this Section, I shall always consider the Object as situated on a horizontal Plane, and the Picture as vertical; most of the foregoing Problems, being general, are applicable in all Positions whatever.

In the following part of this Work, I would advise the young Student, as he proceeds, to draw every Object, or a similar one, at his own discretion. Or, if that be not necessary, in some Cases, let him carefully notice what things are given, in the Proposition, and what is required to be done; and then proceed, step by step; not supposing any thing as done, but what he has passed over; by which means, he will see, clearly, how every Line is produced, by which the Objects are formed.

This precaution would have been necessary in the third or fourth Section, but more particularly so here.

PROBLEM XXIX.

How to describe a Cube, perspectively, or any other right angled Parallelopiped*, having one Side, given, parallel to the Picture; at any Distance beyond the Picture, its Situation and Dimensions being known.

ECE is the Horizontal Line, and PM the Intersection or Ground Line.

C being the Center of the Picture, make CE, on either Side, or on both Sides, equal to its Distance; and let this be understood in future Examples, that CE (E being the Place of the Eye) either on the Horizontal or Vertical Line (the Picture being vertical) always denotes the Distance of the Picture; sometimes C*E*.

First;

* See the Difinition of a Cube, in the General Introduction, Section 1, Page 44.

Plate XV. Fig. 70. First; let the Picture be supposed applied close to a Face of the Cube; in which Case, being parallel to the Picture, it coincides with the Picture.

This is generally the Case, when there is but one Object; as it answers no purpose to suppose it at any Distance, from the Picture, as in Figures 59, 60, &c. or, when there are various Objects to be delineated, it is, most commonly, supposed to be applied close to the nearest, as in this Example (see Fig. 38) so that, the Distance of the Picture is the Distance of a Plane direct before the Eye, and touching the Object, or the nearest Object, if there are more than one.

Let AB, in the Ground Line, be the measure of a Cube, to be represented; having one Face in the Picture.

On AB, describe a Square, ABDF, which, being in the Picture, has the full dimensions of the Side given; wherefore, ABDF is a Face of the Cube required. (Cor. 5, Theo. 10.)

Draw DC and FC; and DE or FE, diagonal-ways, cuting FC in G, or DC in H; draw GH, parallel to FD, which compleats the Figure, AGHB, the representation of a Cube, in that Position, and Situation.

If another, of equal dimensions, be required, situated on the left side of the Station Line, at the Distance LJ; and distant from the Picture equal L*a*.

Make LM equal AB; and, on LM, describe the Square LMNO, as before.

Draw OC, NC, &c. and draw *a*E, cuting LC in a, and MC in e.

Draw a b parallel to LM; and a d, b c, to MN; also, draw c d parallel to a b. i. e. having drawn a b parallel to LM, describe a Square on a b; and, at e, describe the Square e f g h, parallel to a b c d, which compleats the Figure.

Or, having drawn LC, MC, and *a*E, compleat the Plan a b e h. (Prob. 19.)

Describe a Square on a b, and draw cC and dC; draw g h parallel to a d, and f g parallel to c d, which compleats the Figure.

† Cor. 5. Theo. 10. DEM. a b c d being parallel to the Picture (by Hypothesis) is similar to the Original, LMNO †; and its Sides have that proportion to the Originals, as the Distance of the Picture (CE) to the Distance of the Plane a b c d; viz. as EC+La; i. e. ab (or ad) : LM :: EC : EC+La. (Theo. 10.)

But, LMNO is a Square; therefore, a b c d is a Square.

‡ Cor. to Theo. 5. And, since it is parallel to the Picture, the Originals of the Sides a h, d g, &c. are perpendicular to the Picture; consequently they vanish in its Center C ‡.

§ Schol. to Theo. 5. ‖ Th. 3 & 6. † See N. B. Prob. 20. Also, a d g h represents a Square in a vertical Plane, perpendicular to the Picture; for LO is the Intersection of that Plane §, and *ECE* its Vanishing Line ‖; wherefore, C*E*, being made equal to CE, *E* is the Vanishing Point of Lines (in that Plane) inclined to the Intersection in half a Right Angle; consequently, if one Side (a d) be parallel to the Intersection (LO) and OP being made equal L*a*, P*E* passes through a Diagonal of that Side †, therefore, a d g h represents a Square; and J*b* being equal L*a*, *b*E is a Diagonal of the Top, d c f g, which also represents a Square, parallel to a b e h, and ON is its Intersection.

Therefore, a g f b represents a Cube, having three Faces seen, each of which, is the representation of a Square. Q. E. D.

PQ is the Intersection of a Diagonal Plane c d h e, whose Vanishing Line would pass through *E*.

The Cube AGHB being situated on the Station Line (which, on the Picture, coincides with the Vertical Line) has but two Faces seen; the Front, AFDB, and Top, FGHD, the Eye being above it, equal CZ. The other Faces, AFGK and BDHI, seeing they apparently incline towards the Vertical Line, are consequently lost to sight in this Position; as is obvious, supposing the Solid transparent.

E X A M P L E I.

To draw the Representation of a high Wall, the End being parallel to the Picture and at some Distance from it.

This Object, being a right angled Parallelopiped is easily delineated.

Fig. 71. Let PJ be equal to the Distance of the Wall from the Station Line.

Make PQ equal to its thickness, and draw PC and QC; make P*d* equal to the distance of the end of the Wall, from the Picture; draw *d*E cuting PC in R, and draw RS parallel to PQ.

Now, since P is the intersecting Point of RX, the common Intersection of the Plane of the Wall (which is supposed vertical) with the Ground; PT, perpendicular to the Ground Line, or parallel to the Vertical, is the Intersection of that Plane †.

† Cor. 1. Theo. 3.

And, since the Wall is perpendicular to the Horizon, consequently, its Angles or Corners, RU and SV, are parallel to PT or QW the Intersections of the Planes those Lines are in (for they are parallel to the Picture, being vertical) therefore, draw RU and SV parallel to PT, i. e. to the Vertical Line (or perpendicular to the Ground Line.)

Make PT equal to the known height, and draw TC, cuting RU in U, and draw VU parallel to RS; then RUVS, represents the End of the Wall, which is parallel to the Picture.

Lastly. Make PM equal to the Distance of the farther end of the Wall from the Picture, or *d*M equal to its known length.

Draw EM, cuting the indefinite Representation, of its common Section with the Ground Plane, in X; and draw XY, parallel to RU, which compleats the Fignre, required.

Or, if C*F* (on the Vertical Line) be made to CE or *E*, as the height of the Wall to its Length, *F* will be the Vanishing Point of the Diagonal RY, which cuts TC, in the Point Y, for its length; draw XY parallel to RU.

Let it be observed, here, that if there be not room enough, on the Intersection, to set off the whole length of the Wall, equal *d*M, take half the Distance, PJ (equal Pf, in the Plan below) make Ce equal half CE (the Distance of the Picture) and draw Je, projecting the same Point X.

If the half measure be too much, take a third or fourth, or any equal portion, of PM (equal Pm) the same part being taken of CE will answer the same purpose (see Prob. 17.)

EXAMPLE II.

How to represent several Parallelopipeds, as Blocks of Stone, &c. for the Basement of a Building, ranged in a Right Line perpendicular to the Picture.

Let ABD, the Front of the first, be close to the Picture, i. e. in the same Plane; consequently, in its geometrical Proportion, of height and width. Fig. 71.

Draw AC, BC, and DC. Make A*a*, on the Intersection, equal to the length of the first, *ab* equal to the space between the first and second, and *bc* equal to the second.

Draw *a*E, *b*E, and *c*E, cuting AC in F, G, and H; draw FI, GK, and HL perpendicular, cuting BC; and IM, KN, and LO, parallel to BD, cuting DC, in M, N, and O, and compleat them, as in the Figure.

Now, in order to continue them further, to any length, this Expedient may be used, when the measures exceed the limits of the Picture, and, consequently, cannot be applied to the Intersection.

Draw Hg, from the farthest Corner of the second Parallelopiped parallel to the Intersection, and draw *a*C and *b*C, cuting it in e and g.

If it be required to continue them of the same Dimensions, and Space, make Hf equal eg, and draw fE and gE, cuting AC in a and b, and compleat the Parallelopiped abcd as in the Figure.

After the same manner they may be repeated as often as you please by drawing b*g* parallel to gH, cuting fC and *b*C in *f* and *g*, and then proceed as before.

EXAMPLE III.

To draw several long Parallelopipeds, parallel amongst themselves, and perpendicular to the Picture; representing large Joists, supporting a Floor (over Head) parallel to the Horizon.

Let W, X, Y, and Z be the ends of the Joists, of equal Dimensions, and equally spaced; and, since they are supposed to be parallel to the Picture, therefore, they have their geometrical Proportion on the Picture (See Cor. 5, Th. 10.) Fig. 72.

Let them be drawn accordingly, by a Scale of equal Parts.

Plate XV. Fig. 72. Then (becauſe the Joiſts are perpendicular to the Picture, in reſpect of their length; i. e. they are horizontal, and at right angles with the Picture) draw a C and b C, &c. from every Angle, neceſſary, as in the Figure; and determine their length, by Prob. 15, i. e. make AB or *AB* equal to the required length, and draw BE, cuting AC in D; or *A*C in *D*.

Through D, draw FG parallel to the Vaniſhing Line, CE, which determines the under Sides, F e, &c. and the perpendicular Lines, d e, &c. compleats them.

The Diviſions of the Boards are determined by Prob. 8th; i. e. take any meaſure, *Aa*, and repeat it as often as there is occaſion, if the Boards are ſuppoſed equal; otherwiſe *Aa*, *ab*, &c. muſt be made in the ſame Ratio as the Boards, to each other; and at the laſt Diviſion, *e*, draw *e D*, cuting the Vaniſhing Line in H; and draw *a*H, *b*H, &c, cuting *A*C in 1, 2, 3, &c. through which, draw Right Lines parallel to the firſt, *AB*, i. e. to the Vaniſhing Line.

Or, divide *AB* into the ſame number of Parts, and draw Lines to E, which will produce the ſame, as is obvious; ſeeing that it does not depend on the dimenſions of the Parts, but on their number and ratio to each other. (See Pr. 8.)

PROBLEM XXX.

To draw the Repreſentation of a Cube, or other right angled Parallelopiped, any how inclined to the Picture.

Fig. 73. Firſt. Let the Square ABCD be the Plan of a Cube, in the Geometrical Plane, equally inclined to the Picture, and to the Interſection.

Lie down the Diſtance of the Picture, CE^{1}, from the Center, as uſual; E and E are the Vaniſhing Points of all the Lines of its Sides. (Prob. 20.)

Draw AE, both ways, A being the interſecting Point of two Sides, AB and AD.

Find the perſpective Plan A*BCD* (by Prob. 20) or of the two Sides AB, AD only.

Draw AF perpendicular to the Ground Line, and equal to AB, a Side of the Cube, and draw FE both ways.

Draw *B*G and *D*H parallel to AF; and draw GE and HE diagonal-ways cuting each other, at I, which compleats the Cube AGH.

Otherwiſe, without drawing the perſpective Plan, by the Vaniſhing Lines of its Faces.

Draw AF perpendicular to the Interſection, and equal to a Side of the Cube.

Draw AE and FE, both ways; and through E, on either Side, draw the Vaniſhing Line of the contiguous Face of the Cube. (Prob. 3.)

Make E*E* or E*F*, on either Side, equal EE^{1}, and draw A*E* or A*F*, cuting the indefinite Repreſentation FE, in G or H, and draw *B*G, or *D*H, perpendicular.

Draw *BD* parallel to the Interſection, and *D*H parallel to AF; and laſtly, draw GE and HE, interſecting at I, as before.

A*B*GH*D* is the Repreſentation of a Cube, in the Poſition required; ſituated on a horizontal Plane, and its vertical Faces equally inclined to the Picture.

DEM. For, ABCD the given Plan is a Square, wherefore A*BCD* repreſents a Square (Prob. 20) and FGIH alſo repreſents a Square, in a Plane parallel to A*BCD*. (Th. 5, and Cor. 1.)

And, they are between the ſame Parallels, AF, *B*G, &c. therefore equal - - - (18. 1. El.)

But, AF is the Interſection of the vertical Planes, AG and AH; and E*E* is the Vaniſhing Line of AG (Theo. 3 and 11) E is its Center (Def. 19) and EE^{1} its Diſtance (Cor. 2, 4.)

Wherefore, *E* is the Vaniſhing Point of Lines inclined to the Interſection of that Plane, in half a Right Angle or 45 Degrees; conſequently, it is the Vaniſhing Point of the Diagonal of a Square, in that Plane, whoſe Sides AF and *B*G are parallel to the Picture (Prob. 19;) and, A*B* and FG, perpendicular to them, vaniſh in E, the Center of the Vaniſhing Line, EE.

Therefore, AFG*B* repreſents a Square, at right Angles with A*BCD* and AFH*D*, which (after the ſame manner) may alſo be proved to be the repreſentation of a Square; and their oppoſites, *D*HIC and *C*IG*B*, being between the ſame parallel Planes, are alſo repreſentations of Squares; conſequently, A*B*GH*D* repreſents a Cube. (See Defin. Page 44.) Q. E. D.

acfg

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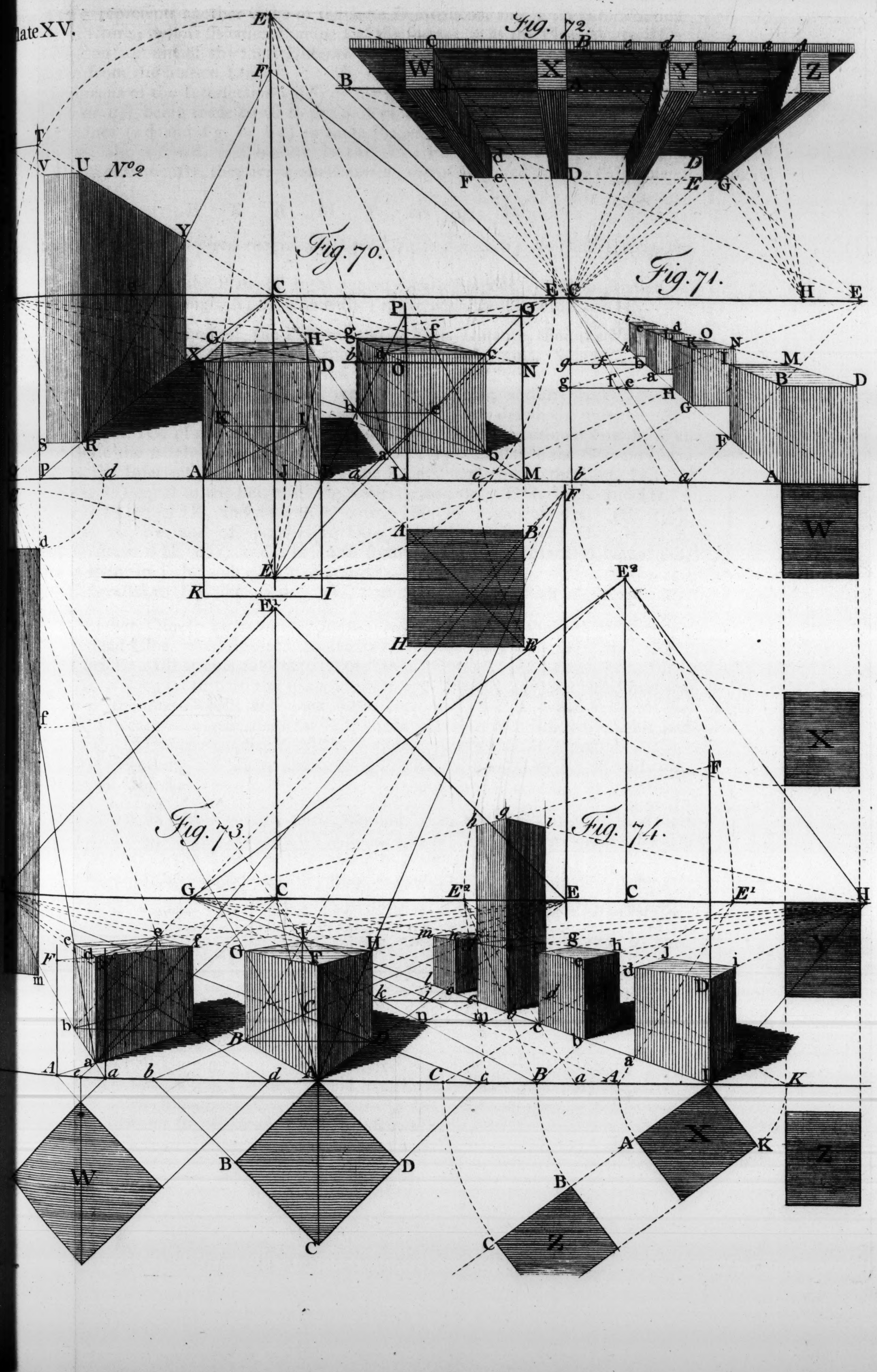
Plate XV.



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acfg represents another Cube of the same Dimensions, and in the same Position to the Picture; whose situation in respect of the former, is as the Plan, W, to BD; which, on account of the short Distance (CE) has a distorted appearance, being remote from the Station Line.

By means of the Intersection (*AF*) of the Face adfg, or *af* of the Face abcd (*AF*, or *af*, being made equal to the Side of the Cube) the proportion of the parallel Lines (ad and fg, or bc) in those Planes are determined. For, *AF*, ad, and fg; also *af*, ad, and bc are in the same Plane, and are all parallel to the Picture; consequently, they are parallel between themselves, and to their Originals†.

† Theo. 10. Cor. 1.

EXAMPLE IV.

How to draw Parallelopipeds ranged in a Right Line, and inclined to the Picture, obliquely.

Let X and Z be the Plans of right angled Parallelopipeds; their inclination to the Picture is the Angle AI*A*, which they make with the Ground Line I*C*. Fig. 74.

C being the Center of the Picture, and C*E* its Distance, find the Vanishing Points, G and H, of the Sides and Ends of the Parallelograms, X and Z, and lie down their Distances on the Horizontal Line at E^1 and E^2.

Transfer all the measures IA, IB, &c. of the Objects, and the spaces between them, to the Intersection, at *A*, *B*, and *C*; and, having drawn the indefinite Representation, IG, of the whole length, draw AE^1, BE^1, &c. cuting it in a, b, & c.

Because the Angle I touches the Picture, IF, perpendicular to the Ground Line, will be the Intersection of the Plane of the Fronts, and also of the End, IK.

Make ID equal to the height of the Objects, and draw DG, DH, and IH.

Make I*K* equal IK, and draw KE^2 cuting IH; Ik represents IK. (Pr. 17.)

Draw ad, be, and cf, parallel to ID; and, where they cut DG, in d, e, and f, draw dH, eH, and fH; and from i, where ki cuts DH, draw iG, cuting them in j, h, and g, which compleats the upper faces.

hl, parallel to ID, &c. cuting kG, compleats the End behl.

If another Object be required, in the same Line, seeing that the measures on the Ground Line, would exceed the limits of that Picture, take GE^2 half GE^1, and draw E^2c till it cuts the Intersection, at *a*.

Make *aB* equal to half the space between the Objects, and *Bj* equal half the width of the next Object, and draw BE^2, &c. cuting IG in *b* and *c*.

If it be required higher than the rest, make IF equal to its known height, and draw FG. Draw perpendiculars from *b* and *c*, cuting FG in *g* and *h*.

Draw *b*H cuting kG in *d*, and draw *g*H and *di* parallel to *bg*, which compleats that Object.

Or, if cn be drawn parallel to the Ground Line, and *B*G cuting it in n; divide cn, in m, as *AC* (equal AC) is divided in *B*; and, from m and n, draw to E^1 cuting IG in the same Points *b* and *c* as before.

By either of these expedients they may be continued, if required, to any length.

Note. G, the Vanishing Point of the front Line IC, and the parallels to it (being much inclined) may be supposed to be beyond the limits of the Picture; consequently, Ic, Df, &c. are tending to a Vanishing Point which is not on the Picture, and must, therefore, be drawn by means of the Expedients, in the 13th Problem; which are very convenient when but few Lines are wanting.

But, a better Expedient than them all is (when the Point is not very remote) to fix a Lath to the Drawing Board, or straining Frame of the Picture, and continue the Vanishing Line as far as is requisite, on the Lath; fix a Pin in the Vanishing Point, and, with a long Ruler, draw the Lines IG, DG, &c. for no other Expedient whatever can be so true, i. e. it cannot be performed so accurately, as to have the Vanishing Point itself.

Prisms of all kinds may be represented after the manner of the foregoing, which are right angled Prisms, having found the Plans of their Bases, on the Ground Plane or other Plane, whether they be triangular, quadrangular, or multangular; of which one or two Examples will be sufficient.

EXAMPLE

Plate XVI.

EXAMPLE V.

To represent a hexagonal Prism, perpendicular to the Ground Plane.

Fig. 75. The Situation, Distance, and Position being determined, or AB being a Side, given or found, in Perspective, whose Radial is EV, find the Vanishing Points, I and K, of the other Sides, and compleat the Plan ABCDEF of its Base (by Pr. 24.)

Draw Aa, Bb, &c. perpendicular to the Ground Plane, i. e. to its Vanishing Line IK or Intersection G*D*.

Produce any Side, as AB, to its intersecting Point, G, and draw GH, perpendicular to the Ground Line, G*D*.

GH is the Intersection of the Plane of the Face AabB, or gh of BbcC (Pr. 3) for it is vertical; and, G, or g, is the Intersecting Point of one Line in the Plane.

† Prob. 13. Make GH equal to the known height of the Prism, and draw HV† cuting Aa and Bb in a and b; the Original of ab being parallel to the Original of AB.

Then, because Aa and Bb are the common sections of the contiguous Faces with AabB, draw aI and bK, cuting the other Perpendiculars Ff and Cc, in f and c; which compleats as many Faces as can be seen.

The other Faces (which are all Parallelograms, See Def. Page 44) Fe, Ed, and dC are supposed to be seen through, the Object being imagined transparent; which, in some Cases, is a necessary Expedient; by which means, the connection of the several Parts are more accurately determined.

AB and *AF*, below the Ground Line, are the Originals of AB and AF in their true place and position to the Picture.

EXAMPLE VI.

To represent a pentangular Prism, laid along on the Ground Plane; the Intersection and Seat of the Object, on the Ground Plane, being given.

Fig. 76. AB is the given Side of the Pentagon, and BD the Seat of one Face of the Prism.

C being the Center of the Picture, and CE its Distance, find the Vanishing Points, I and K, of AB and AD (Prob. 2) by making the Angles JEI and OEK equal, respectively, to *BA*B and *DA*D; or, having determined the one, find the other, by Prob. 4.

Make ab to represent AB (Prob. 17.) and, on ab construct a Pentagon, in a vertical Plane, on the same Principles as in Prob. 23.

I being the vanishing Point of ab; FG, passing through I, is the Vanishing Line of the Plane of the end of the Prism (being perpendicular to the Ground Plane) I is its Center, and EI its Distance. (Prob. 3.)

As there is not room, on the Picture, beyond the Vanishing Line, FG, to find the Vanishing Points of the Sides of the Pentagon, take I*E* on this Side equal to IE, through which draw MN parallel to FG, and, on *E* describe a Semcircle; which, divide into five equal parts.

Make *ab*, *bc*, and *ad*, *de*, each equal to a fifth part, and draw *E*b, *E*c, &c. which produce to the Vanishing Line, producing the Vanishing Points, F, G, &c. by, which the Pentagon abcde is compleated, as in the Figure (by Prob. 23.)

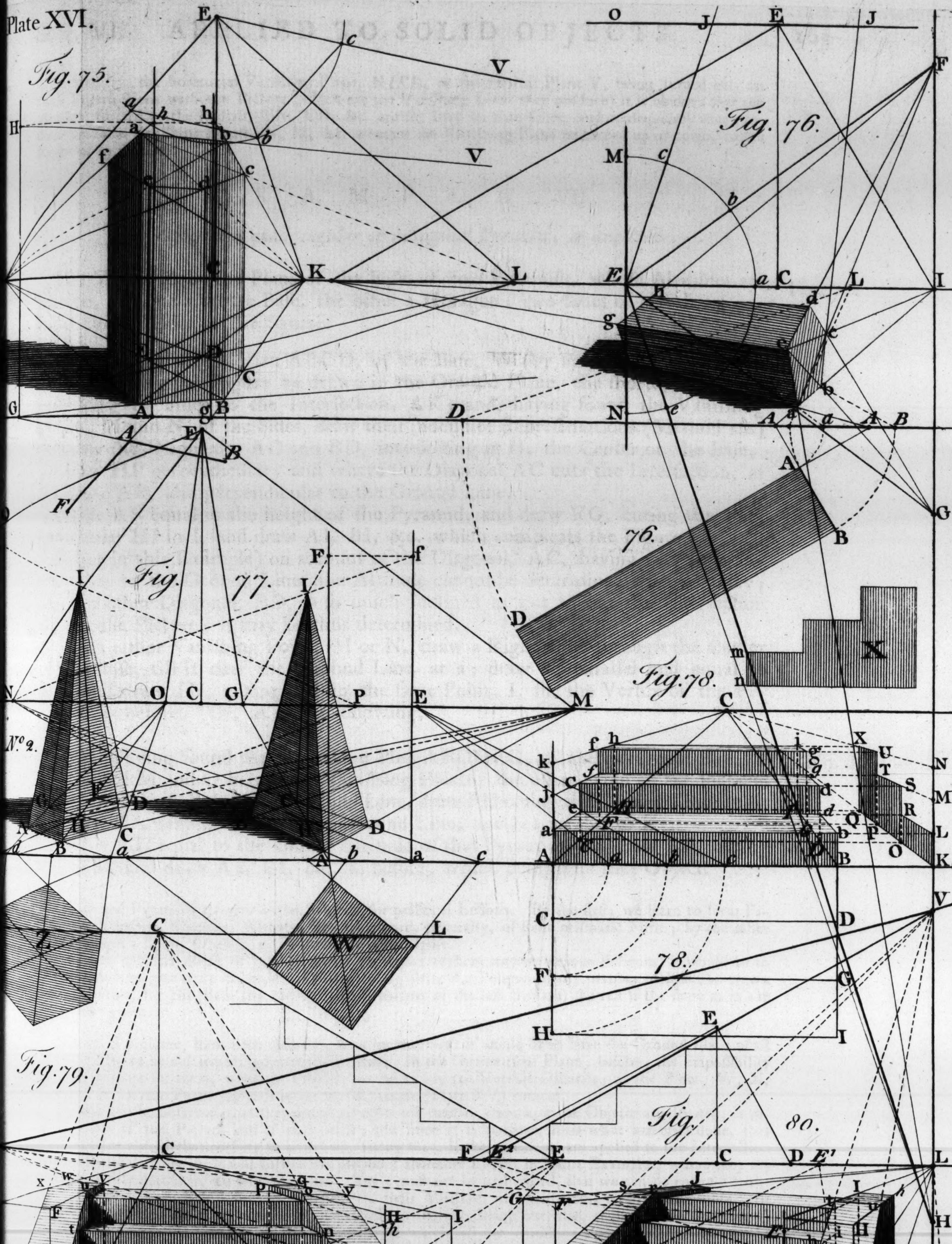
Draw aK indefinite, of the Side AD; and make af to represent a length equal to AD (Prob. 17) also, draw eK and dK.

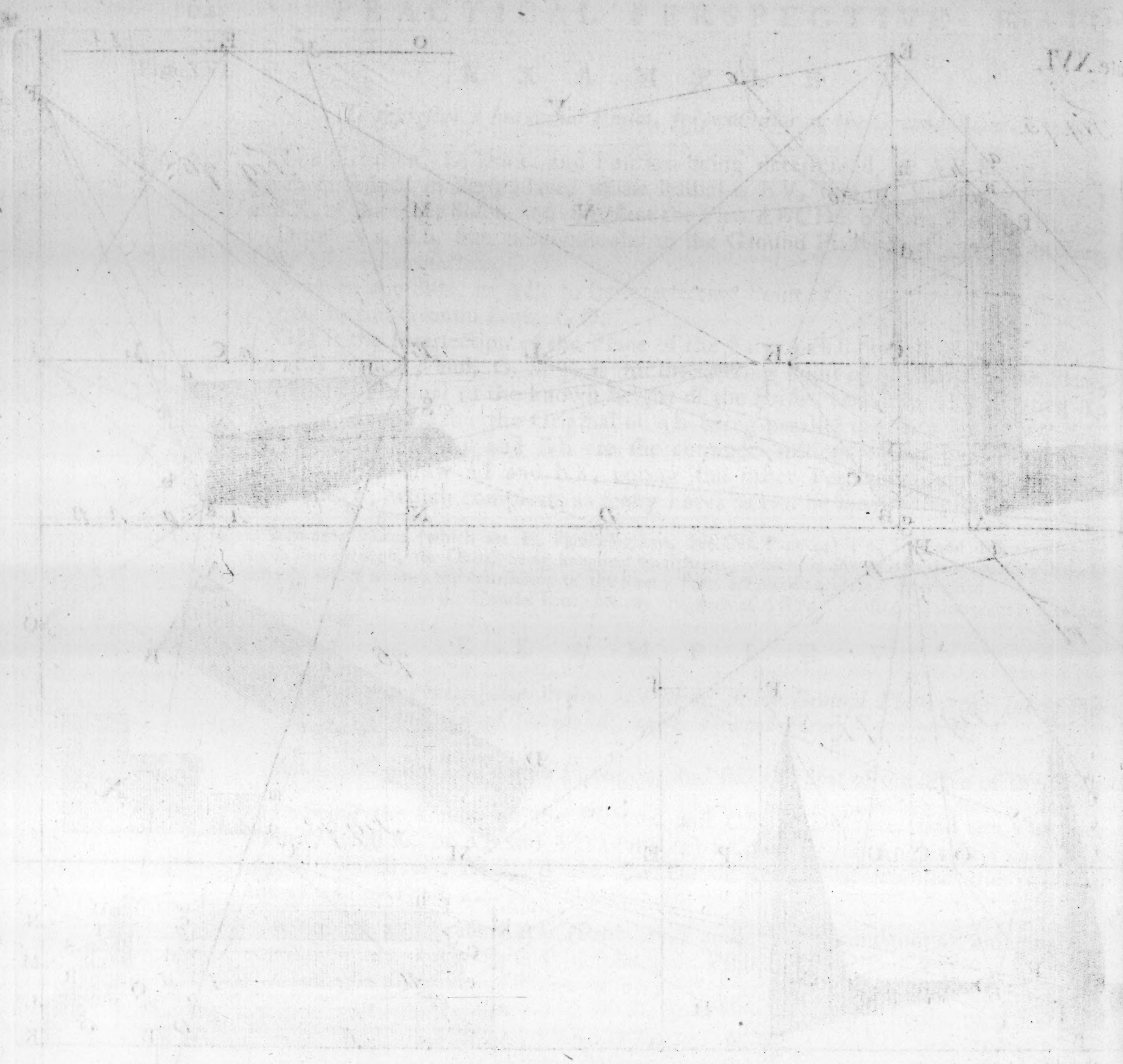
Draw Hg, through f, and gF, cuting dK in h, which compleats the Figure.

For, afge and eghd represent Parallelograms, which are Faces of the Prism afhdb; and abcde represents an End which is a regular Pentagon; to which the other Faces are perpendicular, IEK being a Right Angle (which IaK represents, Prob. 4) equal BAD.

The Expedient, of turning over the Vanishing Plane, F*E*H, to find the Vanishing Points of the Sides of the Pentagon, is a very useful and necessary Expedient; because it often happens, that there is not room for turning it over on the other Side of the Vanishing Line, if it be remote from the Center. And, notwithstanding it interferes with the Object, in this Diagram, yet being done first, and the Vanishing Points found (as in the Figure) all the operative Lines (for finding them) are rubbed out before we begin to draw the Object.

Plate XVI.





In Fig. 37, the horizontal Vanishing Plane, N*IK*L, or the vertical Plane V, being turned over on their Intersections with the Picture (which are the Vanishing Lines they produce) it is obvious that the Angles, formed by the Radials, EN, ED, &c. are the same on both Sides, and consequently they produce the same Vanishing Points, N, M, &c. whether the Vanishing Plane be turned up or down, to the Right or to the Left.

EXAMPLE VII.

To represent a quadrangular or hexagonal Pyramid; or any other.

Let W and Z be the Plans of the Bases of two Pyramids, whose Altitudes are known, the one a square Base, the other a Hexagon; two Sides of which are parallel to two Sides of the Square. Fig. 77.

Find the perspective Plan ABCD, of the Base, W (by Prob. 21.)

If the Original Figure be drawn in the Ground Plane, the shortest way is by producing its Sides to the Intersection, AK; and, having found the Vanishing Points, M and N, of the Sides, draw their indefinite Representations (Method 1st.)

Draw the Diagonals, AC and BD, intersecting at H, the Center of the Base.

Draw HI perpendicular; and where the Diagonal AC cuts the Intersection, at A, draw AF, also perpendicular to the Ground Line.

Make AF equal to the height of the Pyramid, and draw FG, cuting the Perpendicular HI in I, and draw AI, BI, &c. which compleats the Figure.

If (as in this Example) on account of the Diagonal, AC, having very little inclination to the Ground Line, the Altitude cannot be determined with accuracy; and the other Diagonal, BD, is so much inclined as not to cut the Intersection within the Picture; it may be thus determined.

From either Vanishing Point, M or N, draw a Right Line through the Center of the Base, till it cuts the Ground Line, at a; draw af parallel and equal to AF, and draw fN, cuting HI in the same Point, I, for the Vertex of the Pyramid, as before. Or, as in the following.

2nd. Having found the perspective Plan ABCDFG, of the hexagonal Base, Z (Prob. 24) M and O being two Vanishing Points; the other is out of the Picture. No. 2.
From any Point, P, in the Vanishing Line, draw PH cuting the Ground Line, in K.

Draw KL perpendicular to the Ground Line, and HI parallel to KL.

Make KL equal to the known Altitude of that Pyramid, and draw LP, cuting HI in I; and draw AI, BI, &c. as before, which compleats that Object.

Prisms and Pyramids are very useful Subjects for practical Lessons. By the first, we learn to form Pavillions, Temples, Cupolas, Alcoves, &c. which are, generally, of some prismatic Form; by the other we may form a Spire, Obelisk, or Pyramid, of any Figure.

As those Objects admit of infinite variety (as many as there may be various Poligons) it would be to little purpose to give more Examples of them, seeing that, if a Poligon of any number of Sides be drawn in Perspective, for the Base (by the various Problems of the last Section) the rest is the same as in the Examples given.

It may, I presume, have been observed, how inconvenient it would be to have the Ground Plans of all the Objects, to be delineated, geometrically drawn, in the Geometrical Plane; besides, the impossibility of having room for them, when the Objects extend to any considerable Distance; as the Plans, W, X, Y, and Z, of the Parallelopipeds in the 2nd Example (Plate XV) evinces.

It may also be observed, that they are of no other use than to know how the Objects are situated, in respect both of the Picture and of each other; but since it is obvious, from what has been done, that they are not absolutely necessary to be drawn, seeing that, if their measures are applied to the Intersection of the Plane they are in, it will answer the purpose; therefore I shall, in future Examples, where they are not absolutely necessary, do without them. Yet it must not be understood, that we can do entirely without, which is impossible; but, in many Cases, their Measures and Distances, only, are requisite. In others, the Situation and Position of one Object to another, being irregular, require a true and correct Plan to be first drawn (but not on the Picture) from which, all the Vanishing Points may be ascertained; as every Line, in Perspective, is determined from geometrical Proportion, and position of the Object to the Picture; consequently, they must be known or imagined, otherwise we have no foundation to build on.

Plate XVI.

EXAMPLE VIII.

To represent right-lined, or streight Steps, parallel to the Picture.

Fig. 78. Let ABIH be the Plan of three Steps; AB, CD, and FG are the Seats of the Fronts on the Ground Plane. X is a Section of the whole.

The first Step AB is in the Intersection; consequently (being right angled) the Ends, AH and BI, are perpendicular to it.

EC is the Horizontal Line, C the Center of the Picture, and CE its Distance.

Draw AC and BC. Make *Aa*, *ab*, and *bc*, each equal to the width of a Step, and draw *a*E, *b*E, and *c*E, cuting AC in *C*, *F*, and *H*.

Draw *CD*, *FG* and *HI*, parallel to AB, cuting BC, in *D*, *G*, and *I*.

CD, *FG*, and *HI*, are the perspective Seats of the several Steps.

Draw Ak perpendicular to AB; on which take Aa, aj, and jk, each equal to mn (the height of a Step in the Section, X) and draw aC, jC, and kC.

From the extremes of the Seats, *CD*, &c. draw *C*c, *F*f, and Hh, parallel to Ak, i. e. perpendicular to the Intersection, cuting aC, jC, and kC, in ec, *f*f, and h.

Draw e*d*, cd, *f*g, fg, and hi, cuting the Perpendiculars from *D*, *G*, and I, in b, d*d*, &c. and draw b*d*, d*g*, and gi, tending to C, which compleats the Steps.

Otherwise; draw KN perpendicular to the Ground Line, at a convenient distance from the Plan, and draw KC; on which, make KO, OP, and PQ represent the measures of the Steps, perspectively, as at *C*, *F*, and *H*, on AC.

Make KL, LM, and MN each equal to mn, and draw LC, MC, and NC, cuting the Perpendiculars from O, P and Q, in R, S, T, &c.

Make AabB a Rectangle, equal to the Front of a Step, geometrical.

Draw aC and bC; and from R, draw Re parallel to ab, cuting them in *d* and e; draw ec and *d*d, perpendicular; and from S, draw Sc cuting them in c and d; and proceed as before, by drawing cC and dC, and T*f*, cuting them, &c.

By this method of proceeding, the Work may be kept cleaner, as it does not interfere with the Object; but it is not so much to be depended on for accuracy; especially, as the Steps advance higher, and approach near the Vanishing Line; seeing, the Lines intersect so very oblique, that the point of section is not easily distinguished.

The front of each Step, being parallel to the Picture, are similar to each other; and have that proportion to each other, respectively, as their several Distances, i. e. the second Step, cd, is, in length and height, to the first, Ab, as EC is to EC+A*a*; and fg:ab::EC:EC+A*b*; also, fg:cd::EC+A*a*:EC+A*b*.†

† Theo. 10.

For, EC is the Distance of the Picture, and A*a*, *ab*, were made equal to AC, CF.

The Triangles, A*Ca*, E*C*C, also A*Fb* and E*FC* are similar; and so are the Triangles a*C*A, and e*C*C; and also, a*C*b and e*Cd*; and cd is equal to e*d*, &c. (15. 1. El.)

Wherefore, *C*C:A*C*::EC:A*a*, and consequently, *C*C:AC::EC:EC+A*a*; and *C*e:Aa::*C*C:AC, and ec=*C*e, consequently, ec:Aa::EC:EC+A*a*; also, eC:aC::*C*C:AC, and e*d*:ab::eC:aC, i. e. ::*C*C:AC, i. e. ::EC:EC+A*a*; consequently, cd (equal e*d*):ab::EC:EC+A*a*, &c. that is, as the Distance of the Picture (equal to the Distance of the first step) is to the Distance of the second Step; and so of the rest; as affirmed above; by 4. 6. El.

EXAMPLE IX.

To represent Steps with Kirbs of stone at the ends; as at the entrance into a House, &c.

First, when they are parallel to the Picture; and the Picture close to them.

Fig. 79. Take AB, on the Ground Line, equal to the width, or more properly, to the length of the Steps; and, AD and BJ to the thickness of the Kirbs.

Make

Make AF equal to their height, and describe the Rectangles AFGD, and BHIJ, geometrically; which are supposed to be in the Picture.

C is the Center of the Picture, and CE the Distance. Draw AC and BC.

Make B*a* equal to the Distance of the first Step from the Kirb; and *ab*, *bc*, and *cd*, each equal to the breadth of a Step, and draw *a*E, *b*E, &c. cuting BC in a, b, c, &c. the perspective breadths of the Steps, on the Ground.

Make B*e*, *ef*, &c. each equal to the height of a Step, and draw *e*C, *f*C, &c. cuting the Perpendiculars, from a, b, c, &c. in l, m, &c. and transfer them, by the parallel Lines af, bg, &c. on the Ground, to the other End, and compleat the Steps, as by the last.

The upper Step having a greater breadth than the other, make *d*D equal to its breadth, and draw DE cuting BC in e; the Perpendiculars do and ep cut *b*C in o and p; and ou, pv, being drawn parallel to the Ground Line, cut the corresponding Perpendiculars iu and kv, which determine it; viz. opvu.

If the Kirbs are square, they are right angled Parallelopipeds, and are delineated, as described in Problem the 29th. AFGD and BHIJ are the Fronts, AFvk and BHpe are the Sides, which inclose the Steps; and, FvwG and and HpqI are their upper Faces.

If they incline with the Steps (as is common) make the Rectangles AK and BL equal to the Ends; suppose, equal to the height of a Step.

Draw C*C* perpendicular to the Horizontal Line; and, CE being equal to the distance of the Picture, make the Angle CE*C* equal to the inclination of the Steps, cuting C*C* in *C*, which is the vanishing Point of their Inclination.

VL, passing through *C* parallel to the Horizontal Line, is the Vanishing Line of a Plane inclined to the Horizon, in the Angle *C*EC, equal E*C*L, and *C* is its Center (see Fig. 15, No. 3, P. 72, or Sec. 12)

For C*C* is perpendicular to VL; and E*C* producing the point *C*, is the Radial of such Lines in the inclined Plane as are perpendicular to its Intersection KL; therefore *C* is their Vanishing Point. (Def. 22.)

Draw K*C*, L*C*, &c. cuting G*C*, I*C*, &c. in xy, &c. which compleats the inclined Faces of the Kirbs, as in the Figure; by drawing xu and oy.

CASE 2nd. EXAMPLE X.

When the Steps are inclined to the Picture; the inclination being determined, also their Situation, in respect of the Station Line.

In this Example I shall suppose, that there is not room on the Picture for ascertaining the Vanishing Points and their Distances, as usual; in order to apply the 12th Problem.

C being the Center of the Picture, and KCL the Horizontal Line, draw CE perpendicular, and take CE any equal part (suppose one third part) of the determined Distance of the Picture. Fig. 80.

Make the Angle CED equal to the Inclination known; and CD will be one third part of the Distance of the Vanishing Point L, of one Side; which being determined, all the rest are determinable, arithmetically (by Prob. 12.)

Or, make DEF a Right Angle; CF will be a third part of CK; and FE will be a third part of its Distance, by the same, &c.

Having obtained the Vanishing Points, K and L, and their Distances E^1 and E^2 being laid down (CE^1 being equal to three times the difference between FC and FE; and CE^2 to three times the difference between CD and DE) then, proceed as usual; A*E* being the Intersection of the inward Angle of the Kirb.

Draw AK, the indefinite Representation of the Front, and AL of the End.

Let AJ be the length of the Steps (eq. AB, Fig. 79) draw JE^1 cuting AK in *A*.

Make A*a* equal to the distance of the first Step; *ab*, and *be*, &c. each equal to the breadth of a Step, and draw aE^2, bE^2, &c. cuting it in a, b, &c. and, seeing there is not room on the Intersection, A*c*, to apply the measure of the top Step, which, suppose equal to A*c*, draw d*f* parallel to A*c*, and *c*L cuting it in *f*; and draw fE^2 cuting AL in B.

Draw

Plate XVI. Fig. 80. Draw the Perpendiculars a j, b f, &c. indefinite.

On *AE*, take A*B*, *BC*, &c. each equal to the height of a Step, and draw *B*L, *C*L, &c. cuting the Perpendiculars from a, b, &c. as in the Figure.

Draw a K, and *A*L cuting it in k; draw k l perpendicular, and j K cuting it in l; draw l L, and e K cuting it in m, and f K cuting the Perpendicular, m n in n, and proceed, after the same manner, throughout.

Or, having drawn all the Lines e K, f K, &c. indefinite, draw l L, and m n perpendicular to the Ground Line; then n L, and o p, &c. by proceeding after that manner the Steps are compleated.

The Kirb being equal to B J (as in the former Figure) draw B E[1], cuting AK in *D*; *AD* represents the thickness of the Kirb, at that end.

The Original of the Point, G, being supposed on this Side of the Picture, and being in the Ground Plane, the Representation (G) is consequently below the Ground Line, AB. (See N. B. Prob. 8. Fig. 45. The measure being applied on this Side, i. e. below AB, as AH, supposes AB proportioned perspectively; but, being at the other Extreme, as a B, it is projected; and is, consequently, larger than the Original.)

Make A *d* equal to BJ, and draw *E*[1] *d*, cuting K A produced, in G; AG is the projective representation of the end of the Kirb, the Original, of which, is equal to A *d*, equal BJ.

Draw the Perpendiculars GH, *AF*, and *DG*, indefinite; and, A *E* being made equal to the height of the Kirb, draw K *E*, cuting them in *G*, *F*, and H.

*E*L cuts a Perpendicular from B, in I; and, if *F*L and *G*L be drawn, KI, cuting them, compleats the upper Faces, *F*J and AIH. KB and GL intersecting at *g*, and *g b* perpendicular, cuting HL, at *b*, compleats the End.

If the Kirbs are inclined with the Steps, as in the former Case, draw LV, perpendicular to KL; which is the Vanishing Line of the Plane of the End, AEIB.

L*E*[2] being its Distance, draw *E*[2]V, making the Angle L*E*[2] V equal to the inclination of the Steps, cuting L V in V, the Vanishing Point of the inclined Sides*.

KV is the Vanishing Line of the Inclination. (Theo. 11, Part 2nd, Cor. 1.)

A*B* being made equal to the height of the Front, draw K*B*, cuting *DG*, *AF*, and GH, in *K*, *H*, *B*, and *L*; from which Points, draw *K*V, *H*V, &c. cuting the horizontal Lines *G*L, HL, &c. in s, r, u, &c. through which, draw Ku, or join the Points r s, t u, only.

Note. The Picture might, with equal propriety, have been supposed on this side of the Kirb, entirely; but, as it frequently happens, in Practice (the Picture being fixed inadvertently) that some parts of the Object would project on this Side; I thought it necessary to give an Example how to proceed in that Case.

A *B o b* is the Section made by the Picture, with the inclined Kirb, and shews how much is supposed to project on this Side. A *b* is the section of the Bottom, consequently, parallel to the Horizontal Line, *o b* is the section of the End, which is vertical, therefore parallel to VL its Vanishing Line; and *B o* is the Section with the inclined part, consequently, parallel to the Vanishing Line, KV, of the inclination of the Steps. (Theorem 3rd.)

* This will be clearly demonstrated and made manifest, in the 12th and last Section of this Book.

N. B. The Vanishing Line (VW) of the Roof of the Object in the Apparatus, is determined in this manner (as KV) and is in a similar Position, on the direct Picture, MNOP.

V, the Vanishing Point of HG, answers to K, in this Figure; Y answers to L, and W to V.

SECTION VII.

THE foregoing Section contains many useful Lessons in Plane Solids, which may be considered as parts of Buildings, &c. In this, I intend to shew how the more sublime and decorative parts are formed, such as Pedestals for Columns, Cornices and Entablatures, &c. of the various Orders; without which embellishments, a Building seems naked and destitute of Ornament.

The last Lessons, of Steps, are very necessary to the art of delineating Mouldings; but, there yet remains another construction of Steps, which, when we know how to manage well, Mouldings very readily follow; such Steps as return on every Side of a Pedestal, &c. forming mitre Angles. For Mouldings breaking round a Pedestal, or the internal and external Angles of a Cornice, &c. properly considered, are but so many Steps, one above or below another, of different dimensions; formed by the larger or smaller Fillets, between the cylindrical parts, which are, properly, the Mouldings; and are effected only by Light and Shade. The Fillets between them, are small Planes, cuting the curved Surfaces in parallel Lines; which being described, by the Rules given, and the mitre Angles drawn, the business of the linear part is done, and nothing remains but to give the appearance of solidity, convexity and concavity, by a just disposition of Light and Shade.

EXAMPLE XI.

How to delineate square Steps, returning on every Side.

First, when they are parallel to the Picture. AB is their length, ABCD is half the geometrical Plan of the first Step; FGHI of the second, and KLMN of the third; and let OP be the measure of a Cube, on the upper Step. Fig. 81.

C being the Center, draw AC and BC; and AE, which is a Diagonal, represents AQ, in which are the Seats of G, L, and O, the Angles of the Steps.

Compleat the Square *ADCB* (Prob. 19) and draw the other Diagonal B*D*; which, it is evident, tends to a Point on the left hand of C, equal to CE, the Distance of the Picture.

Produce FG, KL, &c. to the Ground Line, cuting it in *g*, *l*, &c. i. e. make A*g*, *g l*, B*h*, &c. equal to the breadth of the Steps, and draw *g*C, *l*C, &c. cuting the Diagonals in *G*, *L*, *H*, &c. the perspective Seats of the several Angles, or Corners of the Steps.

Draw A*d* perpendicular to AB, and make A*a*, *ab*, and *bc*, equal to the height of the Steps, as in the former Examples; and, having drawn the Perpendiculars *G*e, *L*i, &c. draw *a*E, *b*E, and cE, cuting them in c and e, g and i.

Draw *a*b, cd, ef, &c. parallel to AB, cuting the Perpendiculars from B, *H*, &c. in b, d, f, &c. which give the representations of the Fronts, corresponding with GH and LM. A*a*bB, being in the Picture, has its full dimensions, in height and length; to which the other two are similar. (See Example 8th.)

Having obtained the Fronts (by means of an imaginary Plane passing through the Diagonal A*C*, which A*cFC* represents) finish the square of the Top, iklm†. † Prob. 29.

Then, draw fC and bC, till they cut hk and df, which represent the return of the Steps on that Side, so far as they can be seen; on the other Side they are seen to their full extent, the Point of view being on that Side.

Draw *a*C and eC, &c. cuting Perpendiculars from *D*, *I*, &c. in a, j, and m; and, having compleated the Side gmi, from j and a, draw Lines parallel to the Ground Line, cuting the adjacent Step, which terminates their appearance, and compleats the Steps.

To represent a Cube on the upper Step; draw the Perpendiculars *O*r, and *P*q indefinite; make *cd* (on the vertical Intersection, Ad) equal to OP, in the geometrical Plan, and draw *c*E and *d*E, cuting *O*r, in o and r.

Draw op and rq, parallel to the Steps, which compleats the Square orqp.

Or, having obtained op (as above) equal *OP* (the perspective Seat of OP) on op describe a Square, and finish the End orsn, as in the Figure. (Prob. 29.)

I presume that the Reader will, ere this, have discovered, how inconvenient it is to have the perspective Plans, of complex Objects, in the very place where the Object itself is drawn; which, I am sensible, must render it, to young practitioners, perplexing and intricate, to distinguish one part from another. That difficulty shall be obviated in the next Example; but I must observe, that the connection of the several parts, and the agreement between the Plan and Elevation, would not have been so distinctly seen, without the Plan in its true place; hereafter we shall place it either above or below, as is most convenient.

Pl. XVII. Let it be observed, notwithstanding, that this apparent intricacy will vanish at a second process; and that the perspective Plans, &c. being drawn with a fine Pencil (which the dotted Lines represent) are rubbed out, as soon as we have obtained the Perpendiculars of the Corners of the Steps.

Fig 81. Above the Vanishing Line, let *AB* be drawn parallel to it, which let be equal to AB; the Point *A* perpendicularly over A, and *B* over B.

Then, if *A*C and *B*C be drawn, and the Diagonal *A*E, cuting *B*C in *C*; and *CD* being drawn parallel to *AB*, compleats the Plan of a Square, *ABCD*, corresponding with AB*CD* below.

And, if the measure of the Steps be set off, from *A* and *B*, at *a*, *b*, *c*, and *d*, *e*, *f*; *a*C, *b*C, &c. being drawn, cut the Diagonals *AC* and *BD*, in *g*, *h*, &c. which correspond with *G*, *H*, &c. below; as may be seen by the dotted Lines *h*H, *m*M, *p*P, &c.

Wherefore, if there be room above, and not below, the Plan may be formed above; by means of which, the Object may be compleated; drawing the Perpendiculars *g*ec, *l*ig, &c. cuting *a*E, *b*E, and *c*E; and *h*fd, &c. cuting the parallel Lines ef, &c. which terminates the other ends of the Steps.

EXAMPLE XII.

Is an Expedient, or Lemma, by means of which, Mouldings, &c. may be delineated, without having the perspective Plan in the Work.

Fig. 82. Let ABCD be a perspective Plan of a Square, whose measure is AI, on the Intersection (described by Prob. 20 and 21) F and G are the Vanishing Points of the Sides of the Square, and H of the Diagonal; consequently, FHG is the Vanishing Line of the Plane of the Figure; S is its Center.

The Distance of the Vanishing Line, FG, is a mean Proportional between SF and SG (Prob. 11) the Point E, which is the Distance of the Vanishing Point F, from the Eye, is determined by the same; and, the Vanishing Point, H, of the Diagonal, by bisecting the Angle, which the Radials of AB, and AD, make at the Eye; or, by making FH to HG, as the Radial of AB is to the other (3. 6. El.)

See Prob. 10, Case 3rd, Fig. 49. EH, bisecting the Angle DEG, made between the Radials of AC and CB, gives the Vanishing Point H; by which, the Lines AC and CB are cut, perspectively, representing equal Lines. As, in this Example, by drawing AH (H being obtained, as above) AB being made to represent a measure equal to AI (Prob. 17) and BG drawn; BG is cut, in C, representing an equal Line as AB represents, i. e. equal to AI (Prob. 10.) Then, draw AG; and FC, till it cuts AG, in D; ABCD represents a Square, whose Side is equal AI.

It is required to draw, within the Square ABCD, the representations of several less Squares, of certain dimensions in proportion to the other, about the same Diagonals (AC and BD) and consequently, having the same Center; i. e. to draw the representations of Borders or Margins within it.

Make A*a* and *ab* respectively equal to the breadths of the Margins.

Draw *a*E and *b*E, cuting AB in *c* and *d*; then, draw *c*G and *d*G, cuting the Diagonals in e and h, i and m; and draw eF, iF, &c. cuting the Diagonals again, at f, k, &c. and join fg and kl; or, draw fG and kG.

efgh and iklm are the representations of Squares, having the same Center, o; and are about the same common Diameters, AC and BD.

Now, because the Intersection (AI) of the Plane those Squares are in, is so near its Vanishing Line (FG) the Sides of the Squares, AB, ef, &c. cut the Diagonal BD very oblique; for which reason, the Points of Section, f and k, cannot be ascertained with accuracy; nor can the true breadths of the Margins be determined. Therefore, either below or above the Vanishing Line, draw *AB*, or a*B* parallel to AI; in which, take *A* or a, in a perpendicular Line from A, and make *AB*, or a*B* equal AI; and make *A* 1, or a 1, and 12, respectively equal to A*a* and *ab*.

Draw *A*F and *A*G, or aF and aG; and BE, 1E, 2E, cuting *A*F in *B*, p, and q.

Compleat the Square *ABCD*, or abcd, by the same Vanishing Points, F, G, and H, and draw the Diagonals *BD*, or bd. Draw pG, and qG, cuting the Diagonals, in *E*, *H*, *i* and *m*; from which Points draw *E*F, *i*F, &c. and finish the interior Squares, *EFGH*, &c. as in the Figure.

2 If

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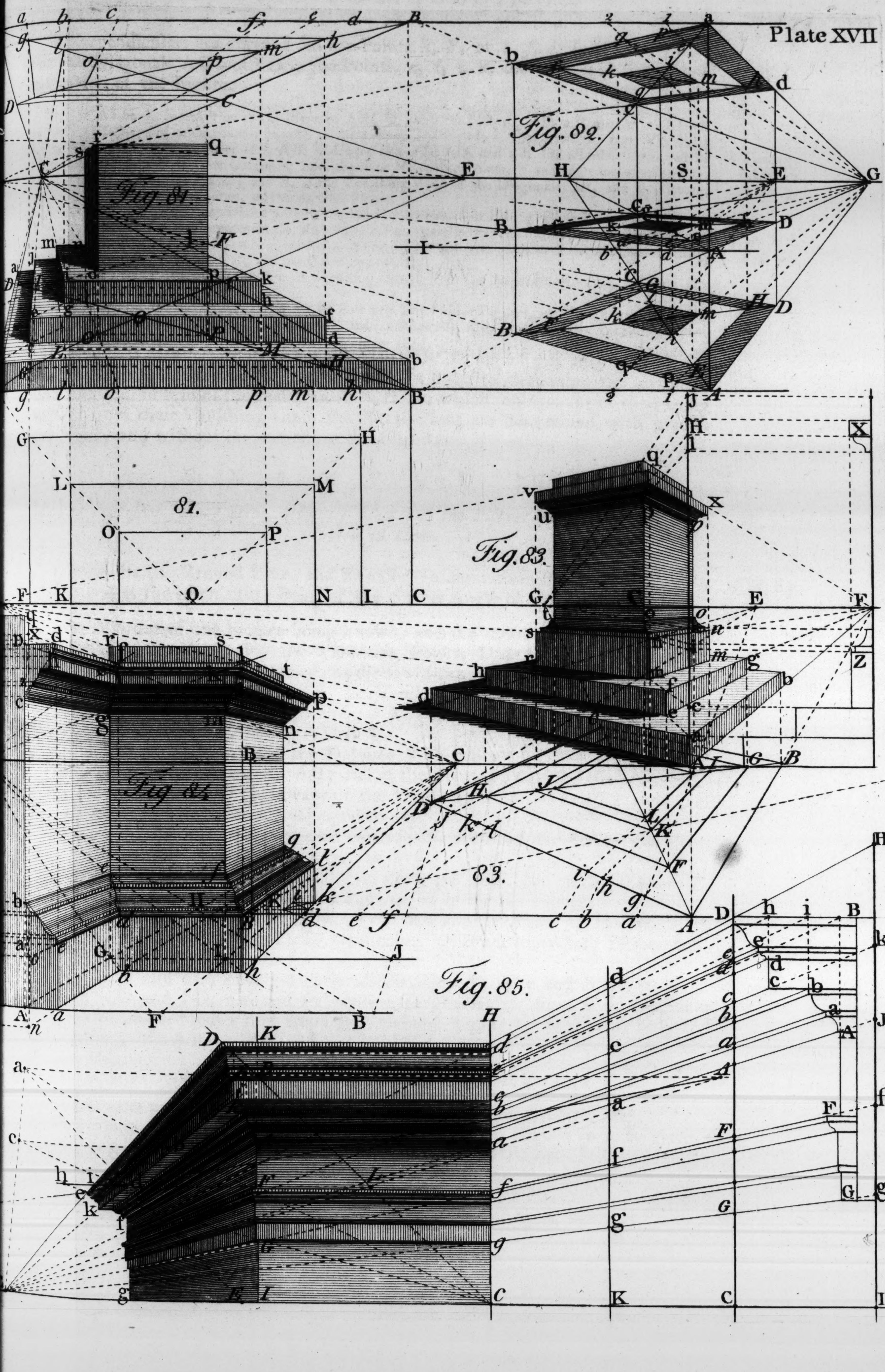
Plate XVII



If Perpendiculars are drawn from the Points *E*, *F*, or *e*, *f*, *i*, *k*, &c. they will cut the Diagonals AC and BD in the Points, e, f, i, k, &c. as it is obvious from inspection of the Figure.

For, if AI be the Intersection of the Plane of ABCD, with the Picture, *AB*, a*B* may be considered as the Intersections of other Planes, parallel to the former, and therefore, they have the same Vanishing Line, FG†; and consequently, *AB*, AB, and ab; also *AD*, AD, and ad, and all other Lines, whose Originals are parallel to their Originals, have the same Vanishing Points, F and G‡; notwithstanding, the Lines are in different Planes; and H, is the Vanishing Point of the Diagonals, *AC* and ac, seeing, that they represent parallel Lines, and in parallel Planes. † Theo. 5. ‡ Cor. to Theo. 5.

Therefore, *AB*bad*D* represents a right angled Parallelopiped or Prism, whose Base, *ABCD*, and Top, abcd, are Squares; consequently, they are equal and parallel to each other§; and similarly posited. § See Def. Page 44.

ABCD may be considered as a Section of that Prism, by a Plane parallel to its Base; and consequently, it is also a Square, equal, similar, and parallel to the other.

The same may be said of all the interior Squares; which, with the Perpendiculars *Ee*, *Ff*, &c. also form Prisms.

In Example 5, the hexagonal Prism has its Base and Top similar Figures, and similarly posited; and being in parallel Planes, their Sides are parallel, and consequently, they have the same Vanishing Points.

Hence it is manifest, that, either the Plan *ABCD*, or abcd, may be used, for the purpose of drawing perpendicular Lines, in order to determine the extremes of Steps, &c. instead of the real Plan ABCD; in which, on account of their being farther from their Vanishing Line, the Intersections are determined with greater exactness, and without incommoding the Object.

EXAMPLE XIII.

How to represent square Steps, obliquely situated to the Picture and having a Pedestal situated on them.

Let AB be the Ground Line, and FG the Horizontal vanishing Line, C is its Center; A is the intersecting Point of the nearest Angle of the Ground Plan. Fig. 83.

The Distance of the Picture being known, and the Inclination of the Sides of the Object determined, find the Vanishing Points, D and F, of the Sides (by Prob. 12) and G of the Diagonal, as directed in the last Example; also, find E, making DE equal to the Distance of the Vanishing Point D.

Draw AD and AF, indefinite; make AB equal to the Side of the Square of the Steps, and draw BE, cuting AD in b.

Draw *AB* parallel to AB at any Distance at discretion*.

Draw A*A* perpendicular to AB; and at the Point *A*, describe the representation of a Square *ABCD* (by means of the Vanishing Points D and F; and G, of the Diagonal) whose Side *AD*, is equal to *AB**.

Make *Aa*, *ab*, and B*d*, *de*, each equal to the width of the Steps; and draw *a*E, *b*E, &c. cuting *AD* in *g*, *h*, *j*, and *k*.

Draw the Diagonals *AC* and *BD*; and draw *g*F, *h*F, &c. cuting them in *E*, *F*, *G*, &c. and compleat the interior Squares, as in the last Figure.

Draw AH, perpendicular to AB. Take Aa and ac equal to the height of the Steps, and draw aD and aF, terminating at b and d, where Perpendiculars from *B* and *D* cuts them.

Draw aG and cG, cuting a Perpendicular from *F*, in e and f.

Draw eD and fD, eF and fF, cuting Perpendiculars from *G* and *H*, at g and h, which terminates that Step; bD and dF cuting the next Step, at g and h, compleats the first.

It, remains, now to delineate a Pedestal upon the upper Step.

JIK is the perspective Plan of its Base, in the place of the third Step.

Set off the geometrical projectures of the Mouldings, from *b* to *c*, or from *e* to *f*, on the Intersection; and transfer them to *AD*, by means of the Point E; and draw *i*F and *l*F; cuting the Diagonals at *K*, *L*, &c.

Then

* Note. This Point (*B*) is in the Plinth of the next Figure, 84.

Pl. XVII. Fig. 83. Then, on the Intersection AH, make cH equal to the height of the Pedestal; make ci equal to the Plinth, ik to the Base Moulding, kl to the Dado, lH to its Cornice or Sirbase, and Hj to the Plinth of the Base of the Column; which, notwithstanding it is a part of the Base, always goes with the Pedestal.

At X and Z, on the left hand, the Profile of the Mouldings and Steps are described, geometrically; from which they are transfered by means of parallel Lines, to the vertical Intersection AH.

Draw cG, iG, kG, &c. cuting Perpendiculars from *K* and *L*, in m, n, o, p, and q; the representations of the corresponding Points in the Profile.

Draw mD, nD, mF and nF, &c. cuting Perpendiculars from *I* and *H*, in r, s, t, *m*, *n*, *o*, &c. and having divided ik and lH into the smaller divisions of the Mouldings (as at X and Z) draw to G, cuting Perpendiculars from the corresponding Parts in the Ground Plan, at *JIK*, by which means, the whole is compleated; as in the Figure.

Now, when this Operation is well considered, what are the several Parts which compose the Pedestal, but so many Steps of different heights and breadths. The Plinth, mn*nm*, is in the place of the next Step, about twice the height; and the Fillet, which follows it; is another of a much less height; the Dado, after that, is another still higher, equal kl; and the Cornice, v, q, x, is but Steps reversed.

The Curve Lines st, no, and *no*, joining the Angles of the several Fillets, are all that have any linear curviture; although no*on* represents a cylindrical Surface, yet the appearance of it is effected by Light and Shade only; which, according as it is supposed to be situated, in respect of the Light, will appear either convex or concave; to which, the curve Lines, at its extremes, do not at all contribute.

And now I might take leave of the Reader, and of this Section, and endeavour (as others have done) to persuade him, that I have, in this Lesson, given him sufficient instructions for delineating all kinds of rectilinear Mouldings; but, I have not quite so good an opinion of his capacity (if it be his first essay) as to suppose any such thing; my intent in this, is only to give him some little Idea of it, by comparing the Process with that of delineating Plane Steps. There remains yet somewhat more to be said on the Subject; therefore I shall not trespass on his time, but give some other Examples for his further Instruction.

EXAMPLE XIV.

How to represent Mouldings round the Basement of a Building, being a regular Pedestal, with an internal and external Angles; whose Planes are parallel and perpendicular to the Picture, and also one to another.

Fig. 84. Let AB be the Ground Line, and AD the Intersection of the Picture with the vertical Plane of the Dado, bcg*e*; on which, set off all the measures of the heights of the Mouldings, and their Distance from each other.

Aa is the height of the Plinth, ab of the Base moulding; bc of the Dado, and cD of the Sirbase, or Cornice of the Pedestal.

Let each Moulding be drawn, geometrical, cdD the Cornice, and ab*c* the Base; draw the Perpendicular *ac*, and draw the diagonal Lines b*c* and cd.

C being the Center of the Picture, and EC the Distance, draw AC indefinite.

Make AF the measure of the first break, and draw EF, cuting AC at G.

Draw GI parallel to AB; make AF equal to the projecture of the Pedestal, and draw FC, cuting GI, at I.

Make IJ to GI as the front of the Pedestal to its projecture; that is, as FB to AF; and draw JE, cuting FC, at K; or BE, cuting AC, at H, and draw HK parallel to AB.

From the several Angles, G, I, and K, thus obtained, draw Gr, Is, and Kt, perpendicular to the Ground Line; which constitute the several Planes, from which the Mouldings are supposed to project.

Draw bC, cC, and DC, cuting Gr, at *e*, g, and e; draw *ef*, gm, and ek, parallel to the Ground Line, and where they cut Is, at *f*, m, and k, draw Lines to the Center, cuting Kt, at *g*, n, and o.

Now, Degc, ekmg, and konm, is the Seat of the Cornice on those Planes; cg*e*b, &c. is the Dado or plane part between; and Ab*e*G, G*ef*I, and I*fg*K are the Grounds of the whole Base.

Draw

Draw *a* C, cuting EF at *b*; draw *b h* parallel to AB, cuting EI produced, at *h*; and draw *h* C, indefinite.

The Angle *k* is determined, by drawing a Line from K to the Eye, on the other Side of the Center, equal CE; for K *k* is a Diagonal of a Square; whose Vanishing Points are, in this Case, the Points of Distance. (Prob. 19.)

Or, if the Vanishing Point of the Diagonal K *k* is not on the Picture, draw K *d* parallel to the Ground Line; on which describe the Square K *i k d* (by Pr. 19) which will give the Angle, *k*, required; i. e. draw *d* E cuting KC; and *i k* parallel to K *d*, cuting *d* C at *k*, the Angle sought.

From the several Angles, *b*, *h*, and *k*, draw the Perpendiculars *b* f, *h* l, and *k* p; and, through e and k, draw E e, and E k, cuting the two first, at f and l.

Or, from *c* and d, draw Lines to the Center, cuting the first at *d* and f; from which, draw *d B* and f l, cuting the second, at *B* and l; and from them draw Lines to the Center, cuting *k* p, in *l* and p; which gives all the extreme Angles of the Mouldings.

Draw the diagonal Lines *e d*, *f B*, and f g, &c. and observe how the parallel Lines in the geometrical Mouldings, cut their Diagonals; as, at h, the great projecture of the Facia goes beyond it.

Draw the lower Line of the Cimma or Ogee, at h; and, where it cuts tl Diagonals, at i and j, draw the perpendicular lines of the Facia, which will te minate it, at the several Angles; by drawing h i, tending to the Center, and parallel to the Vanishing Line, &c. The rest is obvious on inspection of the Figur

The Fillet at Z between the Ovolo and Cavetto, touches the diagonal Lin c d; by which, it is easily described, as above.

It may perhaps be imagined, that b *c*, and c d are mitre Angles, at the Cor ner AD, which they are not. For, suppose AD v u to be a returning Plane, pa rallel to the Picture, and at right angles with AD e G.

Then, because AD was considered as the Section of that Plane, AD v u is i the Picture; and consequently, A b *c a* and c d D are sections of the Mouldings, by the Picture; and if they are continued on the Plane AD v u, they will project on this Side of the Picture.

Draw EA, meeting *b a*, produced, at *n*; and draw *n* x parallel to AD.

Produce f d to x; draw the Diagonal c x, and draw the parallel Lines v x, &c. c x is the mitre Angle of the Moulding, at that Corner.

Also, produce *d c*, till it cuts the Perpendicular *n* x at *o*; and draw *o p* and *n q* parallel to the Ground Line; a diagonal line, *o* b, is the Angle of that Moulding.

It may be observed, that, if AD coincided with u E v, the mitre Angles would be in that Line entirely; or, if that Line was beyond E, the mitre Angle would be on the left hand of it.

AB being the Intersection of the Ground Plane with the Picture, all the Mouldings on this Side, viz. *a c* b y *q n*, and c d x v w project through the Picture, and they are considered as projected to the Picture†. †See Projection, P. 52.

D q is the height of the Plinth of the Base, which determines it, at r, s, t, on the Pedestal.

E X A M P L E XV.

Is a general and universal Rule for delineating Mouldings, parallel to the Picture; whether they are above or below the Eye.

At either extreme of the Picture, if there be room (or it may be done on another drawing Board, and the Scale Line transfered, carefully, to the Picture) describe the Cornice, &c. geometrically.

If the Student be not acquainted with Architecture, or have not a knowledge of the geometrical proportions of Mouldings, let him not attempt the delineation of them in Perspective, before he has studied, and practised himself a little in that kind of Projection; as he will but lose his time; it being impossible to succeed in one without the other; seeing that the projection of Mouldings, in Perspective, depends on the true geometrical Profile.

Pl. XVII. Fig. 85. Let AB be the height, and BD the projecture of a Cornice, intended to be delineated; to which let there be added an Architrave, FG; between which, the plane part, AF, is called the Frize; the whole together composes an Entablature.

Let the several Mouldings be geometrically described, according to their height and projecture; at a, b, c, &c. and, CD being drawn perpendicular, may be considered as a vertical Section of the Picture, applied close to the greatest projecture of the Cornice, at D.

According to the height which the Cornice is supposed to be above the Eye, take the Point C, and draw CE perpendicular to CD, i. e. horizontal; and equal to the Distance of the Picture.

Now, E being considered as the Eye of a Spectator, viewing the Cornice, AD, &c. EA, ED, EF, &c. are Visual Rays, in which direction the several parts are seen; and, CD is supposed to be a Plane, interposed, cuting those Rays; consequently, the several Points, *A*, *a*, *b*, &c. projected by them, are the apparent places of the several edges, or parallel Lines of the Mouldings, on the Picture; which needs no further illustration.

C is the Center of the Picture, and EC is its Distance, by which the perspective Representation of them must be projected; provided that the drawing be required by the Scale of the Original, ABD.

But, it may, from the same Profile, be made either greater or less. e. g.

If the Rays ED, &c. be continued, beyond D to H, and HI be considered as the Plane of the Picture, the projection of the whole Cornice is JH, which is greater than *A*D, and so of the rest; EI is the Distance of that Picture.

Or, if any other Plane be interposed, as at Kd, or *CH*; the proportions of the several Parts on each, viz. *GF*, *FA*, and *AD*; gf, fa, ad, or *gf*, *fa*, *ad*, are to each other, as their several Distances, EC, to EK, or E*C*. (6. 6. El.)

Hence it is evident, that, from parallel Sections of the Rays, EA, ED, &c. any where, the Mouldings will have the same proportion to each other.

The several Sections, CD, Kd, and *CH*, are Scales of the perspective Proportions for each respective Distance.

It is required to delineate an Entablature by the Scale *CH*.

Let E be considered as the Center of the Picture or Point of View, and E*C* the Distance of the Picture; the terms being inverted.

To the farthest Corner (B) of the given Cornice (i. e. to its height, or Seat of the extreme projecture, D, on the Perpendicular AB) draw EB, cuting *CH* at *e*.

Take any length, required, as *CI*, for the Ground Plane of the Cornice.

Draw *IK* parallel to *CH*, i. e. perpendicular to E*C*, the Horizontal vanishing Line; and from *e*, where EB cut *CH*, draw *e B* parallel to the Horizon, cuting *IK* at *B*. Draw *D d* also parallel; and *CB*, cuting it, at *D*, gives the Angle, *D*.

Draw *a A* parallel to *e B*, cuting *IK*; and, having drawn a diagonal Line *AD*, then, from the Intersections of the Rays, Ea, Eb, &c. with *CH* draw Lines parallel to *D d*, till they cut *AD*; and, from *b*, draw a Perpendicular, cuting the Parallel from *c*, at *i*, which compleats the Corona.

Draw *i C*, cuting the Parallel from *b*, at *k*, *i c b k* represents the Planceer, bc, in the Profile. The rest is obvious, for the parallel Side of the Cornice.

The projecture of the Architrave, at *F*, is obtained, by drawing *CF*, after the same manner as the Cornice, at *D*.

If the length of the returning Plane exceeds the limits of the Picture, take any equal part of the whole length, as a*B*, in *e B*, produced, and draw a*C*, cuting *B*E in b; draw aE, and bc parallel to a*B*; and c*C*, cuting *B*E at d, which may be repeated as oft as occasion requires; according to what part a*B* was taken of the whole length. Or, if a*B* be half the length required; make E*E* equal to half E*C*, and draw a*E*, cuting E*B*, at d, as before.

 To

To ascertain the mitre Angle, e, seeing that the Vanishing Point of the other Diagonal is not in the Picture, draw d h parallel to a *B*; and *D*E, cuting it in i; make h i equal i d, and draw h *C*, cuting *D*E in e, the Point sought.

Or, draw d *L* parallel to a *B*, cuting the Diagonal, *DC*, at *L*; draw E*LA*, cuting *Be* produced, at *A*; draw *A*d, and produce it, till it cuts *D*E, at e.

For, d *BAL* represents a Square (Prop. 19) consequently, *BL* and d *A*, being Diagonals of a Square, bisect the Angles, which are Right ones.

Draw a diagonal Line e f, and from all the Angles, *h*, *i*, *k*, &c. on the Diagonal, *AD*, draw Lines to E, cuting e f; observing, that the extreme Angle of the Corona, goes beyond the Diagonal; as at k.

This, I presume, is a clear explanation of this Method of representing Mouldings, which is deduced from the most natural and simple Ideas of it, and cannot fail of being intelligible to the meanest capacity, who has any emulation to attain to the art of Perspective.

EXAMPLE XVI. Pl. XVIII.

To represent a Doric Entablature, with an internal and external Angles.

On account of the variety of Parts, and the many breakings of the Fillets, round the Triglyphs, in the Doric Frize, it is necessary to have it described, geometrically; so that, the Parts may be clearly understood. Indeed every Moulding, delineated in Perspective, ought first to be geometrically drawn, to the Scale of the Drawing; although the measures may be applied from a Scale only, as it will be shewn.

I have shewn, in the 12th Example, how a perspective Plan may be drawn either above or below the Work; by which, Mouldings, &c. may be projected on the Picture with more facility; as exemplified in the 13th. But, that process (though the most to be depended on, for accuracy) is attended with extraordinary trouble, which may be lessened greatly, when we are tolerably acquainted with Mouldings. A Specimen of both will be shewn, in the following Example.

Let AB be the height of the Entablature, according to the Scale of the Drawing. Let it be bisected at C; and AC, or CB, is a Module, or Diameter of the Order. AC divided into 60 Minutes, is the Scale; by which the whole is proportioned. Fig. 86. No. 1.

QM, the projecture of the whole Cornice, is a Diameter; the rest is proportioned as by the Scale AB.

DF is the Architrave, FG is the Frize, with its Triglyphs, and GMQ is the Cornice. HI is the projecture of a Mutule or Modilion, and K of the whole Planceer; which being seen in Perspective (the Eye being below it) has a fine effect, and adds greatly to its august appearance.

RSTU is a geometrical Plan of the Cornice, with an internal Angle at S, and an external, at T; shewing the Planceer of the Mutules, and panneling between them; for, unless we know the true geometrical form, it is not possible to describe any thing, perspectively. Each return of the Cornice is the same, having two Mutules each way, except the last, on the Right, which is supposed unlimited; and, on the Left, it is limited by the bounds of the Picture; which, on account of its distance from the Center, would be distorted, if it was continued much further. No. 2.

BAFG is a perspective Plan of the whole; which is the best method of proceeding, if accuracy be required in the several parts, and we do not grudge the time spent in doing it. It is formed as follows.

At any Distance from the Horizontal Line, *E*CE, either above or below it, draw *AB* parallel to it; one Side, in this Example, being parallel to the Picture.

Having determined on the Place of the Angle a, in the Design, take *A* perpendicular over it, and draw AC, indefinite. No. 3.

Take *AD* equal ST, in the geometrical Plan; and, having made C*E* in the Horizontal Line, equal to the Distance of the Picture (equal EC) draw *DE*, cuting *A*C in *F*; then, *AF* represents the length of *AD* (equal ST) and *F* is the internal Angle of the extreme Moulding in the Cornice.

Draw *FG* parallel to *AD*, and *D*C cuting it, in *G*, the external Angle; for *FG* also represents an equal length as *AF*†, both equal to ST.

† Prob. 19.

Make *AC* equal MQ, the projecture of the whole Cornice, and draw *C*C and *AE* cuting it. The Diagonal *FI* (i. e. *DE)* also cuts *C*C in X; from which, draw XY parallel to *AB*.

The Angles *K*, *L*, and *M*, of the Corona (K, in the Profile) are determined by making *Ad* equal to MN, and drawing *d* C, cuting the Diagonals *A*V and *F*X; by means of which, they are carried round, to *L* and *M*, and, by the same means, the whole Plan is compleated, as in the Figure.

To

Pl. XVIII. Fig. 86. No. 3. To describe every Step, by which the whole perspective Plan is formed, would be as tedious as it would be useless; seeing that, the various Lessons, already given, are sufficient for any right-lined Figure whatever.

The Plans of all the Mutules, Z, Z, are the representations of Squares having one Side parallel to the Picture (found by Prob. 19) their places are determined as follows.

Having made *Ae* equal MO (in the Profile) draw *e*C cuting the Diagonals *A*V and *F*X at *S* and *W*, and draw *W*w parallel to *AB*.

Make *Aa* equal to JM, in the Profile (equal T 1 in the geometrical Plan) make *ab*, *bc*, &c. equal to the width of the Mutules and the spaces between them (12, 23, &c. in the Plan) and from each Point, *a*, *b*, &c. draw Lines to C, the Center, cuting a parallel Line from *S*, in *R*, *P*, &c.

For, the returning Side, *AF*, make *Af*, *fg*, &c. equal *Aa*, *ab*, &c. and draw *fE*, *gE*, &c. cuting *AF*, at 1, 2, 3, from which, draw Lines parallel to *AB*, cuting *S*C at *T*, V, &c. and *H*C at *m*, *n*, &c.

For the other parallel Side, *FG*, make *Dg*, *gf*, &c. also equal to *Aa*, *ab*, &c. from which draw Lines to the Center; cuting *W*w, at *XYZ*.

For the apparent width of each Moulding, &c. in Front, draw M*C* perpendicular to the Horizontal Line; take E*C* equal to *E*C, on the Horizontal Line; and, from every Angle, G, H, K, &c. in the Profile, draw EH, EI, &c. cuting *C*M in the several Points, g, h, k, &c. as in the last Example.

Being thus prepared, we now proceed with the Representation.

Fig. 86. a is the determined Angle of the extreme Moulding; draw the fillet a z of the upper Moulding (which is in the Picture) geometrical.

From V, X, and Y (in the perspective Plan) draw perpendicular Lines indefinite; which are the Angles of the Ground Planes of the whole.

Then, from the several Angles *H*, *K*, &c. in the perspective Plan, draw Lines perpendicular; and, from the corresponding parts on the vertical Section, *C*M, draw Lines parallel to the Horizon, cuting them. e. g.

From *K* draw a Perpendicular, *K*c, and from k (where KE cut *C*M) draw horizontal Lines cuting it, at c, the representation of the external Angle of the Corona.

Draw c C, cuting the Perpendicular from *L* at l, the internal Angle of the Corona; and from l, draw parallel Lines, cuting the Perpendicular from *M* at m, the other external Angle of the same.

Then, from h (where HE cuts *C*M) draw horizontal Lines cuting the Perpendicular from *H* (in the Plan) at h, the representation of the Angle H.

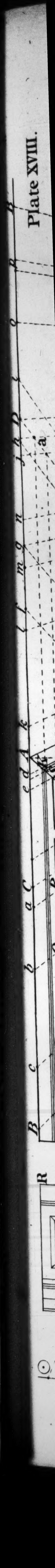
Draw hC cuting a Perpendicular from *I*, at j, and the horizontal Line j n gives the other external Angle, n.

For the Mutules, draw a Perpendicular from *S*, in the Plan (where they would meet if they were continued to the Diagonal *A*V) and from i in the Profile draw parallel Lines cuting it, at s; which must be returned at all the Angles, as if it was a continued Facia.

From *O*, *P*, &c. draw perpendicular Lines cuting them, in the several Points, *o*, *p*, &c. which are the front Faces of the Mutules. Draw *r*C, *q*C, &c. cuting the horizontal Line h h; where they fall against the Facia.

In the returning Side, draw Perpendiculars from *T*, *V*, &c. till they meet *S*C, the returning Facia, and so of all the rest; which are best described by the Figure; observing, that if from the several parts in the first or nearest part of the perspective Plan, *BA*V, perpendicular lines be drawn, and from the corresponding parts in the Section, M*C*, parallel Lines cuting them; and, by carefully remarking the mitre Angles of each Member, the true perspective proportions of them are carried round as many Breaks, in the Object, as are required; by means of the Vanishing Point C (the Center) for one return, the other parallel; the Originals of them being, in this Example, parallel to the Picture, and the Object right-angled.

The



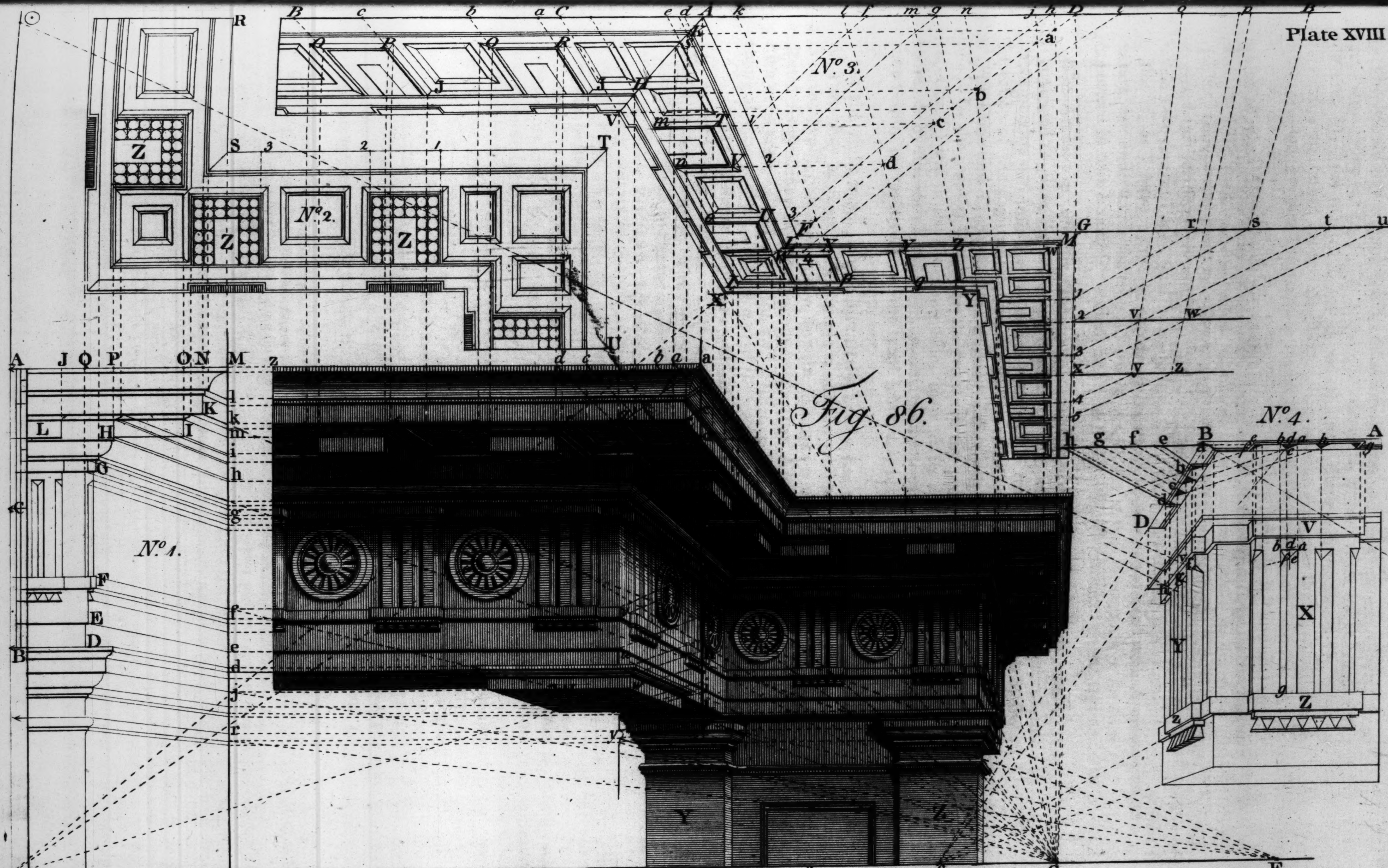


Fig. 86.

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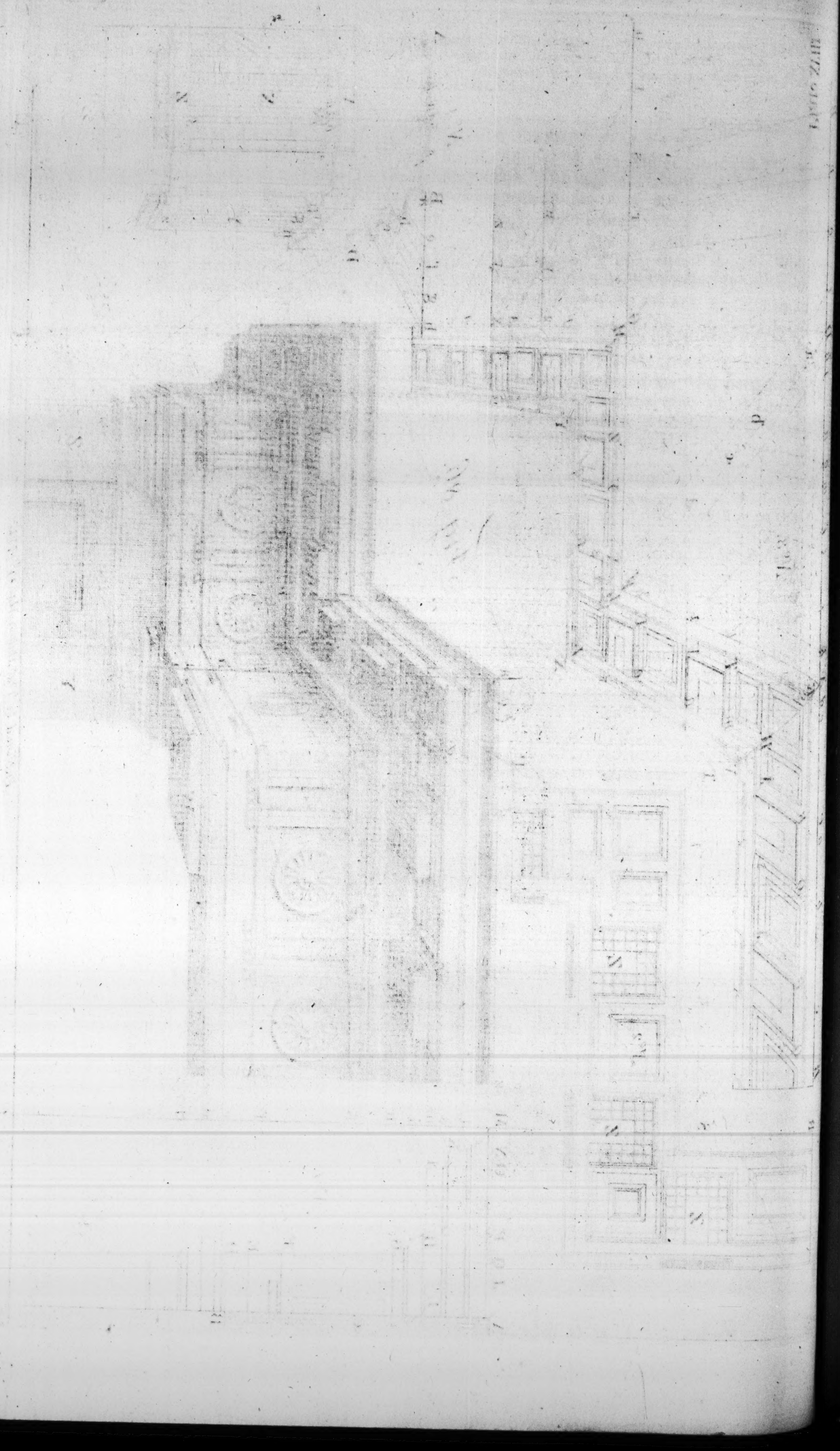


The Mouldings around the Mutules are determined, at their projectures, from the Plan, above (at *O*, *P*, &c.) and the width, in Front, from the Section of the Picture, M*C*, (at m) by means of which, and the Vanishing Point C, they are very easily continued around the Mutules, from one Angle to the next, adjoining. First, drawing parallel Lines, from m, till they cut Perpendiculars from *O*, *P*, &c. in *e*, *f*, &c. and *f* C, *b* C, &c. cuting Perpendiculars from J, at *i* and *j*; through which they are drawn, parallel, to the Angle at *k*, and from *k* they are continued after the same manner, around the whole.

Every thing being correct, and very particular in the Figure, makes it unnecessary to give a further description of it. To describe every Line in the whole Process, would take several Pages, and be extremely tedious and apparently prolix.

For if, as I have before observed, the Student be not acquainted with Architecture, nor understands the several Parts of the Order, it will be impossible for him to succeed in this Example; but if he is, it will be found sufficiently intelligible.

The Architrave, of this Order, it is needless to say any thing about; seeing it is composed of plane Facias, only; projecting, one over another, like Steps, of different heights and projectures, and is managed in the same manner; which, from inspection of the Figure, may readily be described.

The Frize and Triglyphs have nothing difficult in them, especially in this Position. In the parallel Face they are geometrically proportioned and spaced (Theo. 10, Cor. 5) and in the returning Side, their places are determined from the Plan above, or by the 10th Problem; dividing the upper Line of the Frize, s t, in the same Ratio, perspectively, as the front Line, n s, is divided, at o, p, q. (Prob. 8)

On the right hand, I have given two Triglyphs, more at large; in order, that the Parts may be more distinctly made out; in which, after dividing the front one, as in the Profile, geometrically, for the flutings (which are right angled) the indented Angles may be obtained, as in the Plan ABD, above; by drawing a Line from each edge, *a* and *b*, to the two Distance Points, if they are within reach, cuting at *c*.

Or, if one Distance (E) only; bisect the width of the Flute, at *d*, and draw *d* C, to the Center, cuting *a* E at *c*, as before; *ef* drawn, to E, and *fg* drawn parallel, gives *f* and *g*, at the extremes.

For the Triglyph in the returning Side; having drawn the front Line, a d, to its vanishing Point, C; from each angle *a*, *b*, &c. in the front Triglyph, draw lines to E, cuting a d, at b, c, and d.

Or, if it be convenient to have the Distance of the Eye on the other Side, it would be better; draw a h parallel to AB, and take the divisions, from a, at f, g, h, equal to the geometrical measures; and, from them, draw Lines to the other Distance, as in the Figure (supposing it within reach) which gives the same Points more accurately defined. (See Prob. 8.)

From the several divisions, on each Side, draw Lines perpendicular, as in the Figure; by which means, the Triglyphs, X and Y, are compleated. The Perpendiculars, from *c* and *b*, (in the Plan, above) gives the indented Angles in the front Triglyph, in the other they are not seen.

This operation is supposed either at the Top or bottom of the flutings.

The inclined Lines at the Top are determined, truly, by drawing a Perpendicular at the middle point, equal to half its width, as at *e*, and drawing a line to C, the Center, cuting the Perpendicular from *c*, at *f*, and join *af* and *bf*.

It is unnecessary to be more particular, or their vanishing Points may be easily determined; by making the same Angle at the Eye, as the inclined Lines make, either with the horizontal or vertical Line (Prob. 4.) and where it cuts a Line, passing through E, perpendicular (the Vanishing Line of an inclined Face) is the Vanishing Point of *af*. Either being found, determines the other; and, the inclined Planes of Y being parallel to those of X, have the same Vanishing Lines, respectively, passing through the Points of Distance.

The Vanishing Points of the inclined Lines, in the front Triglyph vanish below the Horizon, and those in the returning one (Y) which are seen (at f, g, h) vanish at the same Distance above it, at ⊙.

Y y The

Pl. XVIII. Fig. 86. The other Vanishing Point is below, on the opposite Side; for they are parallel to those in the other Triglyph, as fg, and consequently, they have the same Vanishing Point. (Cor. Theo. 5.)

Although this method will seldom be practised, yet I affirm it to be the best, being by much the readiest and the most correct. Nor, is there any occasion to be at the trouble of forming the Plan, to get the indented Angle; for, the Flutes being proportioned at ab (as above) and the Vanishing Points determined, the Lines, af and bf, being drawn to their respective vanishing Points, determine at the same time, the internal Angle.

The Tenia (v and z) both above and below, breaks regularly around; as it may be seen, and would be needless to explain; similar Subjects having so frequently been repeated. Such minutias are best described by the Figure, only, being accurately drawn, and well defined.

The Abacus W which may be supposed over a Column, is a Square; and the entire Capitals, over the Pilasters, Y and Z, at the internal and external Angles, although they are detached Mouldings, are managed (from the Profile) after the same manner as if they were continued; their perspective Plans being delineated, as above; from which, the dotted Lines, corresponding with the Profile Section, (MC) determines their several Angles.

Method 2nd. EXAMPLE XVII.

How to represent the same thing without having the Profile drawn, on a perspective Plan.

Let AB be the given height, which divide into two Modules, and into Minutes, as before (the rest being supposed not drawn.) The Scale (AM) of the projecture of the Mouldings, &c. at the top, is also necessary.

From a, the determined Angle, draw a k perpendicular, and transfer all the measures of the several Mouldings, from the Scale to a k; at b, c, d, &c.

Make a a, a b, &c. equal to the several projectures MN, NO, &c. and proceed as in the Plan, above; drawing a E for the diagonal Line of the top; and, from the several divisions a, b, c, &c. draw Lines to the Center, C, cuting the Diagonal, at e, f, g, &c. from which draw perpendicular Lines; and, from the measures on a k, draw diagonal Lines, to E; which gives the same Angles c, h, &c. as before; from which they are carried around the several Faces.

The perspective Plan, in this, is supposed to be formed on the Top of the Cornice, the same as above; and since a k is the Intersection of a Plane, passing thro' the Diagonal or mitre Angle of the Cornice (one Side, in this Case, being parallel to the Picture) E, the Distance Point, is the vanishing Point of that Diagonal, consequently, all horizontal Lines, in that Plane (being parallel) vanish in E.

Wherefore, a E, b E, &c. are each, the indefinite Representation of a Diagonal of a Square, in different horizontal Planes, whose Intersecting Points are a, b, &c. and, the distance of each Angle from the Picture, is a a, a b, &c. which being transfered, perspectively, to the Diagonal a g, each Angle, in its proper Place, will be perpendicularly opposite to the Points e, f, g, &c. as in the Example.

And thus may the whole be compleated, when we are a little versed in Mouldings, by transfering the measures perspectively, from one Diagonal to another, which will be further illustrated in the following Example.

EXAMPLE XVIII.

How to represent a Cornice when it is inclined to the Picture; on both Sides.

If the last Example be tolerably well understood, in the last Process, this will be found easy, being performed by the same means. It must be observed, that the first Method, respecting the Profile, cannot be applied here, nor in any Case; but when the Mouldings are parallel to the Picture. The extra Plan may be used in all Positions; but I shall in the following Examples, do without; as it is only supposing the Plan formed at the Top, from which the Perpendiculars are drawn, and the several Angles of the Facias and Fillets determined.

Let

Let ABE be a Profile of the Cornice, to be drawn, and *AC* the Side of one Plane, on which it is to be projected. Plate XIX. Fig. 87.

LM is the Horizontal Line, S is the Center of the Picture, and L the Vanishing Point of *AC*; which is supposed the upper edge of the Plane from which the Cornice is to project. Let the Top be supposed a Square.

S being the Center of the Picture, and L the Vanishing Point of one Side of a Right Angle (the Distance of the Picture being known) find M the Vanishing Point of the other Side; by Prob. 12. SM is a third Proportional to SL and the Distance of the Picture (found by Prob. 32, Geo.)

Also, find N the Vanishing Point of the Diagonal; by making MN to NL as one Side of the Triangle (whose Perpendicular, at S, is the Distance of the Picture) to the other.

Provided there be not room, on the Picture, to draw the Triangle at large, take any equal part of LN, and of the Distance known; as in that Problem; exemplified in Example 10th.

Draw *A*M and CM, and *A*N cuting *C*M at *K*; draw L*K*, till it cuts *A*M at *D*. *ACKD* is the square of the Top. - - - - - - Prob. 20.

From each Angle, *A*, *C*, and *D*, draw perpendicular Lines, which represent the Corners of the Planes of a square Prism; on which, the Cornice, ABE, is to be projected; the Profile being proportioned to the nearest Angle, or Corner *AB*.

From *B* (*AB* being equal to AB) draw *B*L and *B*M, cuting the Perpendiculars, from *C* and *D*, at *G* and *H*. *ACGB* and *ADHB* are the Grounds or Seats of the Cornice on the two Planes, which are seen.

The Projecture of the Cornice, around the Prism, is next to be determined.

Draw the Diagonal *DC*, and produce it both ways; also, produce N*A* indefinite, and draw *A*k parallel to the Horizon; on which, take the diagonal projectures of the Cornice, as on AK, in the Profile; (EK being made equal and perpendicular to AE; consequently, AK, the Diagonal of the Square AJKE, of the projecture AE, is the mitre Angle of a Right Angle.)

From the several Projectures, C, D, &c. draw CG and DF, parallel to AB, and produce them to the Diagonal, cuting it in H and I.

Make *A*h, hi, and ik respectively equal to AH, HI and IK; and having made NO equal to the Distance of the Vanishing Point N, of the Diagonal, draw Oh, Oi, and Ok, till they cut N*A*, produced, in *h*, *i*, and *E*; from all which, draw Lines to both Vanishing Points, L and M, cuting the other Diagonal *CD*, produced, both ways, at *F*, *I*, *k*, *l*, &c. which forms the perspective Plan of the principal Mouldings; and *EB*, *FG*, and *HI*, being joined, are diagonal Lines of each Angle, from the greatest projecture to the bottom of the Cornice.

Let it be observed, that N being the Vanishing Point of the Diagonal of a Square, i. e. of a Line bisecting the Right Angle, is the same whether the Top be a Square or other Rectangle; but, if it be not a Square, *CD*, produced, would not be the Mitre of the other Angles; because it would not bisect them; neither would *A*N pass through the opposite Angle, or Mitre *KL*; yet N would be the Vanishing Point of both, because they would still be parallel. The others are also parallel, and consequently, they have the same Vanishing Point.

Having shewn how the Mouldings are projected forward, on the Diagonal *AE*, and transfered to the other Diagonal; suppose No. 2 the same thing, prepared in the same manner; being divested of several preparatory Lines, which, altogether, render the Work confused, and are supposed to be rubbed out. Fig. 87. No. 2.

On *AB*, take *a*, *b*, and *c*, equal to the several heights of the Mouldings (as at a, b, and c, in the Profile) and from N, project them (as N*a*, &c.) till they cut Perpendiculars, from *i* and *h*, in *d*, *e*, *f*, and *g*; and, from them, draw Lines to both Vanishing Points, L and M, cuting Perpendiculars from *k*, *l*, &c. at o, p, and q, which compleats the Facias and small Fillets; and there remains only to join them by curved Lines, of the same kind as in the Profile.

If the Mouldings were very large, as many Points may be found, in the Curves, as are necessary to describe them with accuracy; but, they may be as well performed by a careful Hand; for after all attempts at exactness, in such minutias, a judicious Person, in Mouldings, and Perspective, would describe them as perfectly by hand; regarding the Position in which they are seen.

Plate XIX. Fig. 87. If the Diagonal Section *AEB* be but little inclined to the Picture, and fall near its Center, the Curves, of the mitre Angle of the Mouldings, approach nearly to Right Lines; i. e. if the Eye be nearly in the Plane of the Diagonal; for if it be in the Plane (however situated) they are Right Lines.

This manner of proceeding is properly Projection; seeing that, the full measures are applied, at *AB*, and projected forward to *E* d, where they are larger than the Originals; that is, than the given measure; as *E* a b c d, than *A a*, *a b*, &c. on *AB*; equal A a, a b, &c. in the Profile

In which Case, if *AB* be supposed in the Picture, *A* e h *B* on one Side, and *A* f g i *B* on the other, are Sections of the Mouldings by the Picture; and, the whole, e f g *B*, of that Section, being on this Side of the Picture, is projected, to the Picture. *AE* d f *B*, *CF* p *G*, and *HID*, are diagonal Sections through the mitre Angles; and, f k l m is a Section perpendicular to the Plane *ACGB*, and to the Horizon.

If what I have advanced be well understood, all that follows respecting right-lined Mouldings, will be found easy, and intelligible, almost from inspection of the Figures.

EXAMPLE XIX.

How to represent an Entablature with Modilions in the Cornice, obliquely situated to the Picture, having an internal, and external Angles.

Fig. 88. ABC is the geometrical Profile of the Cornice; F is the Front of a Modilion, and G is its Profile, or geometrical side View.

CD is the Frize, and DE the Architrave; with two Facias.

In this Example, I shall suppose the whole Cornice to be beyond the Picture, and touching it at the Angle *A*.

AFG is the Intersection of the Plane of the Top, HL is the Horizontal vanishing Line, C is its Center, and CE the Distance of the Picture. H and L are the Vanishing Points of the Sides; i. e. of the horizontal Lines, of the Mouldings; M and N are the Distances of those Vanishing Points H and L, for proportioning Lines which vanish in them, respectively; and O is the Vanishing Point of the Diagonal, bisecting the Angle, made by the Radials of the Sides.

Draw the vertical Intersection, *AE*, of the diagonal Plane, passing through the mitre Angle; and, transfer all the measures of the heights of the Mouldings, from the Profile, to *c*, *d*, &c. equal Bc, cd, &c. also, their projectures, to *A a*, *a b*, *b B*, equal A a, a b, and b B.

Draw *A*L, indefinite; and from all the Points *a*, *b*, &c. draw Lines to N, cuting *A*L at 1, 2, 3; from which, draw Lines to H, cuting a Diagonal, AO, at *f*, *g*, &c. the perspective Seats of the projectures; i. e. of the mitre Angles of the Corona, K, &c. (No. 1.)

Draw perpendiculars from *f*, *g*, and b; and from *c*, *d*, &c. (on the vertical Section *AE*) draw diagonal Lines to O, cuting them, at k, l, &c. the perspective Representations of the mitre Angles, of the several Facias and Fillets, of the Mouldings in the Cornice.

From the several Angles, k, l, &c. thus obtained, draw Lines to both Vanishing Points, H and L, indefinite; and proportion them to their respective lengths, by means of the Distances, M, and N, respectively, of those Vanishing Points.

The length being determined, according to the number of Modilions, contained, or otherwise; make *Ak*, equal to the first Break, and draw *k* N, cuting *A*L at f, the internal Angle; and draw f O, the Diagonal Line of that Angle, in the Plane of the Top; indefinite.

Draw b L, cuting that Diagonal, at g; from which, draw a Perpendicular; and c L cuting it at e, and join e f; the Diagonal of the projecture.

Draw H e, H f, and H g, indefinite; and, make f *F* to represent a Line in the proportion of the Original, of f *F* to *A* f; (by Prob. 10) or, draw M f, and produce it to the Intersection, at *B*; make *BF* to *Ak* in that proportion, and draw *F*M, cuting H f produced, at *F*. (Prob. 17.)

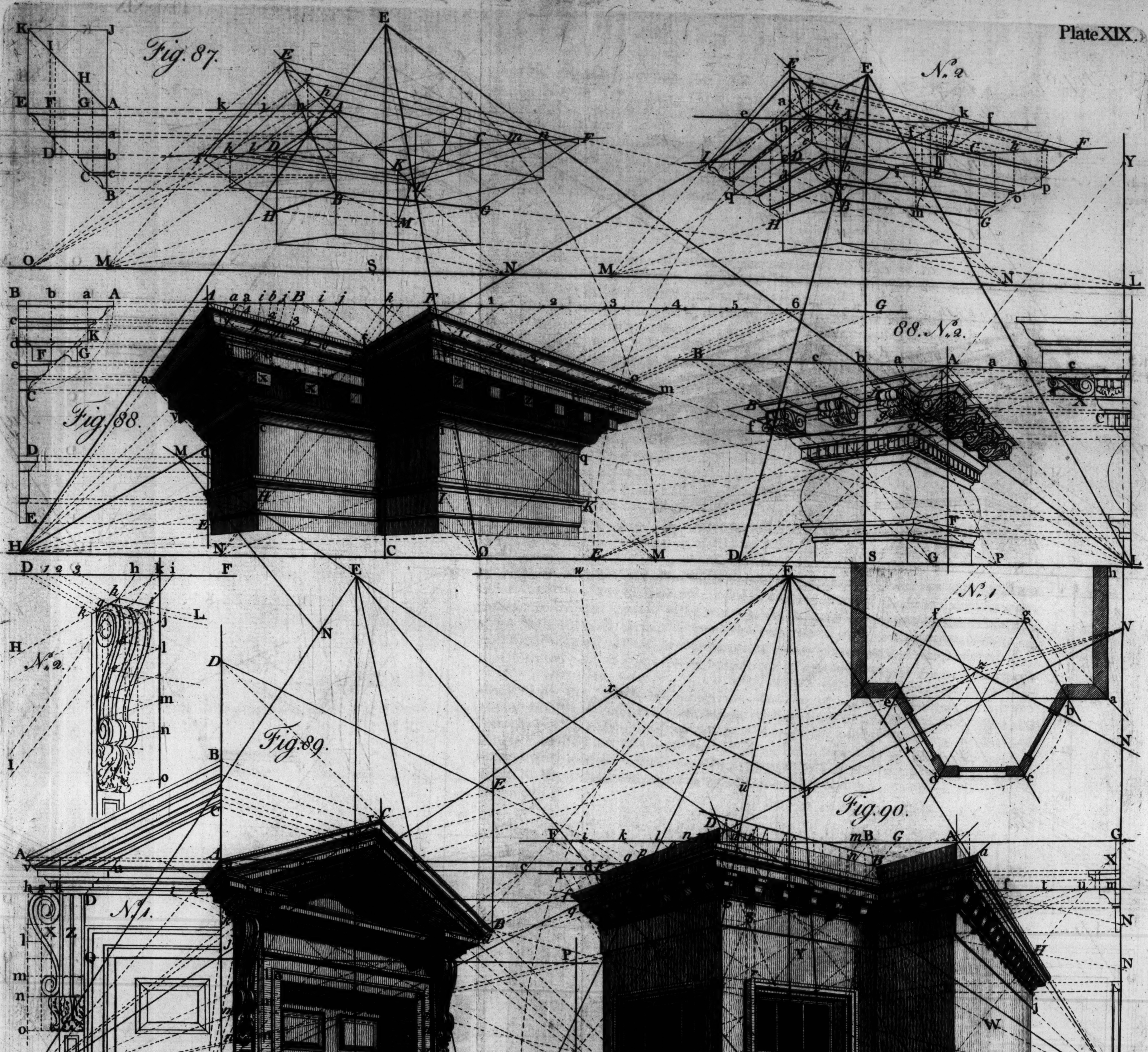
Draw *F*O, cuting H g, produced, at h; from which, draw a Perpendicular, cuting H e, produced, at i; and draw *F* L, h L, and i L, indefinite.

Produce N*F* to the Intersection, and make *FG* equal to the length of the front Line of the Cornice; and *G*N, cuting *F* L at m; and draw m H, cuting h L at n.

Draw O n and produce it to o, and draw o H cuting h L at p; from which, draw a Perpendicular cuting i L at q.

Thus

Plate XIX.



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Thus, are all the Angles at the top and bottom of the Cornice obtained; and having set off the measure of the upper Fillet, at *A*, let it be continued around, by means of the Vanishing Points H and L; then draw diagonal Lines, *F* i, and m q, by which the Mouldings are all determined (as at AC in the Profile) having, before, obtained their places on the diagonal Line, A c, at the first Angle. Fig. 88.

The perpendicular Lines of the Corona fall below it, from the Points where they are cut by the upper Line of the same, at k.

On the returning Side, on the left hand, the Cornice is supposed to fall against a Wall, or cut off streight, and consequently has not a mitre Angle.

Draw *A* H, indefinite; make *A* a represent a length equal to what the whole projects from the Wall (by means of *A* f, Prob. 10; or, by its measure, Pr. 17.)

Draw a L and b H cuting at r; from which, draw a Perpendicular.

Draw c H cuting it at d, and draw d a, which will terminate all the Mouldings cuting the wall; as the diagonal Lines determine the mitre Angles; by drawing Lines from every Angle, on *A* c, to the Vanishing Point H.

The Architrave, *E, H, I, K*, being composed of plane Facias, with one Moulding, only, on each Face, needs no particular description.

The Modilions are determined, in the same manner, as the Mutules in Ex. 16.

Make *A* a equal to A a, and draw a N, cuting *A* f, at 4, and draw 4 H cuting the Diagonal *A* O, at *f*; and, from *f*, draw a Perpendicular, giving the mitre Angle, S, of the front Planes, of the Modilions. Let it be continued around the several Breaks (as the Corona) in Pencil-Lines.

Draw *f* L, cuting the next Diagonal, at *l*. *fl* is the Seat of the front of the Modilions on that Side; which may be continued around, from one Diagonal to the other, by means of the Vanishing Points H and L.

Make *A b* equal to A b, in the Profile, and draw *b* N, cuting *A* f at 2; and draw 2 H, cuting *fl* at *b*. From N, project the Point *b* to the Intersection, at *i*; and make *i j, j i, j k*, &c. respectively equal to the width of the Modilions, and to the space between them, alternately, as often as is requisite; from all which, draw Lines to the Point N, cuting the Seat, *fl*, in their perspective widths, at *m, n*, &c. from which, draw Perpendiculars, giving the several Fronts, x, x, as in the Figure.

The Modilions, y y and z z, in the returning Side and Front, are determined after the same manner, viz. those in the Side, y, y, and also v, v, by means of the Point M, and the front ones, z, z, by the Point N.

Thus, having obtained the Fronts, draw Lines to the respective Vanishing Points H and L, till they cut the Plane from which they project; and, having drawn the Right Lines for the edge underneath, the curved Lines must be drawn by hand, as in the Figure, regarding the different appearance of each, as they recede.

If the Parts are large, and require to be accurately projected, the Moulding, around the Modilions, is managed the same as in the last Example, by a perspective Plan, above the Cornice.

But, a judicious Person will save that unnecessary trouble; for, having obtained their mitre Angles, as at s, t, and u, the front Mouldings are drawn as any other continued Moulding; and they may be returned at the Sides, and meet the inner Moulding, sufficiently correct without the trouble of planing them.

In this Example, there is all the variety that is requisite for such Cornices; but, as the Corinthian Modilion may, to some, appear more difficult than the Ionic, I shall give a specimen how it may be delineated, as briefly as possible.

In the following Lesson, I shall not describe, over again, how the Mouldings are to be drawn, and I have (in order to have the Modilions larger) omited the upper Mouldings of the Cornice, and the Corona; having retained only the Planceer; which I shall consider as the Plane, on which the Modilions are seated, and AB the Intersecting Line of that Plane.

The Vanishing Points, H and L, are also the Vanishing Points of this Figure; but the Center or Point of view is at S, and the Distance is ES. Fig. 88. No. 2.

Plate XIX. Fig. 88. No. 2. Draw the vertical Interſection of the Diagonal, AF; and, by a geometrical Scale of the Proportions, ſet off the heighths of the Mouldings at A*a*, *ab*, &c. alſo, make a b, b c, &c. equal to the width of the Modilions and ſpaces between; alſo the projectures of the Mouldings; and transfer them, by means of the Point P (the Diſtance of the Vaniſhing Point H) to A*B* (the dotted Line) which is the Seat of the fronts of the Modilions; and finiſh the ſquare Blocks *efgb*, &c. which encloſes the Modilion, as in the former Example, by which means, they are truely proportioned, perſpectively, as in the Figure; which would otherwiſe, be ſomewhat difficult.

In the Sides, of each, muſt be drawn, by hand, the Profile of the Modilion, perſpectively, as it is repreſented at X (the geometrical Profile) according as they are contracted, or obliquely ſituated to the Eye.

In the Front, of each, is alſo deſcribed the end of the Scroll, and of the leaf. In ſhort, having obtained the ſquare Block or Caſe which contains them, the reſt muſt be delineated by a judicious hand and Eye; for ſuch Figures will baffle all Rules, nor is it poſſible to ſubject them to any, by which they may with certainty be deſcribed.

The Dentils have nothing particular in them; the proportion being known, of their width and ſpaces between; a Right Line drawn through either Corner, at g, parallel to the Interſection, and divided in the Ratio, and Lines drawn to any Point, G, in the Vaniſhing Line, according as the meaſures are taken, greater or leſs, will give their meaſures perſpectively (as by Prob. 8) but, their true meaſures can only be applied, at the Interſection of the Plane they are in, with the Picture (drawn through C) and projected by means of Viſual Rays, to the Eye, at P.

E X A M P L E XX.

How to repreſent a Door Head, with a Pediment, ſupported by Truſſes or Conſoles.

Fig. 89. Let *A* be the determined Angle of the Cornice of the Object, in the Picture.

At No. 1. is the true geometrical proportion of the whole, in Front.

Through *A*, draw *AD* perpendicular, the vertical Section of a diagonal Plane, paſſing through the mitre Angle, whoſe Vaniſhing Point is G†. H and L are the Vaniſhing Points of the front and ſide Lines, which are horizontal; S is the Center of the Picture, and SE its Diſtance; I and K are the Diſtance Points of each, reſpectively, for proportioning Lines which vaniſh in them‡; EG biſects the Angle HEL; conſ. G is the Vaniſhing Point of horizontal, diagonal Lines. (Pr. 21.)

† Prob. 3.

‡ Prob. 17.

These things, here premiſed or given, are ſuppoſed to be found and determined, from the known poſition and ſituation of the Object, as in the former Examples. (See Prob. 12.)

All the meaſures, from the Profile, being produced, or applied to the Interſection *AD*; and, by means of the Diagonal *A*G, are got the mitre Angle of each Moulding, as in the foregoing Examples.

To determine the breakings of the Mouldings around the Truſſes, draw a d, the Interſection of the Plane they are in, above the Cimma reverſa; and, by means of the Point I, project the Point a to that Interſection.

Make a b, b c, and c d, reſpectively, equal to the Breaks, and to the opening between them, geometrical (as at v u, No. 1.) and draw b I, c I, &c. cuting a L, at *b*, *c*, and *d*, their perſpective Proportions.

Draw *c* H, indefinite; and by means of the Point K (the Diſtance point of H) project the Point *c* to the Interſection, at e.

Make e f equal to the returning Side, and draw f K, cuting *c* H, at *f*; and, through *f*, draw L*f*, cuting *b* H; which determines all the mitre Angles.

The Mouldings are deſcribed, or delineated, as in the laſt Examples.

To determine the proportion of the Truss, or Console; of which X is the Profile, and Z the Front. Fig. 89.

Through D, in the Profile (No. 1.) draw a horizontal Line, DF, and produce *k h* (from H) to that Intersection, cuting it, at k.

Draw k n, perpendicular; and, from k, set off, geometrically, all the proportions of the Truss, at l, m, n, in respect of its height (as in the Profile at X) from which, draw Lines to H, cuting *k n*, at *l, m, n*, from which they are transfered to the other Side, by the Vanishing Point L, at *v, u, x*; and, by the Point H, projected forward; which proportions the heighths in the other Truss.

The geometrical projectures are perspectively proportioned, in the same manner as the returning Moulding, at *c f*, by projecting the Point *k* (from K) to the Intersection DF, at 1, and make 1, 2, 3, &c. equal to the projectures in the Profile; from which, draw Lines to K, cuting k H at *g*, *h*, &c. and from them, draw perpendicular Lines, as in the Figure; by which means, finding as many Points as are necessary, the true perspective form may be determined.

At No. 2, is shewn how the Truss is described, separate from the Work, having all the same Letters of reference; and, in proportion to the other, as 3 to 2.

If the width be not already determined, by the Mouldings; draw I *h* till it cuts the Intersection, DF, at h.

Make h i equal to the width of the Truss, and draw i I, cuting *h* L at *i*.

Draw *g* L, *h* L, and *k* L, cuting a Line drawn from *e* to H, at *r, s, t*; from which, perpendiculars being drawn, determines the projecture of that Truss.

Having, by those means, obtained as many Points as are requisite, as at 1, 2, 3 (No. 2.) the Curves must be described by a judicious and steady Hand; for, no Rules can possibly describe the Curves, otherways.

To determine the Pediment. Having obtained the Angle *B*, its perspective length (Prob. 17) equal twice A*A*, geometrical; at I (the distance of the Eye from the Vanishing Point L) make the Angle, LIV, equal to the Angle of the Pediment, cuting VL (the Vanishing Line of the front Planes, Prob. 3) at V, and set off an equal Distance on the other side L; from which Points, Lines drawn through *A* and *B*, determines the middle, at *C*; and all the Mouldings vanish in those Points; and is the only means, by which they can be projected with accuracy.

The height and true pitch, or inclination, may be thus determined.

Make B*D* equal *A*B; i. e. make *AD* equal to twice the height of the Pediment. Draw *BE* also perpendicular, and draw *D*L, cuting it at *E*.

Draw *AE* and *BD*, intersecting at *C*; *ACB* is the true perspective outline of the Pediment.

Or, having bisected *AB*, perspectively at *G* (by Prob. 8) draw a Perpendicular; and, from B, draw BL, cuting it, at *C* as before; and draw A*C* and *BC*.

From all the Points, between B and C, draw Lines to the Vanishing Point L, cuting *CG*; and the projectures of the Mouldings being also perspectively proportioned, on *C*H, at r, s, t; perpendicular Lines, from them, will cut the others, from *CG*, to H, in the true Mitres of those Angles; and, from those Angles, draw Lines to both Vanishing Points, V, of the Pediment, which will give all the Mouldings, with accuracy, beyond any other method whatever.

The horizontal width of the Frize and Architrave, are proportioned to the Truss and Mouldings, on *k n*; by drawing Lines to L; the perpendicular widths of the Architrave (being parallel to the Picture) and the opening of the Door, by the Vanishing Points of the Diagonals of a Square (as at Y) in the Vanishing Line VL of that Plane; regard being had to the projecture of the Mouldings (see Pr. 26.)

The recess of the Door, p q, is determined (by Prob. 10) by its width o p; determined above.

Produce

Plate XIX. Fig. 89. Produce the Radial LE; make EM equal to the width, and EN to the recess; or, make EM and EN, in the Ratio of one to the other.

Join MN, and draw EO parallel to MN. O is the Vanishing Point of a Diagonal of the Soffit, or head of the Door Case; and o O being drawn, cuts p H at q, as required. The rest is obvious, from the Figure.

This Method, when it is applicable, is preferable to any other; for, having obtained the true proportion of one Line (o p) in any Plane, any other Line (p q) in that Plane is easily determined, by the 10th Problem. Indeed, it may always be applicable (provided, the Diagonal Line o q (i. e. M N) be not very oblique to the Picture; and consequently, its Vanishing Point O, (very remote) for, if the whole Distance (SE) cannot be used, half, or any other portion, may be taken, with equal propriety; and the Point, O, ascertained the very same (by Prob. 12.)

Otherwise; if no Line, in the same Plane, be found or determined, there is but one general Method; which was applied, to proportion the return of the Moulding *c f*; or the front Line a *d*, at *b* and *c*, viz. by the true measure applied to the Intersection of the Plane it is in (Prob. 17) which being frequently used, would be needless to repeat; particular regard being had to the true Intersection of either Plane, o p q, of the Soffit (which is horizontal) or of the vertical Plane, W, of the Door Jamb.

For, since p q is the common Intersection of both Planes, it is consequently in both; and therefore, either Intersection will answer the same purpose.

PQ is the Intersection of the horizontal Plane o p q; its true geometrical Distance, from any other, being known, equal DQ; a Line drawn through Q, parallel to the Horizon, is its Intersection with the Picture; for, all parallel Planes have parallel Intersections (8. 7. El.) And, if H p be produced, till it cuts PQ; a Line drawn through P, perpendicular, is the Intersection of the vertical Plane W.

For, p q cuts the Picture at P; therefore, P is its intersecting Point (Def. K) and consequently, PR is the Intersection of the vertical Plane, W, that Line is in. (Prob. 3.)

EXAMPLE XXI.

How to delineate a Block-Cornice, and to break the same, or any other, around a Bow Window; which is half a hexagonal Prism.

Fig. 90. HL is the Horizontal vanishing Line, C the Center of the Picture, and CE is its Distance; H is the vanishing Point of the Front of the Bow, and L of the Side of the Building, at right angles with it.

Draw EH and EL making a Right Angle, HEL; and, with any Radius, on E, describe an Ark, *x*, *y*, *z*, cuting EH, at *x*.

Make *xy* and *yz* each equal to E*x*; and draw E*y*, cuting the Horizontal Line, at M; and E*z*, which, would cut it, if produced; M, and N (the supposed Point where E*z* would cut HL, produced) are the vanishing Points of the Sides and Diagonals of the Hexagon.

Bisect the Angle *v* E *x*, at *u*, and draw E *u*, to P; which is the Vanishing Point of the Diagonal of the Right Angle.

Let AD be the Intersection of the nearest Right Angle of the Object with the Picture; and A the determined height of that Corner; let A a be the height of a Plinth, above the Cornice; & a c the height of the Cornice, of which X is the Profile.

The Dimensions of the Bow window, and other parts of the Building, being known, it would almost be superfluous to describe how it is to be determined in Perspective; sufficient instructions for that purpose are contained in the 4th and 5th Examples. However, as the application of them to real Objects, may not, to some, be familiar, I shall, briefly, describe it.

At No. 1. is a Plan of half the Building, to a Scale of half the measures applied in delineating; and about one sixtieth part of the real Object.

Draw AF the Intersection of the Plane of the Top, parallel to the Horizontal Line; make HD equal to EH, the Distance of the Vanishing Point H; and draw AH and AL indefinite.

Make AB equal to the short returning Plane (V) which is at right Angles with W, the long Side of the Building (twice a g in the Plan No. 1.) i. e. make AB equal twice a b in the Plan, and draw BD, cuting AH, at *B*.

Draw N*B* indefinite (Prob. 13) and having found the point O (the distance of N, by Pr. 12) draw O*B* to the Intersection AF, cuting it at *G*.

Make

Make *Ge*, on the Intersection AF, equal to twice bc (No. 1) and draw O*e*, cuting *NB* at *D**. Draw *D*H, indefinite; and draw *DD* cuting AF at *d*; make *d*F equal to *Ge* (i. e. to the Front) and draw FD, cuting *D*H at *F*.

From all the Angles, *B*, *D*, and *F*, draw Perpendiculars, which gives all the Faces of the Hexagon, that can be seen (Y, and Z) i. e. bc, and cd (No. 1.)

The Plane W, which represents the long Side of the Building is thus determined.

As there is not room, on the Intersection, to set off the whole measure (equal four times ah, No. 1) take AG a third part of it, and L*E* one third of EL, and draw G*E*, cuting AL at *H*; from which draw a Perpendicular.

Thus, having determined the Object, we now proceed to the Cornice.

From X, the Profile, transfer all the measures, to AD, of the heights of the Mouldings, at ab. Draw the Diagonal P*a*; and determine the mitre Angle of the Moulding (as in Example 17th) at a*ab*b.

Draw aH and bH, cuting *B*h, at c and h. Draw Nc and Nh, till they cut *D*g, at d and g; and draw dH and gH, cuting *F*f, at e and f. Also, from b and i, draw bL and iL, representing a Facia, below the Cornice, and from i draw iH, cuting *B*h, and continue it around each Face of the Hexagon.

The mitre Angles of the Hexagon are thus obtained.

In every regular Poligon, having an even number of Sides, the Diagonals, which pass through the Center, bisect the Angles of the Poligon; and in a Hexagon, they divide it into six equilateral Triangles, each Diagonal being parallel to two opposite Sides. (See the Figure, No 1.)

*x*E*y* and *y*E*z* are two such Triangles; consequently E*x*, E*y*, and E*z*, are parallel to all the Sides and Diagonals of the Hexagon†, producing the Vanishing Points, H, M, and N, of the Diagonals or mitre Angles. † Cor. 1, Th. 8.

But, the Angle, at b (No. 1.) where the Hexagon joins with the Building is internal, and its Mitre is gb, produced; consequently parallel to fc; therefore, they have the same Vanishing Point, M.

ab is the Angle of the Building (which is a Right one) where the Mouldings are first projected; ch is the internal, and dg, ef, two external Angles of the Hexagon.

Draw dM, cuting aH, at S, the representation of the Center of the Hexagon; and eM, cuting it, at l; cdel is the Plan of the Top of the Cornice, around.

Produce Se, Sd, and Mc, indefinite; and draw *a*H, cuting Mc, produced, at *c*.

Draw N*c*, cuting Sd at *e*, and *e*H cuting Se, produced at *f*; *a*, *c*, *e*, and *f*, are the extreme Angles of the Cornice; which being obtained, the rest is managed as in the former Examples; by transfering all the measures, of all the heights of the Mouldings, from ab to ch, and from ch to dg, &c. also, the projectures of the Mouldings, from the Diagonal a*a* to c*c*, from c*c* to d*e*, and from d*e* to e*f*; by means of the vanishing Points H and N; from which Diagonals, perpendiculars being drawn, giving the Corona, at *b*, which is carried around in the same manner; and also the small Mouldings, at bhgf.

The Blocks, are obtained after the same manner as the Modilions in Ex. 19th.

Or; having, from the internal Angles, made by two Blocks (at n and o) drawn the sides of each, to their respective Vanishing Points, H, and L; and Q of the Side Y (making NEQ a Right Angle) by which means, the Fronts are obtained, as at p. Draw pq parallel to the Vanishing Line; and having, also, found the point *q*, of the farthest Block; from any Point, whatever, in the Vanishing Line, as P, draw P*q*, till it cuts pq at q.

Divide pq, geometrically, into the number of Blocks and spaces between them, at 1, 2, 3, &c. from which, draw Lines to the same point, P, cuting p*q*, at *g*, *h*, &c. the true perspective proportions of the fronts of the Blocks, on the Face Z.

On the other Sides, they may be determined after the same manner, or otherways; as on the Side Y, by their Seat, *BD*, on the Top; how they are finished is best explained by inspection of the Figure.

* Because the point *D* (in *BD*) is beyond the Intersection, AF, it is considered as projected on this Side of the Picture,

Plate XIX. Fig. 90. On the Side W, they are determined from their true measures, by drawing m n, through n, parallel to AF; m n is the Intersection of the Plane they are in with the Picture (which p q is not) therefore, having (from the point Q) projected the first to m n, at r, set off the true geometrical measures, from r towards m, at r, s, &c. and, by means of the same point Q, project them to r L; i. e. draw r Q, s Q, &c. cuting r L, which gives their true places, on that Side.

The Blocks, in the returning Side, V, are determined, after the same manner, from the true measures; by means of the point D, the Distance point of the vanishing Point, H, of that Side.

The Windows, and the Mouldings around them, are determined as the Door, &c. in the foregoing Example; by setting off the true space and measure of the Moulding from i to k, and transfering them from one Plane to the other, as in the Figure.

G *e* and *d* F, on the Intersection AF, being the true geometrical measures of each Plane; divide them in the true measures of Piers, Mouldings, and Sash Squares, at *i*, *k*, *l*, and *m*; from which draw Lines to the Points D and O, respectively, cuting *BD* and *DF*, at *n*, *o*, *p*, *q*; and, from them, Perpendiculars being drawn, determine their perspective Proportions on those Planes, at *r*, *s*, &c. and the proportions of the Sash Squares are projected to their true places, from *t* and *u*, by means of the Vanishing Points L and Q, respectively.

SECTION VIII.

OF CURVILINEAR OBJECTS.

THE Perspective of curve-lined Objects, the Subject of this 8th Section, is the most difficult, of all other; seeing that, Curve Lines cannot be projected as Right Lines, by means of intersecting and vanishing Points, indefinite; neither can any portion be taken, or cut off, perspectively, otherwise than by drawing a Chord Line from one Point to another in the Original, and finding its Vanishing Point; or, by any means, finding the representation of the extreme Points, from their corresponding Points in the Original.

There are various ways of obtaining the representation of a Circle, in Perspective; all which, do no more than find the representations of various Points in the Circumference. For, by the Theory of curvilinear Perspective, it is supposed, that the description or delineation of a Circle, or Sphere, in Perspective, is some one or other of the Sections of a Cone; as in Fig. 28, Plate 7; it is obvious, that if Right Lines, EA, EB, &c. from the Eye to the several Points A, B, C, &c. in the Curve (which is supposed to be a Circle) in the Plane Z; be cut by another Plane, X or Y, the Points a, b, c, &c. being joined, carefully by a steady hand will generate a Curve, a d g, which, to the Eye at E, will (as it is obvious it must) exactly coincide with the original Curve; seeing that, it is in the surface of the same Cone, of which, the Eye is its Vertex.

Hence it is manifest, that, the more Points there are found in the Representation, the more exactly may the curve be described; but after all, it depends greatly on the Hand and Eye; insomuch that, without great nicety in both, the Representations of curve-lined Objects will have a lame, and very disagreeable Appearance. It is, therefore, no wonder to see such bad Representations of round Objects, as are to be met with; but it is a matter of surprize, that any Person, who attempts it, should have so little judgement as to turn the Curves the contrary way; or to make them flatter in those parts, where, it is obvious that, they would have a greater curviture.

Without having the least notion how to project Curves, perspectively, is it possible for a Person, who has been used to sketch at all, to place a cylindrical Object; of any kind, or a Vase, &c. before him, and not see, imediately, how the curves are to be described? or, what part of the Object appears more or less curved than others? is it not obvious to a common Eye? yet may we frequently see, not only on the Pannels of Coaches, &c. Vases represented, whose greatest swell, at the Top, is a Right Line, whilst the Curves of the lower part take a contrary direction, to each other; indicating, that the Eye is between them; in which Case the Top would be the most curved. Would not such attempts at Perspective be better let alone? and content themselves with a perfectly geometrical Representation? in which they would all be Right Lines; save only, the external Figure of a vertical Section through the middle.

But, what can be said for the performances of those Artists, whose Works are an honour to their Country and to the Age they lived in, to see them, often, greatly deficient in those particulars. I could wish to see the Works of the present Age more perfect; which, in other respects, seem to vie with the most celebrated amongst the Antients; yet do not pay sufficient attention to those necessary Appendages, which are essentially requisite, to a perfect Picture.

As a Circle is the first and principal of curve Lines, so it is the only one that can be reduced to any certain Rules, in delineating it perspectively. And, of all the various ways to project the representation of a Circle in Perspective, the best, and most practicable, is to suppose a regular Octagon to be inscribed, or the Circumference divided into eight equal Parts; or, if very large, into sixteen.

Irregular curved Objects are not Subjects for Perspective; all attempts at a Spiral, or twisted Column, &c. by Perspective Rules, would be in vain. Various other Objects, as Rocks, Mountains, Rivers, Trees, &c. are not fit Subjects for Perspective; and consequently Landscape Views cannot be taken or delineated by its Rules; because it is impossible to have the true geometrical Figures and Proportions of such Objects, as before mentioned. I shall however, in the Appendix, describe the use and application of an Apparatus for taking views, with ease and great exactness; the best calculated, for the purpose, of any I know or have ever heard of. For, notwithstanding what many imagine and affirm, of the possibility of taking Landscape Views with Accuracy, by Sight only, I know it is impossible to be done; and cannot conceive it to be any way derogatory to the abilities of the most eminent Artist, to make use of any expedient; by means of which, he may be enabled to make a more correct Portrait, and Picture. I do not mean that he should, rigidly, describe every minutia of the Objects, as in Trees, &c. by it; but I must affirm, that he would take the Magnitudes of the several Objects, and their Bearings in respect of each other, with infinitely greater accuracy than it is possible to do by Sight.

P R O B L E M I.

To find the Representation of a Circle; the Original being given, in any Plane, whose Vanishing Line, its Center and Distance are given; according to Brook Taylor.

First; by means of the Vanishing Line, and one Vanishing Point only.

Let AFG be a Circle, in the Geometrical Plane; of which *GK* is the Intersection, and V*E* the Vanishing Line; *C* is its Center, and *C*E its Distance. Plate XX. Fig. 91. No. 1.

Draw AB, at pleasure, cuting the Intersection at a; and through C, D, G, &c. (Points assumed at pleasure, in the Circumference) draw Lines parallel to AB, cuting the Intersection, at a, c, d, &c.

Draw

Plate XX. Fig. 91. Draw EV parallel to AB, producing the Vanishing Point, V, of those Lines.

Draw aV, cV, &c. and, to E, draw Visual Rays from every Point, A, D, G, &c. cuting the indefinite Representations of the Lines passing through those Points, at *a*, *d*, *g*, &c.

Having thus obtained as many Points as are necessary, a Curve *adifb*, described carefully through those Points, will be an Ellipsis, and it is the true Representation of the Circle AFG.

Or, having drawn the parallel Lines AB, &c. and their indefinite Representations, as before; make V*E* equal to VE; also, make ab equal to aB, ge equal to gH, &c. and draw b*E*, e*E*, &c. which will give the same Points as before.

This needs no Demonstration; seeing that, the Points *a*, *b*, *c*, *d*, &c. are projected the same, as in Prob. 6th, where it is fully demonstrated. And, that the Curve is an Ellipsis, is demonstrated in Th. 2, Sect. 5, of Curvilinear Perspective; for it is evident, and manifest, that the Visual Rays EA, ED, &c. from the Eye to every Point in the Circumference, would cut the Picture in the corresponding Points *a*, *c*, *d*, &c. therefore the Curve *adf* is the true Section of the Cone of Rays, and consequently it is an Ellipsis.

Secondly; by the Directing Line and Director. No. 2.

Fig. 91. No. 2. Let No. 2. be a Circle, nearly in the same Position to the Eye (at E) as before; having the same Intersection, GKI, and the same distance of the Eye.

Draw CD parallel to the Intersection, and distant from the Eye equal to the distance of the Vanishing Line, VL, from the Intersection GKI. CD is the Directing Line. (Def. 10.)

Draw AD at pleasure, cuting the Directing Line at D; and, from several Points, B, F, &c. in the Circumference, draw Right Lines to D, cuting the Intersection at *c*, *d*, *e*, &c. the Intersecting Points of those Lines.

Draw ED; and, from the Intersecting Points, K, c, &c. draw Right Lines parallel to ED, which are the indefinite Representations of those Lines, respectively.

Draw the Visual Rays EA, EB, &c. as before, cuting those Lines at *a*, *b*, *f*, &c. and describe a Curve through them, which is the true Representation of the Circle AFH. Draw VL, the Vanishing Line of the Plane of the Circle.

DEM. For, because the Lines AD, BD, &c. have the same Directing Point, D, their Representations are parallel between themselves (Cor. 1. Theo. 14.)

Consequently, seeing that Ka, cb, &c. pass through the Intersecting Points, K, c, d, &c. parallel to ED, they are the indefinite Representations of those Lines; and consequently, the Visual Rays EA, &c. will cut them, in the same Points as before, which is manifest; for, if Ea, Eb, &c. be drawn, parallel to AD, BD, &c. respectively, their Vanishing Points, a, b, c, &c. are produced; by which, the affinity between the different Methods of producing the same thing is accounted for.

PROBLEM II.

To describe the Representation of a Circle, having the Representation of one Diameter given.

Fig. 92. Let AB be the given Diameter; let *ECE* be the Vanishing Line of the Plane it is in; *C* is its Center, and *CE* its Distance.

Produce AB to its Vanishing Point, V; and bisect AB, perspectively (Prob. 8) e. g. through A draw A*bc* parallel to the Vanishing Line, and take two equal Divisions A*b*, *bc*, at pleasure. Through *c*, draw *c*B, and produce it to the Vanishing Line at G, and draw *b*G cuting AB, at C, the Center of the Circle; through which, draw DF, parallel to the Vanishing Line.

Make VG equal to VE, and draw AG, cuting DF at D; and, make CF equal to CD; or, through B draw GF. DF is a Diameter of the Circle.

Make *CE*, on both Sides, equal to *CE*; draw *CC* indefinite; and, through D or F, draw *E*D or *E*F, cuting *CC* produced, at H; through which, draw ab parallel to DF; and through D and F, draw *C*D, *C*F, cuting it at a and b.

Draw

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Plate XX.

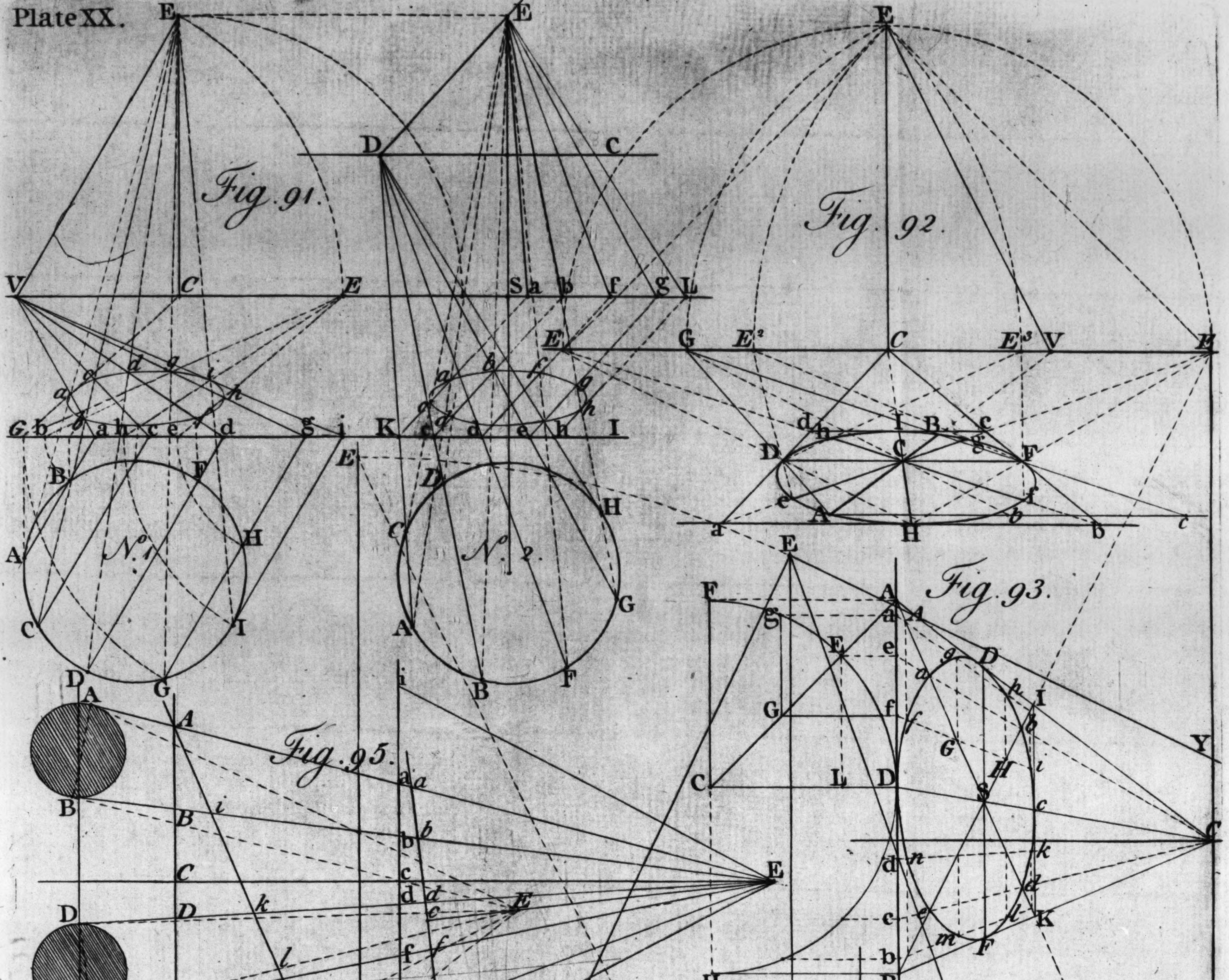


Fig. 91.
No. 1.
No. 2.
Fig. 92.
Fig. 93.
Fig. 94.
Fig. 95.
Fig. 96.
Publish'd May 14th 1774.

Draw the Diagonals aE and bE; which will pass through C, the Center of the Circle, and cut aC, and bC, at d and c; draw cd, cuting CH at I; IH is a Diameter, and abcd is the representation of a Square, circumscribing the Circle.

Make EE^2, and EE^3, each equal to EE; from both which Points, draw Lines through D and F, cuting the Diagonals at e, f, g, and h; which Points are also in the Circumference.

Through the Points, A, H, f, F, g, I, h, and D, if a Curve be described, it will be the true Representation of a Circle, whose Diameter is equal to ab.

DEM. For, suppose ab the Intersection of the Plane the Original Circle is in; and ECE its Vanishing Line. E, E are the Vanishing Points of the Diagonals of a Square†, and the two Diagonals, ac and bd, cut each other in its Center, C‡; which is, consequently, the Center of a Circle, inscribed. † Prob. 19. ‡ 16. 1. El.

And, because HI passes through C the Center of the Circle, HI represents a Diameter perpendicular to ab seeing it vanishes in C; and DF, passing through C, is also a Diameter, parallel to the Picture.

Also, because CD is equal to CF, and EE^2 is equal to EE; E^2 and E^3 are the Distance Points of the Diagonals; and consequently, seeing CD is equal to CF, the diagonal Diameters, eg and fh, represent Lines equal to DF. (See Prob. 8th and 10th, Case 3rd, for a further Demonstration.)

PROBLEM III.

How to describe the Representation of a Circle, in any Plane whose Vanishing Line, Center and Distance are given; and Intersection.

Let AB be the Intersection of a vertical Plane, and ECE its Vanishing Line; its Distance is CE. Let AB be the Diameter given. Fig. 93.

Bisect AB, at D; and draw CD perpendicular to AB.

Make DC equal to AD, and describe the Ark DEF, a fourth part of the Circumference. Draw AC, cuting the Circumference, at E; from which, draw a Perpendicular, Ee, to the Intersection.

Make Bc equal to Ae, and draw AC, BC, and DC; also cC and eC; and draw the Diagonals AE and BE, cuting AC and BC, at I and K; draw IK; and DF, through S, parallel to AB.

Through the Points, D, D and F, also through a, b, c, d, and e, where the Diagonals, and IK, are cut by cC, eC, and DC, if a Curve be described, accurately, it will be an Ellipsis, and the true representation of a Circle, viewed oblique.

If the Circle be large, and eight Points are not sufficient; bisect the Arks, DE and EF, at f and g; from which, draw Perpendiculars to AB; and make b and d, equally distant, as a and f, from D; or from A and B.

Draw aC, fC, dC, and bC. From A and G, &c. where the Diagonals are cut, draw Af and Gg parallel to AB, cuting fC and aC at f and g; and the others at h, i, k, l, m, and n, through which Points, the Representation will also pass, and may be described with more accuracy.

The affinity between the Original and the Representation, which the corresponding Characters particularize, is sufficient Demonstration, S represents the Center of the Circle, C.

This Method of projecting the Representation of a Circle is, of all others, the best and most practicable. It is nearly the same thing as Prob. 25th; for, the eight Points a, D, b, c, d, F, e, and D being joined by Right Lines, will be the Representation of an Octagon, which is circumscribed by the Circle; as, in the other, the Circle is inscribed, i. e. touches every Side, as this passes through its Angles.

Notwithstanding, the Methods, in Prob. 1st by Brook Taylor, are facile and simple, yet I believe, they are scarce ever used in Practice. If the Circle be large, and Distance adequate thereto, they are utterly impracticable; because there is a necessity for having the whole Circle and Distance, at once, in their true places. Whereas, by the last, the Distance is applied on either, or on both sides of the Center, as in all other Cases whatever. Nor is AB necessarily the Intersection of the Plane of the Original Circle. For, if the place of the Circle be determined on the Picture (either its Center, or the nearest part of the Circumference, at D) a Line drawn through S or D, parallel to the Intersection, or Vanishing Line, answers the same purpose. AB or DF being made equal to the known Diameter, in those places, and a quarter of a Circle described, as DEF, of that proportion (which, it must be obvious, is as sufficient as the whole) the rest is as already described. Other Points, if requisite, may be obtained, as f and g, and more, if necessary.

 In

Plate XX. Fig. 93. In common Practice (in a Circle not very large) eight Points being sufficient, there is no real necessity for describing an Ark, geometrically; for e (or c) the Point where a Perpendicular, from E, cuts the Diameter, is distant from A or B somewhat more than one seventh part of the whole Diameter, AB; so that, in all common Cases, it may be ascertained near enough.

The 2nd Problem is useful in many Cases. Having obtained a Line, AB, by any means, on the Picture; which, being known to be the Diameter of a Circle, the whole Circle may be projected by the means there described; or by Prob. 10th, Case 3rd, with the greatest exactness. Having obtained the parallel Diameter, DF, the similarity with this last Problem is discernable.

E X A M P L E XXI.

How to represent a plain, circular Arcade, casually inclined to the Picture.

Fig. 94. Let VM be the Vanishing Line of the Horizon, C the Center of the Picture, CE its Distance, and A the Intersecting Point of the hither Angle of the Steps.

Through A, draw AJ parallel to the Horizon; which may be considered as the Ground Line; also, draw AG perpendicular, the vertical Intersection of a Diagonal Plane; on which, set off, from A, the several heights and proportions of the Object, at D, G, &c.

† Ex. 13. The Angles of the Steps being first obtained, as at A, a, b†; their length may be acquired by Prob. 7th. If they exceed the bounds of the Picture, take any equal portion of their length, as AJ, one third part; also, V being the Vanishing Point of that Side (see Prob. 21, Meth. 3rd) and VE its Distance, make V*D* one third of VE, and draw J*D*, cuting the indefinite Representation AV at N. AN represents a length equal to thrice AJ. (Prob. 17.)

The Steps are finished as in the 13th Example.

Next, the proportion of the Plinth, ecf, is determined by the same; and the others, gh, ik, and l, by Example 4, which, with the Piers, are so many Parallelopipeds, of equal magnitude and equally spaced.

Their measures are set off, at 1, 2, 3, on the Ground Line; and projected to their Seats, at 4, 5, 6, &c. by means of the Point *E*, on one Side; the Distance, VE, of the other Vanishing Point, V, falls out of the Picture.

Their height, and the upper Fillet at H, are obtained, by their geometrical height, AD (as the Pedestal in Example 13th.) These others are perpendicularly over the Plinths below; and the Fillet at the Top is determined from G; and, by drawing Lines to both Vanishing Points, V and Y. The last is not in the Picture, and must be drawn by Prob. 13, or the Point must be ascertained; where, EY, produced, would cut the Vanishing Line VM.

To represent the Arches; which are, in this Example, inclined to the Picture; and consequently, their Representations are Ellipses.

The last Figure and Problem is adapted to this Example.

D on the vertical Intersection, AG is the height of the spring of the Arch.

Make DE and EF equal to the measures on AB (Fig. 93) i. e. make DF equal to the height of the Arch, half the space between the Piers (equal CF) and DE equal De (equal LE) and, by means of the Vanishing Point of the Diagonal (M) transfer them to the Angle of the Building, at H, I, K; or, if HK be the determinate height of the Arch, at that Angle, the rest was unnecessary.

From H, draw Lines to both Vanishing Points, V and Y, cuting the Piers, at *a*, *f*, m, n, &c. Bisect mn, perspectively, at S‡, the center of the Arch.

‡ Prob. 8.

Draw Sp perpendicular, cuting a Line, from K to the Vanishing Point, Y, at p, the crown of the Arch; also, draw no and mq perpendicular; mqon is half a Square, circumscribing the Arch.

Draw the Diagonals So, and Sq, cuting a Line from I, at r and s.

A Curve described through m, r, p, s, and n, is the front of the Arch, the Representation of a Semicircle. If more Points are requisite, see the last Problem.

The inner Curve is described as the outer; n *n* being the thickness of the Pier.

From all the Points in the outer Curve, draw Lines to the Vanishing Point V; and, *m n*, the inner Diameter, answering to m n, being drawn, SV cuts it at *s* the Center; from which, the Diagonals *so*, *sq*, being drawn, in the Rectangle *mqon*, give the Points *r*, and *s*; and a Perpendicular from *s* gives *p*; then, a Curve drawn through *m*, *r*, *p*, *s*, and *n*, determines as much of the Soffit of the Arch as can be seen.

After the same manner, the Arches *a d f*, &c. are described, in the returning Side; which would be superfluous to repeat over again; as the Lines themselves shew their use and application the same.

E X A M P L E XXII.

How to determine the true Diameters of Columns from any Station, and on a Picture in any Position.

Let AB, DF, and GH be the Sections of three Columns by a horizontal Plane, in which the Eye may be supposed to be, at E; and, let *AH* be a Section of the Picture, i. e. suppose a Plane standing upright on *AH* parallel to the Columns; EC perpendicular to *AH* gives its Center. Fig. 95.

The Visual Rays EA, EB, &c. being drawn, are in the same Plane; and the Points *A*, *B*, *D*, &c. where they cut the Picture, it is evident, are the places, where the apparent edges of the Columns will appear, and consequently, *AB* is the apparent width of the Column AB, *DF* of DF, and *GH* of GH; which, on account of the Rays, EG, EH, intersecting the Picture more oblique than those from DF, has a larger Representation; notwithstanding it is really further from the Eye, and from the Center of the Picture, at C.

If the Eye be moved to *E*, the dotted Lines shew the difference in their Proportions, from the two Stations or Points of View; where the difference between the Representations of DF and GH are much larger than at E.

If the Picture be at a h (the Eye being at E) parallel to *AH*, the Proportions of the Representations are in the same Ratio, as their Distances, E c to EC.

The true position of the Picture from that Station, is *a h*, whose Center is at *c*, E*c* perpendicular to *a h*, bisects the Optic Angle, *a* E *h*; which Optic Angle of the other Pictures, is i E h; c i being equal to c h.

And thus the Diameters may be obtained on any Picture, situated to the Columns in any Angle, at pleasure, as *HAK*, the apparent Diameters on *AK*, are where the Visual Rays cut it, at *A*, *i*, *k*, *l*, *m*, *K*.

See this matter more fully treated, in the sixth Section, of the Theory, Fig. 34.

E X A M P L E XXIII.

To represent an upright Cylinder, of any given Dimensions.

Let A B be its Diameter. The height of the Cylinder is supposed to be known. Fig. 96.
Let V be the Center, and VC the Distance of the Picture.

Describe a Square ABDg (Prob. 19) and inscribe a Circle a c e g (by Pr. 2 or 3.)

At the two Extremes of the Ellipsis, a and d, draw Perpendiculars, which represent Sides of the Cylinder. Their height must be determined by a Perpendicular either from A or b, or any Point in the Circumference.

Make A *A* and b *b*, each equal to the determined height of the Cylinder; the Point *b* is in the Circumference at the Top; and, if AC be drawn, it will cut Perpendiculars from a and e, in *a* and *e*, which are in the same Circumference. *b* V will cut a Perpendicular from f at *f*; and thus, as many Points as you please may be obtained, as *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h*, corresponding with a, b, &c. below; through which, the upper Curve may be described.

Or,

Plate XX. Fig. 96. Or, having obtained any three Points, the Curve *a*, *b*, *c*, *d*, may be determined, by Prob. 11, with the greatest accuracy.

The Plinth, ABD is parallel to the Picture; and the Abacus, *B*FG, is equally inclined on both Sides (see Prob. 29 and 30) the Curves are still the same; and if they are equally distant from the Vanishing Line, VC, they are equal and similar Ellipses.

EXAMPLE XXIV.

How to represent several Cylinders, in the same Right Line; inclined to the Picture, at pleasure.

Plate XXI. Fig. 97. Let X, Y, and Z, be the Seats of three Columns, in the Ground Plane, between the parallel Lines, AB and CD; inclined to the Picture in the Angle BAF, at discretion. AF is the Intersection; and HL, the Horizontal Vanishing Line; C is the Center, and CE the Distance of the Picture.

Find the Vanishing Point, H, of the inclination of the Line of the Columns, AB (Prob. 17) and make HEL a Right Angle; which bisect by the Right Line EO.

L is the Vanishing Point of the other Sides of the Squares; in which the Columns, i. e. their Plans are inscribed; and O of the Diagonals.

It would be superfluous, after so many Examples of the same kind given, to shew how the Representations of those Squares are obtained; either by applying their several Distances from A to a, b, c, &c. (as in Ex. 4) or, as the Square *ABFD* is found (Prob. 19) severally, at discretion.

The dotted Lines shew how the last is managed; and it is the best, when there are Circles to be inscribed; as they are represented at *a f*, *c g*, and *e h*.

Which being obtained, severally, by the last Example, draw AK, perpendicular, and equal to the height of the Columns.

Find the corresponding Squares of the Tops of the Cylinders, in which inscribe Circles perspectively†, and draw perpendicular Lines, *a i*, *g k*, *e m*, &c. representing the apparent edges of the Cylinders. If they were to represent Columns, diminishing at the upper ends, the Squares containing the Circles must be represented less, in proportion to their respective Diameters; and the Lines, which are, in this Case, perpendicular, will be gently curved; having obtained the Diameters above and below, a curved Ruler is the best expedient, for drawing the apparent Lines of the sides of the Columns.

† Prob. 3

Now, although this Method of obtaining the apparent Diameters, if it be accurately performed, is strictly true; for, the representations of their Bases and Tops being true Ellipses, there cannot possibly be any error; yet, the performance of it is liable to great error, because it is impossible to describe the Ellipsis accurately; and therefore, the Diameters cannot, by this means, be truely obtained; especially at the bottom, being so near the Vanishing Line; but, much more accurately, by Example 22.

Then, since EC is the Distance of the Picture, produce it to the Intersection at G; make SG equal to EC, and conceive the Picture to stand erect, on its Intersection AF, and S to be the Station Point, at the foot of the Spectator; for, SG the Distance of the Picture is equal to EC, from the Eye to its Center; the space between the Intersection and the Horizontal Line not being a part of the Distance, seeing they are both on the Picture. (See Fig. 37, No. 1.)

It is equally the same, if S be considered as the Eye, and AF the Horizontal Line; and suppose X, Y, and Z, sections of the Columns by the Horizontal Plane.

Right Lines drawn from S, as the Station Point; or being, considered as the Eye, in the Horizontal Plane, are Visual Rays to the extreme edges of the Columns; which, by their Intersections with the Picture, at 1, 2, 3, 4, 5, and 6, give the apparent proportions of the Columns, on the Picture; from which, Perpendiculars, being drawn, give their true places, beyond any other means whatever.

EXAMPLE

EXAMPLE XXV.

How to represent the Tuscan Base, its Plinth parallel to the Picture, and seen obliquely.

C is the Center, nearly as in the last; but the Distance is greater, equal CH: the Horizontal Line is the same; and the Intersection, or Ground Line, is AB. Fig. 98.

At X is a Profile of the Base, consisting of one Torus and Plinth.

Being parallel to the Picture, the Plinth, A a b B, in Front, is geometrical; compleat the square of the Base, perspectively †; and within that another Square (a b c d) equal to the Circle at the bottom of the Torus, in which describe an Ellipsis ‡; the nearer part, e f g, will only be seen. † Prob. 19. ‡ Prob. 3.

Produce the corner of the Plinth, from A; A J may be considered as the Intersection of a Diagonal Plane. Make A J equal to the height of the Fillet above the Torus; and having made a Perspective Plan below (the geometrical proportions, from the Profile, being set off, from A and B, as A *a*, B *b*) from the Angles *A*, *B*, and *D*, draw Perpendiculars *A*I, *B*K, and *D*L.

From J, on the vertical Intersection, draw JH, cuting *A*I at I; also draw IK parallel to the Ground Line, AB, and compleat the Square IKL, of the Fillet; in which, describe the Ellipsis *d e f*, and another parallel to it, perspectively, equal to its width.

Make AG equal to BE; or, through E, the middle of the Torus, draw EG, and compleat the Square EF; in which describe an Ellipsis, as at *g h i k*, which gives the greatest swell of the Torus; and, if greater accuracy be required, describe another Ellipsis, above that; and thus, as many representations of Circles as you please being described, a Curve, *d g* e, on the left side, and *f k* g on the right, falling into the Ellipsis on the Plinth, at e and g, will be the true Contour of the Torus, in Perspective.

Now, although I can hardly suppose that any Person would go through this process, who would delineate a Base; yet is there no better or readier way of doing it, with certainty. Nevertheless, this dissection of it will greatly assist the imagination; and is the best method of acquiring a true knowledge of the nature of curvilinear practical Perspective.

h i k, at the bottom of the Plinth, is an Ellipsis, which is the perspective Seat of the Shaft of the Column; from which may be obtained the edges of the Column, at MN; but it is of no other use, seeing the Curve of the Ellipsis, at MN, is very different from that below, on account of its being nearer to the Vanishing Line; in which, the Curve would be lost totally.

On account of the oblique situation of the Eye, in respect of this Object, it must necessarily appear distorted, when the Eye is opposite to it; but, let the Eye be placed opposite to the Center, at C, and at the Distance CH, the Eye being then in the true Point of view, it will appear perfectly round, and without any distortion.

EXAMPLE XXVI.

How to represent a Doric Capital, inclined to the Picture, casually, or at Discretion.

S is the Center of the Picture, and C is the Vanishing Point of one Side of the Abacus; the other is out of the Picture; the Distance is equal to CH. Fig. 99.

E is the Eye, i. e. the Distance of the Vanishing Point of the other Side (found by Prob. 12) and F is the Vanishing Point of a Diagonal, bisecting the Angle.

Let A be the place of the nearest Angle, in the Picture; and let Z be the geometrical Profile. AB is the Intersection of the Plane of the Top; and AB is the measure of the square of the Abacus.

A*B* being drawn indefinite §, draw BE cuting it at *B*, and draw AC, and *B*C, and a Diagonal AF, cuting *B*C; compleat the Square A*BC*. (Prob. 21.) § Th. 12.

Draw AD the Intersection of the mitre Diagonal, and transfer all the measures from the Profile to AD; from which, draw Lines to F; and having got the Points *c* and *e*, on the Diagonal AF, the Seats of the Projectures of the Facia, Z, and the Fillet at x; obtained as usual, by seting off the measures at *a* and *b*; from which draw Perpendiculars, giving the Points a and e.

Plate XXI. Fig. 99. Compleat the Squares c d and f g, in which describe Ellipses; a b being drawn parallel to AB, may be considered as the Section of the under Face of the Abacus.

From E, or any other Point in the Horizontal Line, draw E c to b; bisect a b, at *d*, and make a 1, b 2, each somewhat more than one seventh part of a b; or with the Radius a *d* describe an Ark of a Circle, and get the Points 1 and 2, as by Prob. 3rd, by which means, the Ellipsis, i. e. the Points *e*, *f*, *g*, *h*, *i*, by which it is described, are obtained.

After the same manner, in the Square e f g of the fillet at x, describe the Ellipsis j, *k*, *l*, and another parallel to it, equal to the width of the fillet; and also, another above that, for the thickness of the small Bead, at x; which, being so little distant from each other, will not vary much in their Curves; otherwise, every different representation of a Circle must be severally described, as the foregoing.

As it is somewhat more difficult to get the necessary Points, when the Square is inclined than parallel; let the remainder of the Curves be thus obtained.

Through y draw a Line parallel to AB; and (because this Object is viewed centrally) where it cuts the Vertical Line, at i, set off i f and i k equal; making f k equal to the Diameter of the fillet, at y, geometrical; draw f S and k S.

From the Point j, where f d cuts AD, draw j F, cuting i S at o, the representation of the Center of that Circle; and, through o, draw L o, cuting f S and k S at h and m; through which, draw g h and l m parallel to f k; g h l m is the Square of that Circle, in which an Ellipsis must be described, as before; with the Fillet and Astragal, as was done above; particular care being had, to make them lead true, at the Extremes, as if they were continued around.

The Square of the section of the Shaft (p q) is obtained in the same manner; and, an Ellipsis being described in it, from the Extremes of which, perpendicular Lines are drawn, for the Shaft or apparent edges of the Column. How they fall into the Cavetto is impossible to describe, seeing the Line is lost insensibly.

The large Ovolo can only be described, truely, as the Torus, by means of various representations of Circles, and describing, carefully, a Curve over the Extremes of them all; for it must be obvious, that the Contour of such Mouldings, though circular, is not in a Plane, but is continually changing its place, from the upper Circle to the lower.

Let it be observed, that the Curves are the very same, whether the Squares, in which they are described, be parallel to the Picture or oblique, in any inclination whatever.

EXAMPLE XXVII.

How to represent a large Crane, or Water Wheel, vertical.

Fig. 100. Let ABCD be the geometrical Figure, and proportion of the Wheel; and S, its Center or Axle. Its Position is accidental, as the vertical Line, l m, passing thro' the Center, indicates.

Let C be the Center of the Picture, and CE its Distance, in the Horizontal Line.

Let *AB* be the Intersection, of the Plane of the Wheel, with the Picture; which, the Wheel is supposed close to; and let *BD* be the Ground Line.

Transfer the measure of the Wheel to the Intersection, *AB*, at *FG*.

Draw CV perpendicular to the Ground Line, which is the Vanishing Line of the Face of the Wheel. Make C*E* equal to CE; draw *F*C and *G*C; and *FE*, cuting *G*C at *H*, and draw *HI*, parallel to AB.

FGHI represents a Square, containing the Wheel; in which, describe the Ellipsis, a b c d; and within it another, representing the Rim of the Wheel.

Make *BK* equal to its determined width, and draw *KL* parallel to *AB*, by which describe the Curve *OPQ*, representing the cylindrical Surface of its convexity; and there may be two other Curves described, equal in width, representing the breadth of the Rims, and another, shewing their thickness.

2 The

Plate XXI.

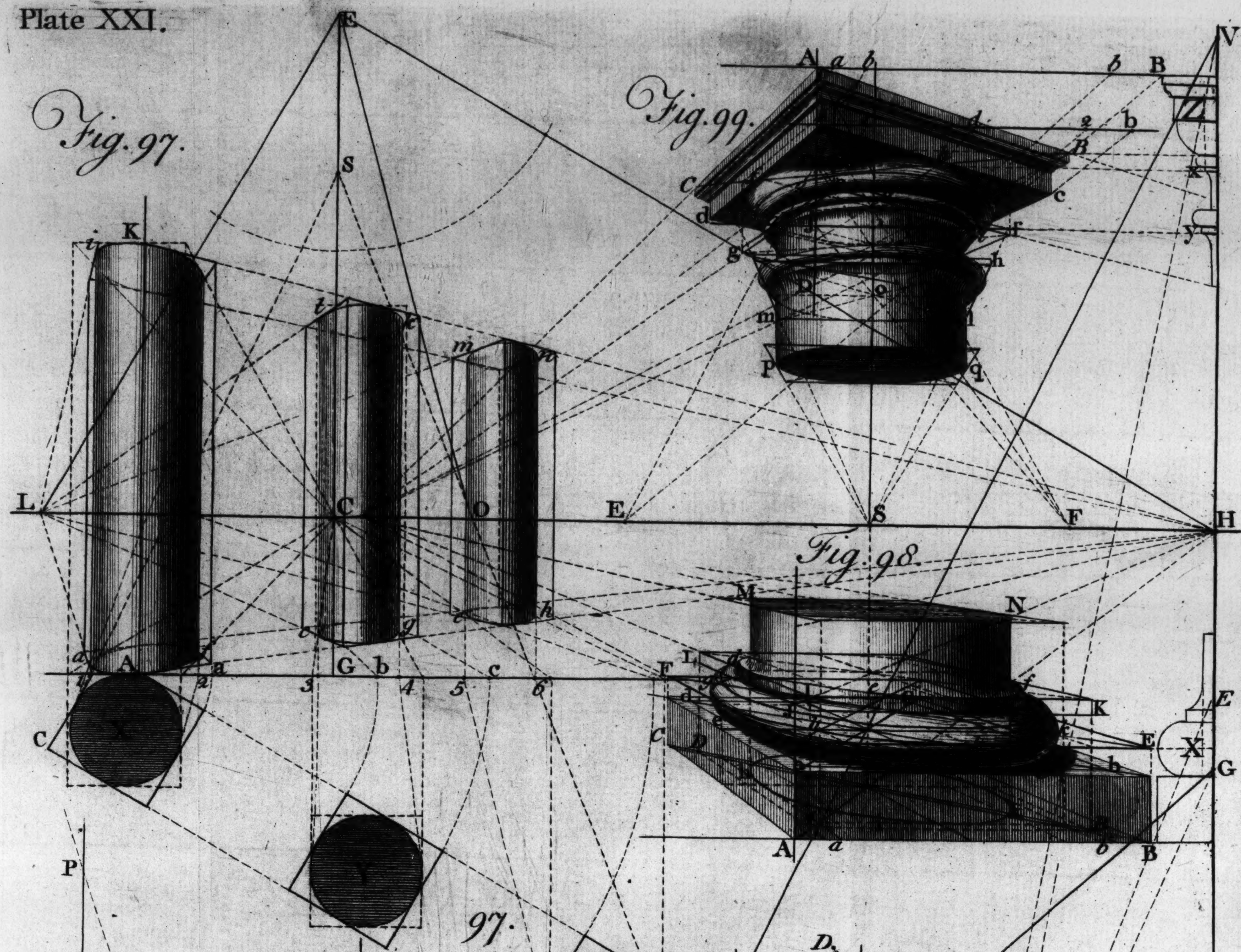


Fig. 97.
Fig. 99.
Fig. 98.
97.
Fig. 100. No. 1.
100.
Publish'd by T. Malton Feb 6. 1775.

The Levers* a, b, c, d, &c. may be thus obtained.

Make *AG* and *BF* each equal to their width; and draw *A*C and *BC*; produce *EF*, cuting C*B*, at *C*, and draw *CD* parallel to *AB* cuting C*A*.

On *CD* describe, perspectively, the exterior Square, *CDHI*, in which describe, an Ellipsis, by a Pencil Line; which, with the interior Curves, represent concentric Circles (as ABCD, abcd, No. 1) this last will limit all the Extremes of the Levers. Their places are found, by drawing Perpendiculars, from each, to the Ground Line; as ip, nq, &c.

Draw a Perpendicular from the Rim, at J (No. 1) cuting the Ground Line at *B*; from which transfer all the measures *B*p, *B*q, &c. to *Bp*, *Bq*, *Br*, &c. from which, draw Lines to E, cuting *B*C at 1, 2, 3, &c. from which, draw Perpendiculars cuting the outer Ellipsis at *b*, *f*, &c. as in the Figure, where the corresponding Characters particularize the several Levers.

Any one being obtained, a Line drawn through S, which represents the Center of the Wheel, gives its opposite, as *bSd*.

By the same Method the cross Timbers, &c. may be obtained.

This Method, though the same as all the old Authors practised, is not repugnant to the new Principles, by Brook Taylor; and may do, accurately enough, in common Cases, and where room is wanting; but, as a contrast, I will also shew, how much more simple, masterly, and correct, they may be obtained, by means of their respective Vanishing Points; and, when they are not very remote, Intersecting Points.

AB was considered as the Intersection of one Plane of the Wheel, with the Picture. A Right Line, CV, drawn through C, the Center of the Picture, parallel to *AB*, is the Vanishing Line of them both, and C is its Center. (See Th. 5 & 6.)

The Horizontal Line is of no use, in this operation; no regard being had to any other Vanishing Line or Intersection, whatever, but that of the Plane of the Figure.

CE, perpendicular to CV, the Distance of the Picture, is also the Distance of this Vanishing Line, which, is the Vertical Line of the Picture; and E is the true Place of the Eye, for that Plane.

Find the Vanishing Points, D and V, of the Inclination of the Timbers, parallel to AC, and BD (No. 1) i. e. make the Angles CED, CEV, respectively equal to the known or determined Inclination of AC, and BD, with the Horizon †; cuting the Vanishing Line at D and V, the Vanishing Points of those parallel Timbers.

† Prob. 2. Sect. 3.

Or, having found either, make DEV a Right Angle, it will give the other; because the Timbers are at right angles with each other. (Prob. 4, Sect. 3.)

Bisect the Angle DEV by the Right Line EF; F is the Vanishing Point of the other Timbers, E, F, &c. (No. 1.) which make half Right Angles with these. In the same manner, i. e. by bisecting the Angles all the other Vanishing Points, G and H, &c. of the Levers are acquired; which, seeing they tend to the Center of the Wheel, may be drawn without more preparation; as GS and HS, &c. which gives the Levers, at *i* and *k*, &c and FS gives the Timbers at EG.

The Timbers at I are parallel to them, and have the same Vanishing Point; the others, at O, and F, H, are at right angles with them. Make FEY a Right Angle, and they will tend to that Point, where EY would cut the Vanishing Line.

The cross Timbers, do not pass through the Center of the wheel, but may be thus obtained; with the greatest accuracy.

Draw KL, at the farther extreme of the Wheel (No. 1) and produce the Timbers, &c. till they cut it, at M and N; also the Lever, at w, cuting KL at K. KL may be considered at the Intersection of the Picture, coinciding with *AB*, or *KL*.

Transfer the measures KM, MN, &c. to *AB* and *KL*, at v, x, y, & z, & K to *A*.

Draw vD and xD, giving the representations of the Timbers, parallel to AC; whose thickness are obtained at v1, x2. yD and zD give those on the other side of the Wheel, as they are seen between the hither Timbers.

From *A* draw a Line through the Center S, which gives the two Vanes, at n and o, whose Vanishing Point is not in the Picture, nor necessary.

The

* They are, by some, called Ladles; but I think that Term very improper, unless they were intended to scoop the Water away.

Plate XXI. Fig. 100. The Intersecting Points of the other cross Timbers are below the Intersection, *BD*; which, if they were remote could not be acquired.

Therefore, draw *r* F and *s* F cuting the inner Curve of the Rim at *t* and *u*; and, through them, draw V*t*, and V*u*, which gives those Timbers.

Or, describe a Square, perspectively, at the Center S, equal to the width of the other two, and use the Vanishing Point V, as before.

The Levers, it is obvious, are all between the two Ellipses, as their Originals are between the Circles, at No. 1.

The Points a, i, e, &c. being obtained, draw a a, i i, &c. parallel; for the Front is, supposed, at right angles with the Picture.

The parts where they are hid, by the Rim of the Wheel, need not be drawn; which, with the Supporters, or Stays behind them, are best described by the Figure; to particularize every minutia, would take above another Page to little purpose. The Square of the Axle being got, at S, the Sides are parallel; and the Timbers at I and O form a Square, whose Vanishing Points are F and Y.

Or, those at I may be produced to the Intersection KL cuting it at J and L; and one of the other, at P. The rest is obvious, on inspection.

E X A M P L E XXVIII.

How to represent circular Steps, in Perspective.

Pl. XXII. Fig. 101. Let CE be the Horizontal Vanishing Line; C the Center, and CE the Distance. Let AB be the Intersection of the Plane of the first Step; and let AB be the measure of that Step.

Describe a Square ABDF (Prob. 19th) in which inscribe the Ellipsis a b c d e f g h. Make c G equal to the height of a Step, at at X (No. 1) and by drawing Perpendiculars, b 1, c 2, &c. and, from *A* and *B*, Diagonals cuting them, the lower Curve may be described.

Draw GH, or AI perpendicular; and on them, from A or c, set off all the measures of as many Steps as are required, viz. G c, c i, or *A*A A*C*, equal (as in the Profile, for two Steps) and from them draw to C, or E, as in the Figure.

Find the Square of the next Step JKLM, in which describe the Ellipsis *abcd*, &c. as before; and the lower Curve, *g h i k l*, may be obtained, likewise, in the same manner. And thus, as many Steps may be represented as are required.

AB is not necessarily the Intersection of the Picture, but parallel to the Intersection; and AB the known or determined measure of the first Step, in that Place.

E X A M P L E XXIX.

How to represent a circular Pedestal, on the Steps.

Fig. 101. At No. 1 is the Profile of the Pedestal; from which transfer all the measures of the Plinth, Mouldings, &c. to the Vertical Intersection, GH or AI, or both; the one considered as the Section of a vertical Plane perpendicular to the Picture; thro' the Axis of the Pedestal, and the other a diagonal Section.

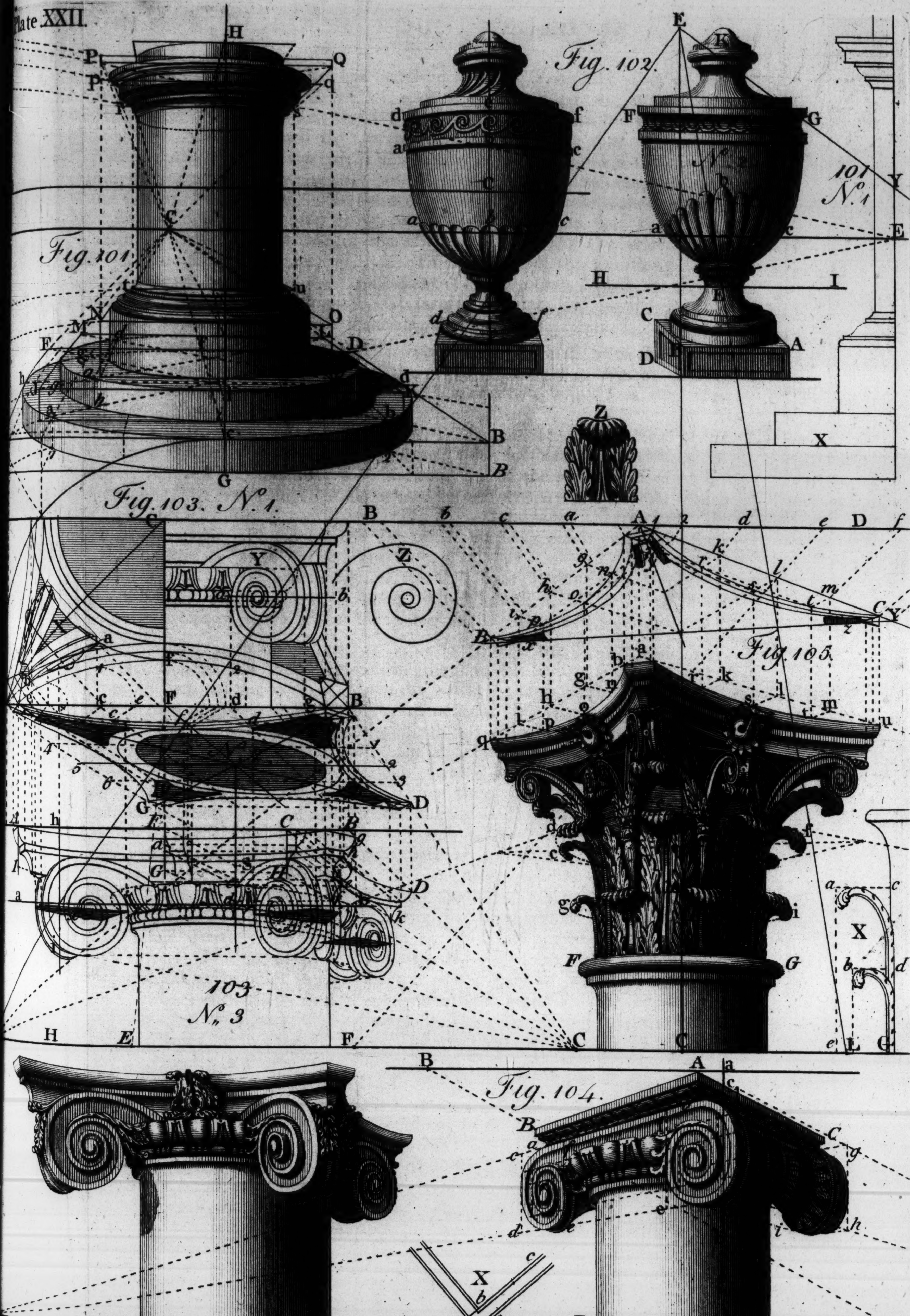
Make *CD* equal to the Plinth of the Base; *C* I the height of the Cornice, and at 1, 2, 3, &c. set off the proportion of the Mouldings.

The square of the Plinth being obtained, at NO, and an equal one at PQ, in both which describe Ellipses; as, for the Steps below.

The other Curves must all be obtained after the same manner, by inscribing Ellipses in Squares; as p q of the Facia, r s of the lower line of the Cornice, and t u, of the upper Line of the Base Moulding.

For, let it be observed, that, in curved Objects, if the Plane of the Curve falls near the Vanishing Line, consequently, the Curve will be more excentric and flatter; yet there is no possibility of describing them in another Plane either above or below, as in Right-lined Objects, so as to be of any service, in respect of the Curve, seeing

Plate XXII.



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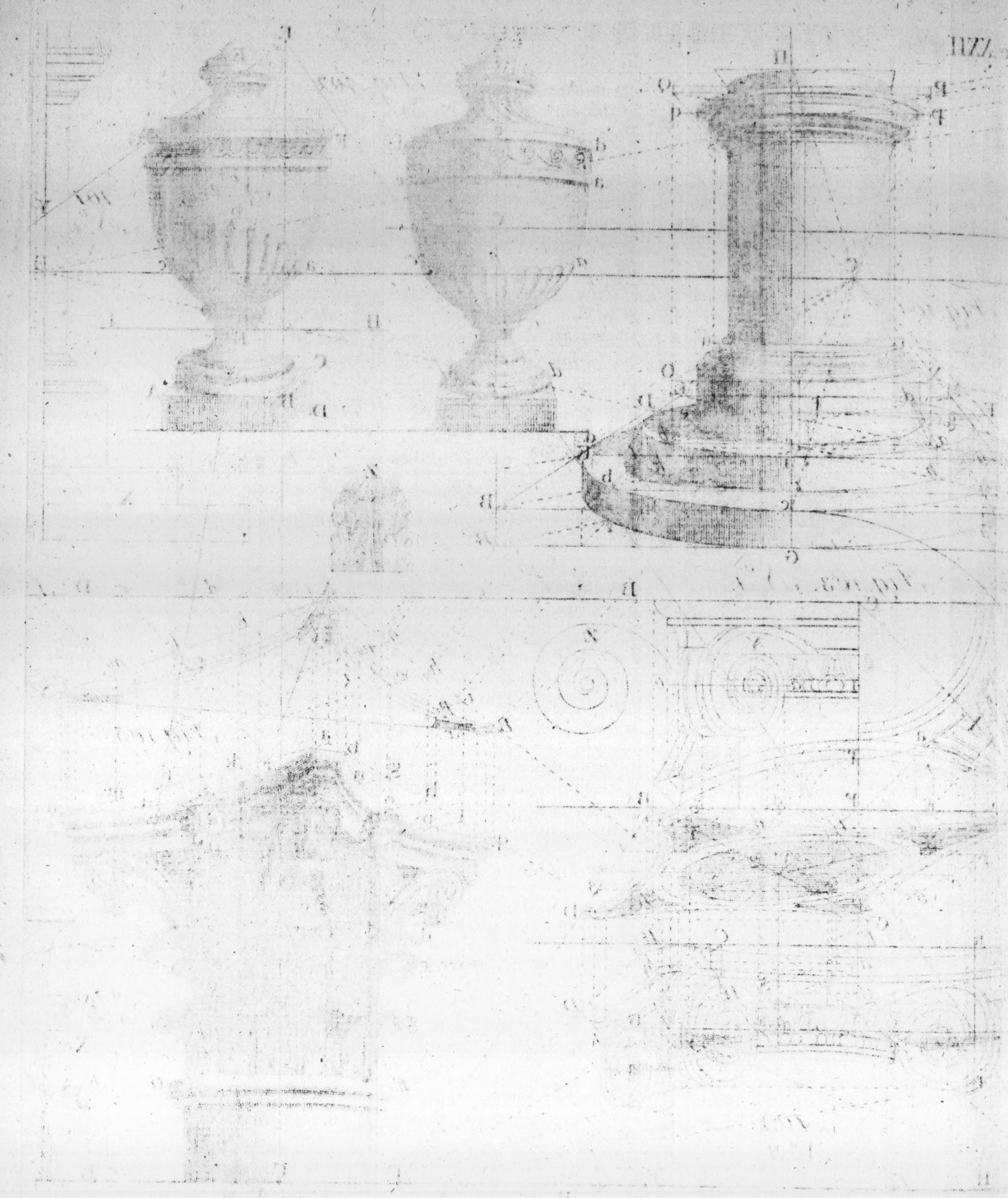


ſeeing that every Curve, the farther it is removed from the Vaniſhing Line of the Plane it is in, becomes more curved, i. e. it is more convex; or, as workmen phraſe it, quicker; which implies, that it falls off from a Tangent, more ſuddenly.

Let it alſo be obſerved, that if the Curves, that is, the Planes they are in, be equally diſtant above and below the Eye; being equal and ſimilar Curves, of any kind, and perpendicularly one over the other, their Repreſentations will alſo be equal and ſimilar, but inverted. For, let it be particularly remarked, that, where-ever the Eye is ſituated, in reſpect of a convex Object, as a Vaſe, &c. (Fig. 102) the Vaniſhing Line of the Plane of the Circles, CE, cuting the Object, the Curves a b c, d e f, &c. or *a b c, d e f,* &c. on either ſide of the Vaniſhing Line, are always concave towards it; and, conſequently if a concave Object be repreſented (as the Abacus, of the Ionic or Corinthian Capital, Ex. 30 and 32) the Curves will be convex, towards the Vaniſhing Line. Fig. 102.

How is it poſſible then, for any Perſon, who has any pretenſions to the Art of delineating, to run into ſuch groſs abſurdities as are exhibited in No. 2. I have ſeen a Vaſe ſo repreſented, on the Pannel of a Coach Door, which was very well painted, and, in other reſpects paſſable.

The Plinth, AB, was parallel, and the Side, BCD, was ſeen; but, inſtead of the Lines CB and DB tending to a Vaniſhing Point, they inclined to each other towards the Eye; and conſequently, gave the top, ABC, the appearance of an inclined Plane. The Mouldings, at E, where curved contrary to the Baſe, which plainly indicates the Horizontal Line, HI, to paſs between them; and, the Curves, being regular, alſo evinces the Eye to be at E, and conſequently the Side, BCD, of the Plinth, could not be ſeen. No. 2.

The Lines at FG, the greateſt ſwell of the Vaſe, were Right Lines, which alſo evinces the Eye to be on a level with them; whilſt the nulled, or fluted part, a b c, was a parabolic Curve, riſing as in the Figure; and the Top, at K, gently curved.

Is it not ſurprizing, that any Artiſt could be ſo regardleſs of his Reputation, as to ſuffer ſuch a medley of inconſiſtencies to go out of his hands; and to palliate it by ſaying, in ſuch things, it is not worth the while to ſtand conſidering about it. I grant it would not be worth the while, to delineate the whole by the Rules of Perſpective; but ſurely, the Lines in the Plinth are as eaſily made to incline the right way, as wrong; and if, by curving the horizontal Lines of the Vaſe, it was intended to give an appearance of roundneſs; would not that be better effected by curving them properly? it would take no more time; and, if the Delineator was not inclined to think much about it, I would adviſe all ſuch, to place an Object before them, and their Eye in the Point of View, as they intend the Object to appear; they will then aſſuredly ſee, how unpardonable ſuch miſrepreſentations are, becauſe they are as readily, without loſs of time, done right as wrong.

I have alſo ſeen full as groſs abſurdities on the ſtage; which ought to be regulated by the ſtricteſt Rules. In Theatres, Perſpective might be diſplayed to great advantage; and certainly, its Rules ought not to be diſpenſed with, on any account whatever. I do not mean in every minutia, but in the whole Deſign; for, if liberties be taken, in order to appear better in one Point of View, it will appear worſe in another; particularly in Scenes repreſenting a grand Palace, &c. or other regular piece of Architecture, there is no poſſibility of departing from the Rules; in detached Planes, as in Scenery, they could not form one entire Building, without adhering ſtrictly to Perſpective.

To the beſt of my remembrance, in that Maſter-piece, by Servandoni, in the Pandemonium (though I think, that the groteſque Stile, of that Deſign, is not altogether conſonant to the grand deſcription of it, in Milton's Paradiſe loſt) the Eye, in that Scene, is on a level with the Baſes; and conſequently, the Ruſtics appear curved. It is a long time ſince I ſaw it; but I believe, the firſt, from the Baſe, is as much curved as the upper one. The whole has a grand and ſtriking effect, chiefly occaſioned by the magnificent diſplay of Lights and Colours; and which, being left out, I am perſuaded that this Deſign would loſe much of the applauſe it has obtained.

Pl. XXII. Fig. 103.

EXAMPLE XXX.

How to represent an Ionic Capital, perspectively, having one Face parallel to the Picture.

This Example, and more particularly the next, of the Corinthian Capital, is a matter of real difficulty. To represent such Objects as the Volutes (and other decorative parts, of which the latter is composed) will indeed baffle all Rules. I shall nevertheless attempt to lay down some, which will give their true places, with certainty; and, if the Practitioner will carefully observe how such Objects appear, according to the position in which they are seen, and intended to be delineated, he will not be much at a loss; if he has any hand at Ornaments, Foliage, &c. and carefully adheres to the Rules I shall prescribe.

At Fig. 103, No. 1, is the true geometrical Plan and Elevation of the Ionic Capital; not according to the Antique which is not difficult, seeing that the Volutes or Scrolls are in one Plane. On the left (at X) is the Plan of the Volute, shewing its position in respect of the Abacus and Ovolo, &c. On the right (at Y) is the Elevation; which appears elliptical, on account of its Position, which is known from the Plan.

If a b be the Line of its direction (which is somewhat curved) to the front, AB; it is obvious, that it makes a considerable Angle with it; to which Perpendiculars being drawn, give e c, its apparent width in Front; which measures being transposed to the other Side, at d and g, and perpendiculars drawn, give its apparent geometrical Front; of which Z is the true.

Having thus described the geometrical Plan and Elevation, I now proceed to shew how it may be delineated, perspectively.

Let AB be considered as the Intersection of the Picture, with the Plane of the top of the Abacus. Let HL be the Horizontal vanishing Line, and C the Center of the Picture. The Distance is CE; which, seeing it is greater than can be applied on the Horizontal Line, take C*E* half CE, and describe the Square ABDG (Prob. 19) which contains the whole Abacus.

The Curve e f g, which is the sixth part of the Circumference of a Circle, may be easily obtained; as thus.

From c, F, and d, any Points at pleasure, in AB, draw Perpendiculars, c 1, d 2, &c. to the Curve; and draw c C, FC, and d C, which represent those Lines, indefinite. Make c *c*, F *f*, &c. represent the finite Parts; through the Points e, *c*, *f*, *d*, and g, describe a Curve, which will be a portion of an Ellipsis, and represents the Curve e f g.

The returning Side, BD, is obtained by means of Ordinates, after the same manner, which are parallel on that Side. The interior Curves may be projected by the same means, and the Plans *X*, *Y*, and *Z* of the Scrolls, as any other geometrical Figure (in Sect. 5) from which, their Representations are projected, as below; the returning Angles at A and B, being perpendicular and parallel to the Diagonal AC, tend to the Distance Points.

This preparation is sufficient for an extra Plan; I shall next shew how far it is useful in finishing the Representation below. This Plan may be supposed to be the Top of the Abacus, as a Plane simply; but I shall suppose it to be above, as in Example 12th, aud that the Representation itself is below it.

Let *AB*, No. 3, be now considered as the Intersection of the Top, instead of AB; to which Line transfer all the measures from AB; and form the out Line of the Plan *ABD*, as before; by means of the same Vanishing Points.

To compleat the Abacus; bisect *AB*, at *F*, and draw a Perpendicnlar *FG*.

From *F*, set off all the measures of the heights of the Mouldings, and draw Lines from them to C; and suppose *FG* a vertical Section of a Plane through the middle of the Capital.

The Center, S, being obtained, draw a Perpendicular, from S, cuting those Lines; through which, draw the parallel Lines *c e*, *d f*, &c. till they cut Perpendiculars from the corresponding Mouldings, in the Plan above. Or, for greater

accuracy, make *BC* equal to F f, the receding of the Curve from the Picture; where, make a true Profile of the Moulding, of the Abacus; from which, draw Lines to the Center of the Picture cuting *c e*, *d f*, &c. from the Axis, S *d*, at *e* and *f*. Having obtained the returning Moulding, at the Corner *B*, by means of the perspective Plan, above, and a perspective Section at *F G* (observing, that it is a mitre Angle touching the Picture, and consequently, recedes more than the true Moulding, as may be seen by the geometrical Plan, No. 1, at A or B) from which, Lines tending to the Point of Distance, on the right, cuting Perpendiculars from the Plan, give the other Mitre, *g i*.

From *g*, *h*, and *i*, draw Lines to the Center, C, cuting Perpendiculars from the Angle D, in the perspective Plan; by which means, the several Angles at *D* are obtained; and the curve Lines may be drawn by a careful hand, as in the Figure, nearly parallel amongst themselves; otherwise, if great accuracy be required, each Curve must be got, severally, in different Planes.

The Abacus being compleated, proceed to the Scrolls or Volutes.

Having carefully obtained the perspective Plans *(X, Y, and Z)* of the Scrolls, which are regular Trapezia (as at X, No. 1) not regarding the hollowing.

In this Diagram, it is too near the Vanishing Line; but may, for greater exactness, be at a greater Distance from it (as in Ex. 13.)

Draw a b parallel to *AB*, the Intersection of a Plane passing through the middle of the Scrolls (as at *a b*, No. 1) its Distance from *AB*, is equal to the distance of *a b* from the top of the Capital, geometrical.

By means of the Intersection, a b, the representations of those Trapezia, *x*, *y*, and *z*, are easily obtained; transfering the geometrical measures to this Intersection, first; Perpendiculars, from No. 2, will cut Lines tending to C, the Center of the Picture, and give all their Angles, sufficiently correct; but, if greater accuracy be required, their Vanishing and Intersecting Points may be had, as in other Figures, which are in the Horizontal Line; but, on account of their Inclination to the Picture, they are beyond its limits.

Now, the great use of these Figures is very obvious; by which means, the several apparent Faces, with their several Divisions, are determined, passing through the Eye of the Volute, &c. But, if an extra Plan be drawn, as above, Perpendiculars from *X*, *Y*, and *Z*, give the same, in respect of their widths.

Hence may be seen the unavoidable distortion of the Volute on the left; which, on account of its oblique Position to the Picture, and the Eye, is dragged out beyond its geometrical proportion; and consequently, cannot please the Eye but in the true Point of View. It is true the Distance is too short; nevertheless, every oblique parallel Representation of such Objects, must be more or less distorted, according to the Distance, and oblique situation of the Eye.

Having got their Dimensions, of height, by means of a Perpendicular, h i, from the middle of their geometrical Plan (X, No. 1) and the place where they fall on the Ovolo (which, must be drawn as in the Doric Capital, without its Abacus, Ex. 26) the rest must be delineated by a steady hand, guided by a nice Eye; for, without great nicety in both, it is not possible, by the Rules of Perspective, to project such Objects; seeing that, the Curve is not only continually varying from the Center to its utmost extent, but it is also continually growing forward, from the utmost extreme to its Center, and is not described on the same Plane, or other Surface; which circumstance renders it utterly impossible to be described perspectively, by Rule; nevertheless, by the Rules prescribed and judgment in drawing, it may be projected so as to please the most critical Eye.

Below this outline is a finished Capital, divested of the necessary Lines for projecting it. It is somewhat further removed from the Horizontal vanishing Line, but, in other respects, it is the same; excepting the decorative parts, of foliage, &c. which, according to the position in which the Capital is viewed, will differ in Figure; they were omitted, above, for the sake of continuing the Lines of the Abacus, which they hide, in this. No Rule can possibly be of any use in delineating such Ornaments; a real Object, before the Eye, is the best and only means to effect it.

EXAMPLE

Pl. XXII. Fig. 104.

E X A M P L E XXXI.

Is an Ionic Capital, according to the Antients, which is inclined to the Picture.

This Capital is delineated with much more eaſe than the other, for ſeveral reaſons; becauſe its Abacus is a Square, with only a Cima reverſa; and, the Volutes are both in one Plane, which is vertical.

The Abacus, is ſuppoſed to cut the Picture, at A, through which draw AB, parallel to the Horizon; V is the Vaniſhing Point of the front Lines, the other is out of the Picture.

Make AB equal to a Side of the ſquare of the Abacus, and project the Square ABC, perſpectively (Prob. 21) and finiſh the Abacus, as in Ex. 13. of the Pedeſtal.

The hither Scroll projects through the Picture; becauſe the corner of the Abacus, at A, cuts the Picture, and the Scroll projects beyond it, in this Capital.

Draw a b perpendicular, the Interſection of the Plane of the Volutes; which may be thus determined.

At X is an Angle of the Abacus, geometrical, being inclined to *AB* as the Object is ſuppoſed inclined to the Picture.

The whole Moulding projects before the Plane of the Scrolls; therefore, produce *cb* to *a*, cuting *AB*. Make A a equal *A a*, draw a b perpendicular, and produce *ab*, cuting a b at c; all that part of the Scroll, on the right hand of a b, projects through the Picture.

Make c d equal to the height of the Scrolls, or a d of the Capital, and draw d V.

Deſcribe a Rectangle, incloſing the Scroll, at each extreme; as c d e f, *c d e f*; and, within theſe, others may be deſcribed, as in the Figure; by means of which, the Scrolls may be very accurately delineated, perſpectively.

The other Front, or rather the End of this Capital, is different, and is not eaſily delineated, truely. Having obtained the outline of the oppoſite Scroll; at *g b i*, biſect the returning Line of the Abacus, at *k*; where, deſcribe the repreſentation of a Semicircle, *k l*, in a Plane parallel to the Front, but different in dimenſions to the Scrolls. The proportion, and figure of it, may be had in Books of Architecture.

This End repreſents a kind of Balluſter, ſpreading at its extremes, to the width of the Scroll; but the Axis of it does not paſs through their Centers.

The Ovolo is the ſame in this as in the Modern Capital; ſave only, that it is not ſeen at the Ends, at all, being wholly hid by the Balluſters.

E X A M P L E XXXII.

To repreſent a Corinthian Capital inclined to the Picture.

Fig. 105. This Object is ſuppoſed to be direct before the Eye; and conſequently, the Axis of the Capital paſſes through the Center of the Picture, at C.

CE is the Diſtance, VL the Horizontal Line, and V the Vaniſhing Point of one Side of the Capital, or rather of its Poſition; for, the Object not being right lined, ſave the ſhort returning Angles, it has indeed no Vaniſhing Point; yet they are neceſſary, in the Delineation.

Let a be the Point in which a Square, incloſing the Abacus, touches the Picture; through which, the Interſection of the Top may be drawn; but it will be more eligible to take another, above, at BD, parallel to the Horizon.

The Abacus of this Capital is the ſame, in Proportion, and every other reſpect, as the modern Ionic; therefore, the meaſures may be taken from Fig. 103, No. 1, being delineated by the ſame Scale; and excepting its ſuppoſed Poſition, being different, the delineation of it would be the ſame.

Take A perpendicularly over a, and draw AV; and AY to the Point in which EY would cut the Horizontal Line (Prob. 13) indefinite.

Biſect the Angle VEY, by the Line EL, L is the Vaniſhing Point of a Diagonal. Find

Find F, the Distance of the Vanishing Point Y (Prob. 12) and, because the Distance of V cannot be in the Picture, take VG three fourths of VE (or any other Part, at discretion.)

Take AB three fourths of AB, No. 1, which bisect at *c*, and take all the other measures, at *a*, *b*, &c. on that Side, each three fourths of the real measures on AB (No. 1) but, on the other, take *d*, *e*, *f*, &c. the full measures.

From all which Points, draw Lines to F and G, respectively, as in the Figure; cuting AV, and AY, at *g*, *h*, *i*, &c. from which, draw Lines to both Vanishing Points, V and Y, indefinite.

Take A 1, and A 2, equal F f, &c. (No. 1) and draw 1 F, 2 F, cuting AY; from which Points draw Lines to V, cuting the Lines from *g*, *h*, &c. to Y, at *n*, *o*, *p*; and from *j*, where 1 V, 2 V, cut the Diagonal AL, draw Lines to Y, cuting *k* V, &c. at *r*, *s*, *t*; through which Points, on both Sides, draw the Curves of the Abacus.

At the Corner A, describe an Equilateral Triangle (x) perspectively, and the same at each Corner, *B* and *C*; which are the mitre Angles of the Moulding, as at A and B. (No. 1.)

This operation done, draw a V, and a *Y* indefinite, to which draw Perpendiculars from *g*, *h*, *i*, *k*, &c. also, from *r*, *s*, *t*, &c. and draw k V, &c. cuting them at r, s, t; through which Points, the Curve b n o p q may be described.

The corresponding Letters, on the other Side, shew the Operation the same.

N. B. The Operation above would have done here, if the other be not absolutely necessary; on account of its being near the Vanishing Line.

After the same manner, the Abacus may be compleated; i. e. by projecting the Curves of the Fillet, in the middle; and also, the bottom Line, in their respective Planes.

The places of the Leaves are obtained by describing an Ellipsis in a Square, inclined to the Picture, as the Abacus.

This may also be done above, at v, u, x, y, z. The Diagonals and the two Diameters, whose Originals are perpendicular to the Fronts, give the middle of each Leaf, in the first Row; and they also give the widths of the upper Leaves, between each, allowing a little space between them.

The Heads of the Leaves may also be obtained, by describing the representation of a larger Circle, concentric with the other.

Their Profiles and projectures, are represented at X: with the Front of one Leaf, over the Plan, at Z.

Describe two Ellipses, c e f and g h i, giving the greatest projecture of the Leaves, in their true places; their heights above the Astragal, *FG*, are taken, from the Profiles, at X, to the greatest projecture of the Heads at *a* and *b*; which projectures, *a c*, and *b d*, respectively, are added to the Diameter of the Column at the Astragal, for the Diameters of those Circles, represented by the Ellipses c f and g i.

Those Ellipses being obtained, in their true places, the representative Diameters of the upper one, give the true middle of each head of a Leaf; the lower Row falls between them; as may be seen in the Plan above.

The Volutes, at the Corners of the Abacus, are, except in dimensions, the same as the Ionic, and must be obtained the same, by perspective Plans, at *x*, *y*, *z*.

How they fall into the Caulicola, with the other decorative parts of the Capital, the Figure only can describe; for, I fairly own that it is not in words to describe it, so as to be of any Service in delineating. Being well acquainted with the several parts of the Capital, the Rules I have given are sufficient; the rest is best described by a careful perusal of the Figure.

Those who are not versed in it, would do well to draw from a real Capital, first, in all positions; after which, they will be enabled to delineate it by Rule.

Pl. XXIII.
Fig. 106.

SECTION IX.

Shews the application of the whole, to entire Buildings and regular pieces of Architecture.

IN this Section, I shall apply the whole of what has been already done to complex Objects, and particularly to Architecture; as being, of all other, the fitest Subject, and gives the greatest Lustre to Perspective; on account of the regular disposition, and arrangement of the several parts of a Building, and being composed, chiefly, of plane Surfaces, which generate Right Lines.

It is, to me, surprizing, that Artists, in general, have no better notions of Perspective; if they would give themselves time to think about it, seriously, they could not commit such palpable mistakes and gross absurdities; even suppose they were not acquainted with the Rules, having a true Idea of the meaning and intent of Perspective; which cannot but be obtained, from the Apparatus.

Whatever we are about to delineate, it is supposed that the Object, itself, is on the other Side of the Picture, generally (it may be on this Side.) Is it not then rational to consider, and thence to conclude, that the Picture we are delineating should be directly between the Eye and the Object; or what reason can possibly be assigned that it should not? and yet, it is, almost generally the Case. Some, not considering it properly, would tell us, that it is in order to see the End or Side of an Object, as well as the Front; as if that could not be effected otherwise; true, it cannot, on their Hypothesis, that one Side must necessarily be parallel to the Picture.

I shall illustrate the whole of this matter in few words.

Figure 106 exhibits the Plan and Elevation of a plain Building geometrical, which is to be delineated; nearly the same as the Object, BFIL, in the Apparatus; in the same Position, and at the same Distance, nearly, by a Scale of one to two, or half the Proportion.

In the first place, I shall shew how to determine the true Position of the Picture, the Station being previously determined.

Let S be the Station fixed on, from which it is intended to delineate the Building; X is the geometrical Elevation of the Front, Z is the Plan, or Seat of the Building on the Ground; in the Position, and at the Distance, the Building is to be represented, as seen from the Station S.

The Station, and consequently the Point of View being determined, it is evident, that the Building cannot vary in its appearance at that Station; but, it is nevertheless manifest, that there may be a great variety of Pictures, or Representations of the Object; seeing that, every different Position of the Picture, or Section of the Pyramid of Rays (forming a solid Angle, under which the Object is supposed to be seen) will produce a different Picture. All which, Representations, will affect the Eye alike, in the true Point of View; nor is it possible it should be otherwise, seeing that, when the Eye is in the true Point of View, every Point or Line, on the Picture, is seen under the same plane Angle, and consequently, the whole Representation under the same solid Angle, as the Original.

Hence, it is easy to account for the many distorted and preposterous Representations which are frequently to be seen; and which, is mostly owing to the absurd Position of the Picture, in respect of the Object and of the Eye; or frequently to the Distance of the Object, itself; a Circumstance which ought to be particularly considered; for, if it be too near, it is impossible, in such Case, to produce an agreeable Picture.

† 1. 8. El. Then, since every different Section of the Pyramid of Rays produces a different Picture (for all parallel Sections produce similar Representations †) certainly, that Section which is perpendicular to the Axe of the Eye, or the Station Line, SL, will be the most natural and agreeable Representation; seeing that, the parts of the Representation will differ but very little (if the Optic Angle ASC be not large) from the true Appearance, i. e. from a Section made by the Surface of a Sphere, of which S is the Center.

Consequently,

Consequently, then, if the Station be fixed, from which it is determined to delineate an Object, the Position of the Picture is also determined, as follows.

ABC is the place of the Object, its Seat on the Ground Plane; and S is the Station determined on. Draw the Right Lines AS and CS, which determine the Optic Angle of the Object, ASC; for, S may as well be the Eye, and AS, CS Visual Rays, the Eye being perpendicularly over S.

Bisect the Angle ASC, by the Right Line SL; SL is the Station Line for that Object; to which, the Picture must (if placed properly) be perpendicular; and all such Sections are similar Representations.

VY, perpendicular to SL, is the true Position for a Picture of that Object, delineated from the Station S; and if Right Lines are drawn, from S, to the several parts of the Plan, as a, b, c, &c. their Sections with the Picture, *a*, *b*, *c*, &c. determine the apparent widths of each Part on the Picture, perhaps with greater accuracy than by any other means.

Here the Picture is supposed close to the nearest Angle of the Object, at B; and consequently, the perspective Section of the Rays, by VY, is the largest that can be made on this Side the Object. Now if an Ark, *ABC*, be drawn on the Center S (Radius SB*) it is obvious that the Section of the Rays by that Ark (or a Sphere of that Radius) differs but little from the Section by the Plane, on VY; because the Optic Angle, ASC, is but small, and therefore the extreme Rays, AS and CS, cut the Plane not very oblique; which, being increased, occasions Distortion.

SV parallel to AB, and SY to BC, determine the Vanishing Points of those Sides of the Building. 1, 2, 3, &c. may be considered as Intersecting Points of several Lines in the Object.

Having thus determined the true Position of the Picture from the Station S, answering to the Picture, MNOP, in the Apparatus; which is directly between the Eye and the Object; I shall, next, shew the great absurdity of placing it otherwise.

Let every thing remain in the same Position; S is the determined Station for the same Object; and consequently it must appear the same, whatever Position the Picture is supposed to have, in respect of the Object. But let it be observed, that the Representation may again, differ very widely.

It is usual with all (who know a little of common Perspective only) to make one Face of the Object parallel to the Picture, in which Case (the Object being right angled) the Center of the Picture, is necessarily, the Vanishing Point of the horizontal Lines in the other Side†. Consequently, if a Side of the Object be seen, the Picture, must necessarily be obliquely situated, in respect of the Object and the Eye.

Now, can any Person conceive it rational or eligible, to place the Picture in the Position of ED, parallel to the End of the Building, to be seen from the Station S; and yet, this is the very supposition, in the Representation exhibited by Fig. 107; whose Center is C, and Distance CE (equal twice SE, 106) which Position answers to the Picture MN*OP*, in the Apparatus.

How egregiously absurd is the Idea of viewing a Picture in that Position; and yet, in any other Point of View it cannot appear like the Object represented. The Distortion of the Front of the Building is obvious, but much more so, is the Face X, of the Bow Window; whilst the other (Z) is geometrical, as well as the end of the Building.

The Position being determined, and the Center (C) fixed; and also, the Intersection BG, of the Front Plane (its distance from the Center, equal twice EF, 106) the Representation is delineated by Prob. 19; as by Ex. 1 and 2; in which Case, C, the Center of the Picture, is the only Vanishing Point. Fig. 107.

Now,

* BS perpendicular to the Picture does not necessarily fall on the Angle B; in the Apparatus it is somewhat on the Right hand; for, if the Station was ever so little on either Side it would fall differently, as it must be obvious from the method given of determining it.

† See Theorem 6th, and the remark after it.

Pl. XXIII. Fig. 106. Now, those Artists, who make a kind of necessity of delineating their Objects, after this method, cannot, I conceive, give a reason, why the Picture ought not to be placed parallel to the Front, AB, as well as the End, BC; excepting, that they rather choose the one to be parallel than the other; for, there is as much propriety in one, as in the other, but the Representation will be much more distorted; because the Distance, SG, of the Picture standing on AG, is less than SE in the former; and the much greater length of the Front, AB, occasions a greater obliquity of the Rays, with that Picture. For, SG being the Direct Radial of that Picture, ASG is but half the Optic Angle under which it is seen; for it is manifest, that the Eye can take in as much on the Right; in which Case the whole Angle will be very obtuse.

CASE THE SECOND.

Fig. 108. Fig. 108 Exhibits a Representation of the same Object and from the same Station, in that Position, of the Picture; in which, the Front is geometrical; but the projectures of the Cornice, Steps, &c. are dragged out preposterously; and the Face X, of the Bow Window, is worse distorted than before.

C is the Center of the Picture, and CE its Distance, equal to twice SG (106) The whole of the Object ought by no means to exceed the Point E; whereas it extends above twice CE, which is preposterous.

KL is the Vertical Line; and the Angles, CEK, CEL, being made each equal to the Inclination of the Roof, determine the Vanishing Points of the Lines FG, and F*D* in the Roof; K is also the Vanishing Point of HI parallel to FG; the rest is determined as the former.

In both these Pictures, the horizontal Lines, in the Face, X, of the Bow Window, vanish in the Point of Distance, in the Horizontal Line; equal CE, on the other Side of C, because the Figure of it is a regular Octagon. (See Prob. 25.)

CASE THE THIRD. EXAMPLE XXXIII.

Fig. 109. Fig. 109 Exhibits a true and natural Representation of the same Object from the same Station, in the most judicious position of the Picture; and, in order to shew the affinity between this and the Picture MNOP, in the Apparatus, I have made use of the same Letters for Reference, where they can be properly applied.

VY is the Horizontal Line, and C the Center of the Picture.

Draw CE perpendicular to VY, and equal to the Distance (equal twice SB, 106) and make the Angles CEV, CEY equal, respectively, to the Angles BSV, BSY (106) or, make CV equal twice BV, and CY twice BY; V and Y are the Vanishing Points of horizontal Lines in the Side and End of the Building†.

† Th. 9.

Then, because the Angle B touches the Picture, BG is the Intersection of both Planes (in the Apparatus, the Object being at some Distance from the Picture, each Plane has a separate Intersection, *BG* and *BG*) on which all the measures, of the heights of the Windows, &c. are set off, geometrically (as in the Elevation, X, 106; each being double) as Bm, mn, &c. from which draw Lines to both Vanishing Points, V and Y, indefinite.

Make V*E* equal to VE, and Y*E* 2 equal YE. From B, set off Ba, ab, &c. as in the Plan (106) and draw a*E*, b*E*, &c. cuting BV and aV, in the Points *a*, *b*, &c. the perspective proportions of the Windows, or Steps; from which, draw Perpendiculars, cuting Lines drawn from the several heights, on BG to V, in the Representations of the Windows, &c. AB is the length of the Front, which gives B*A* its perspective length.

Or, if B1, 2, 3, &c. be set off, as from B to V (106) from which, Lines drawn to Y determine the same Points; on BV, or aV.

By which means, they are projected in the Apparatus, viz. by the Intersecting Points; and the Steps by the same, which may be finished by Example the 13th. Or, the place of each may be taken, from VY, 106, where the Lines, AS, &c. cut the Picture; which for the Face, X and Z, of the Bow Window, will be the readiest; observing to take them double, because it is but half the Scale of Proportion.

Otherwise,

Plate XXIII.

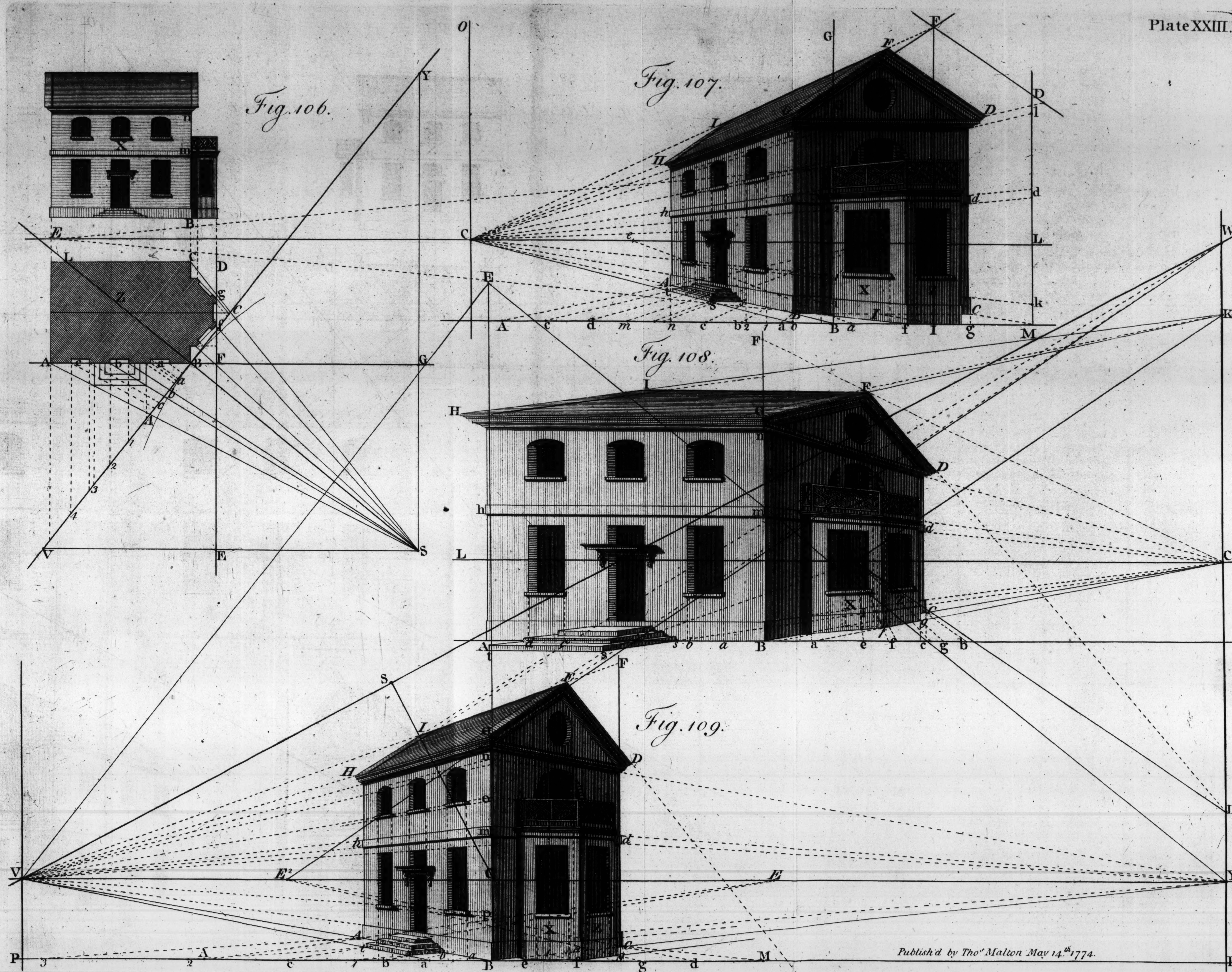


Fig. 106.
Fig. 107.
Fig. 108.
Fig. 109.
Publish'd by Thos. Malton May 14th 1774.

Otherwise; the Bow Window may be projected, by Example 5th, observing the difference in Figure; and, being too near the Vanishing Line, another Intersection may be taken, at pleasure, at a sufficient Distance (Ex. 12.) as it has been exemplified in various Cases.

The Cornice, being plain, is managed by Ex. the 18th, and the Pediment, by the 20th. The Vanishing Points are determined by making the Angle YE²W equal to the inclination of the Pediment; cuting the Vanishing Line YW, of the Plane of the End, at W; the other will fall as much below the Horizon.

W is therefore the Vanishing Point of *GF* and *HI* (as in the Apparatus) and *FD* tends down to the other, in WY produced.

The semicircular Window, and the Circle in the Pediment, are determined on BY, in respect of their widths; the heights on BG; as the Arches in Ex. 21, Sect. 7; for, the Plane being inclined to the Picture, they are consequently Ellipses; as also in the former Position; in the other they are Circles. (Theo. 10, Cor. 5.)

Thus, are delineated three different Pictures, or Representations of the same Object, from the same Station and Point of View; and, notwithstanding they are so very different, yet they are all true Perspective. It is obvious, which is the nearest to the true Appearance, but, when the Eye is in the Point of View, of each, they will appear the same.

In Fig. 107; let the Eye be placed opposite to the Center, at C, at the Distance CE; and having attentively observed the Appearance, then let the Eye be moved to 108, viewing it, at its proper Distance, CE; and lastly, to 109, at its respective Distance. I say, that, each being seen in the true Point of View, exactly (which can only be done perfectly, by fixing a small Pin hole at the true place of the Eye) every Part of each being seen under the same Angle as the Original, must necessarily appear the same.

Now, because the Eye is perpendicularly on the Picture, at 109, and at the proper Distance, the Angle it subtends is equal to ASC (106) and, because the Angle does not exceed 25 Degrees, the portion of a Sphere, of that Radius, will not differ much from the Plane of the Picture; and consequently, this Representation is the nearest to the true Appearance.

This Lesson, properly digested, will give the Student a clear Idea, of the Cause of distortion, in the Representation, and teach him how to avoid it; for notwithstanding the Distance may be sufficient for the Object (the Picture being judiciously placed) yet if the Picture be placed according to the two first, it must necessarily be distorted. Although the real Optic Angle, respecting that single Object, is not varied; yet the Angle under which the Representation is seen, may be varied and increased infinitely, from 25 Degrees to any quantity, less than 180, two Right Angles.

I shall, in this Place, take occasion to display the whole Theory, and shew the immediate and absolute dependance of the Practice on the Theory.

Observe, that No. 15 also refers to the Apparatus.

In Theorem the 1st. it is asserted, and proved, that the Representation of every Right Line, in Perspective, is a Right Line.

Any two Visual Rays, EF, EI, or EF, ED, together with the Original Line, form a Triangle, which are all in the same Plane, passing through the Original Line and the Eye; which, by its Intersection with the Picture, generates the Right Line fi, or fd, its Representation; and if the Original Line was extended, infinitely, its representation will still be a Right Line. (1. 7. El.) Fig. 15.

The second Theorem is exemplified in the Picture MN*OP*, which is parallel to the Plane BGFDC. This is applied to Practice, by the 10th Theorem.

The third demonstrates, that the Vanishing Line, and Parallel of the Eye, &c. are always parallel to the Intersection of the Original Plane.

This has been exemplified, throughout the whole; but as it has been chiefly in horizontal and vertical Planes, I shall illustrate it generally.

Fig. 15, No. 2 and 3, illustrates the Theory for all Planes perpendicular to the Picture; or inclined in a certain Position, respecting the Horizon and the Picture; and if a Plane passes through the Spectator's Eye (at E) parallel to the Original Plane HIFG, its Vanishing Line, VW, is parallel to the Intersection, *FH*. (8. 7. El.)

The great uſe of this knowledge is obvious, for, if either be determined or found, as *FH*, and any one Point, as V (the Vaniſhing Point of AB, &c.) the Vaniſhing Line, VW, is determined; and conſequently, the Parallel of the Eye is alſo determined, its Diſtance being known.

Theorem 4th. A Line drawn through the Center of the Picture and the Center of a Vaniſhing Line is perpendicular to the Vaniſhing Line.

The uſe of this Theorem is to find the Center and Diſtance of every Vaniſhing Line, the Center of the Picture being determined, and Diſtance known.

To do which, there is only to draw a Perpendicular, as CS (109) from the Center of the Picture, to the Vaniſhing Line; and, forming a Right angled Triangle, of the Direct Radial, or Diſtance of the Picture, CE, and the Diſtance between the two Centers; the Hypothenuſe is the Diſtance of the Vaniſhing Line. Def. 20.

Theorem 5th has been frequently exemplified, and beſpeaks its uſe and application ſufficiently; particularly in Example 12. The Corollaries deduced from it, eſpecially the firſt, is derived from the ſecond general Theorem, Book the firſt.

The 6th is a generally uſeful Theorem; becauſe in Practice, eſpecially in common Perſpective, more Planes are perpendicular to the Picture than in any other Poſition. The Horizontal Line, the Vaniſhing Line of the Ground, and all other horizontal Planes; alſo the Vertical Line, of all which are perpendicular to the Picture and to the Horizon; both paſs through the Center of the Picture.

The Plane HIFG is perpendicular to the Picture MN*OP*; its Vaniſhing Line, CV, alſo, paſſes through the Center, C, of that Picture.

The 7th; that Planes, whoſe common Interſection is parallel to the Picture, have parallel Interſections and Vaniſhing Lines, is exemplified in the Apparatus.

The Line BG, in the Object, is parallel to both Pictures, and it is the common Interſection of the two Planes, AHGB and BFC.

If thoſe Planes are produced they will cut the Picture in *BG* and *BG*; which are their Interſections; the Planes being vertical and the Picture vertical, their Interſections are vertical and conſequently parallel. For the ſame reaſon, their Vaniſhing Lines, OP and RW, are parallel; and becauſe they are parallel to their Interſections, reſpectively, by Theo. 3rd.

The uſe of this knowledge is evident; having obtained, by any means, one Interſection or Vaniſhing Line; and alſo, by any means, the Interſecting or Vaniſhing Point of a Line in any other Plane (whoſe common Section, with it, is parallel to the Picture) the whole Interſection or Vaniſhing is conſequently determined.

In the laſt Example the common Interſection (BG) of the contiguous Planes is in the Picture; conſequently, it is the Interſection of both.

By the 8th Theorem, Vaniſhing Lines, of contiguous Faces of Objects, are determined according to the Inclination of the Original Planes, to one another.

In the laſt Example they were at right angles; but, being inclined in any Angle whatever, their Vaniſhing Planes being parallel to them, conſequently they make the ſame Angle with each other, at the Eye; and their common Interſection, which paſſes through the Eye, is parallel to the Interſection of the Originals; and conſequently, when it is not parallel to the Picture, produces its Vaniſhing Point (Def. 22) which, Vaniſhing Point, is the Interſection of the Vaniſhing Lines.

How the Vaniſhing Lines are found, in particular Caſes, is determined by Fig. 15, No. 2 and 3; viz. when the common Interſection is either parallel or perpendicular to the Picture. When it is perpendicular by Problem the firſt; and when parallel, by the third; and, when it is inclined to the Picture, in general, by the fifth. In all which Caſes, it is obſervable that the Interſection of the Vaniſhing Lines, is the Vaniſhing Point, and the Interſection, of the Interſecting Lines, is the Interſecting Point, of the common Interſection of the Original Planes; which is an eſſential thing to be well underſtood; for Practice.

Theorem 9th is of the greateſt uſe; inſomuch that, without it, we cannot draw the repreſentation of one Line, inclined to the Picture, except by means of expedients; making uſe of Lines, otherwiſe, unneceſſary in themſelves.

I Notwithſtanding

Notwithſtanding this is the 9th Theorem, yet its application is prior to ſeveral others, reſpecting Vaniſhing Lines; ſeeing by its means, only, they can be determined; as in Prob. 2. It being impoſſible to determine any one Vaniſhing Line, before the Vaniſhing Point of ſome Line in the Plane be determined, through which the Vaniſhing Line muſt neceſſarily paſs.

For firſt, the Center of the Picture muſt be determined, before the horizontal and vertical Vaniſhing Lines can be drawn; or that of any other Plane, perpendicular to the Picture; or indeed of any other whatever; becauſe it is by means of Lines, in horizontal Planes, others are determined; whoſe Vaniſhing Line is always determinable from its known Poſition, the Center of the Picture being fixed; or which is the ſame thing, the height of the Eye being determined. Notwithſtanding the Center of the Picture (being the Vaniſhing Point of Lines perpendicular to the Picture) is fixed arbitrarily, yet it is as much ſubject to the 9th Theorem as any other; and is only determinable from the fixed and invariable Poſition of the Lines, which vaniſh in it, to the Picture.

As this Theorem has been applied in almoſt every Example, or Problem, where Vaniſhing Points of Lines, inclined to the Picture, are required; I ſhall only illuſtrate it by an Example or two, from the Apparatus.

The Parallel of the Eye muſt be imagined, paſſing through the Eye of the Spectator parallel to the Vaniſhing Line. Def. 9.

For the parallel Picture; EC cuts them both perpendicularily; therefore it makes equal Angles (i. e. Right Angles, with the Parallel of the Eye and the Horizontal Line; as the Original Lines, AB, HG, &c. make with the Interſection of a horizontal Plane paſſing through them; or any other, whatever.

In the direct Picture; EV, producing the Vaniſhing Point of the ſame Lines, alſo makes equal Angles with the Parallel of the Eye and the Vaniſhing Line, of either Plane, AHGB, or FGHI, as the Original Lines, make with the Interſection.

And laſtly; EW, makes equal Angles with the Parallel of the Eye of the Plane FGHI, and the Vaniſhing Line, VW, of that Plane, as FG, or HI, makes with *FH*, the Interſection of that Plane with the Picture. Or, with WY, of the Plane BFC, as FG makes with BG, its Interſection.

This is practically exemplified, in Problems 5, 21, Method 3; 23rd, &c.

The 10th Theorem contains the whole Practice, relative to Planes which are parallel to the Picture.

Firſt; all Lines in ſuch Planes, are parallel to the Picture; conſequently they have no Vaniſhing Point (Cor. 1. Th. 2.) but their Repreſentations are (by the 10th) parallel to the Originals; that is, to the Vaniſhing Line and Interſection of whatever Plane they are in; and their Proportions to the Originals are alſo there determined; viz. as the Diſtance of the Picture, to that, of the Plane they are in.

The Corollaries, of this Theorem, are moſt uſeful Leſſons; which need only to be read over, with attention, to apply them to Practice. The fifth ſums up the whole; that the Repreſentation of every Figure, in ſuch Poſition of the Original Planes, is ſimilar to its Original; that is, it has the ſame geometrical Figure.

Theorem the 11th is ſelf-evident, being read attentively; and the Corollaries are uſeful practical Leſſons deduced from it, which are fully explicit in themſelves.

By Theorem 12th, the Indefinite Repreſentation, of a Right Line, is a Line drawn through its Interſecting and Vaniſhing Points.

This Theorem has been exemplified frequently; but I ſhall in this place illuſtrate it further, it being moſt eſſential in Practice.

Whatever Plane any Right Line is in, if it be not parallel to the Picture, it will cut it, ſomewhere, in the Interſection of the Plane it is in; IF, HG, and FG, are all in one Plane; which being produced, the two firſt cut both Pictures, at *F* and *G*; the other (FG) is parallel to one Picture, and therefore cannot cut it; but it will cut MNOP at *H*; *F*, *G*, and *H* are, therefore, the Interſecting Points of thoſe Lines; and conſequently, the Plane FIHG, being produced, would cut that Picture, in the Line *FGH*; and the other in *FG*; as by the 11th.

But,

* But, E V and E W being parallel, reſpectively, to thoſe Lines, the Points V and W are their Vaniſhing Points; wherefore, the Lines *F* V and *G*V, alſo *H*W, are the indefinite Repreſentations of thoſe Lines; by this Theorem.

The ſame thing is applicable in all Planes whatever; as *B* V, *G* V, &c. in the Front and End, which are vertical; and a V, *B*V, MV, &c. on the Ground Plane.

By changing V for C, it is all applicable to the parallel Picture (MNOP) ſave only, FG is parallel to it, and therefore has no Vaniſhing Point; for the Indefinite Repreſentations of all Lines, in ſuch Caſe, are infinite, ſeeing they have neither Interſecting nor Vaniſhing Point; and are, conſequently, parallel to the Vaniſhing Line of the Plane they are in.

In Theorem 13th, is contained the whole knowledge of proportioning Right Lines, indefinitely drawn on the Picture (by the 12th.) It may, to ſome, appear too mathematical for Practice; but, notwithſtanding that Theorem is more ſtrictly and rigidly demonſtrated than any other, it is nevertheleſs applicable to practice. For, whether it be conſidered mathematically or not, 'tis certain that it is put in practice, by every one, in proportioning Lines; even without its ever entering into his Head what Proportion means; on the contrary, if he really underſtood what is there demonſtrated, with how much more pleaſure, ſatisfaction, and certainty, would he proceed in the application of it to Practice.

The Theorem tells us, that the Diſtance of the Repreſentation of any Point (in the indefinite Repreſentation) from the Interſecting Point (of the Original Line) is in that Proportion to the whole Indefinite Repreſentation; as the Diſtance, between the Original Point, and the Interſecting Point, is to its Diſtance from the Directing Point of the ſame Line.

Now when the Premiſes of this Theorem are clearly underſtood (for which purpoſe, the words are adapted, for brevity and perſpicuity, as much as may be) what is there of Myſtery in it? For the firſt, in reſpect of the Proportion, ſimply. If the Diſtance of the Original Point, from its Interſecting Point, be to its Diſtance from the Directing Point, as 2 to 3, or as 3 to 2; as 3 to 4, 5 to 6, 7, &c. in any Ratio, whatever; then, the Diſtance, of the Repreſentation of that Point from the Interſecting Point, will be in the ſame Ratio to the whole Indefinite Repreſentation†.

Can any thing be more ſimple and eaſy? it does not depend on the real Meaſures, but on the Ratio of one to the other. Wherefore, ſince the Diſtance of ſome Points, in Original Lines, cannot eaſily be obtained, from their Interſecting and Directing Points; I have ſhewn, further (which Dr. Brook Taylor has not) that the Diſtance of the Original Point, either from the Interſection of the Plane it is in, or from the Picture, is in the ſame Ratio to its Diſtance from the Directing Line or Plane, a Circumſtance of great utility; becauſe we may, with eaſe, have the meaſures of one, when we cannot, without applying Geometry, or by Calculation, have the other. For, the Diſtance of any Original Point from the Picture may be had; and the Diſtance of the Directing Plane, from the Picture, is nothing more than the Diſtance of the Eye from it; hence it is, with the greateſt facility imaginable, reduced to Practice, in any Poſition of the Lines whatever. It is moſt frequently done by the laſt; and commonly, by its Diſtance from the Interſection of the Plane the Original Line and Point are in.

And further; the Repreſentation of any Point in a Line being obtained, the Repreſentation of any other Point in the ſame Line, or any other, cuting the former in that Point, may be obtained by the ſame Theorem; when the Interſecting Point cannot be had, nor is in the leaſt Degree neceſſary.

* Let the triangular Piece be fixed, by the two Pins, to the Picture MNOP, of which it is a continuation; and bring the Piece, which folds back, to the ſame Plane; the continuation of the Vaniſhing Lines, &c. on them, ſhow their affinity with Figure 109. If the thread, at W, be drawn through the Eye of the Spectator, EW will be parallel to the Lines, FG and HI, in the Roof of the object; and conſequently, W is their Vaniſhing Point. (Def. 22.)

By the ſame, the thread EV determines the Vaniſhing Point of AB, GH, &c. on both Pictures; to the firſt, it is perpendicular; therefore C is its Center, or Point of View; whilſt another, EC, perpendicular to the other Picture, gives its Center (Def. 17) directly in the middle of the Repreſentation, as it ought always, where there is but one Object.

† See this fully and practically illuſtrated in Prob. 6. Sect. 4. and indeed in almoſt every Problem and Example, throughout the whole Work.

The 14th Theorem, which is more theoretic than practical, or really useful, is fully explicit in itself; and the Corollaries, deduced from it, needs no further Comment.

Thus I have, briefly, taken a transient retrospect of the whole Theory; and I think it cannot fail of answering the End I aimed at, which is most obvious. As the Reader, who has advanced thus far, must now be perfectly acquainted with every Rule given; yet, being eager in his pursuit of what he deemed really and only useful, he may not have given that attention to the Theory (which is frequently referred to) as I could wish, for his more perfect knowledge in it.

He cannot now be insensible of the great advantage resulting from a well-founded Theory, the very Essence of all useful knowledge; for, without it, like a blundering Workman in any mechanic Art, he may continually be runing into Error; which, for want of a just knowledge of the Theory, he must, unavoidably; nor can he, with ease and certainty, extricate himself from the Difficulties he will frequently be immersed in.

E X A M P L E XXXIV.

Is a Tuscan Arcade, with Pilasters; having one Face parallel to the Picture.

The scale of Proportion being determined, and a Profile, of the Order, proportioned to that Scale (at AD) as in the previous Examples, let AB, the Intersection of the upper Face of the Step, be considered as the Ground Line; and, equal to the height of the Eye (in proportion to the Object) let CE be the Horizontal vanishing Line, C its Center, and CE the Distance. Pl. XXIV. Fig. 110.

Then, the Object being right angled, the horizontal Lines in the returning Side vanish in the Center of the Picture †; in the other Face, every part of it, which are in Planes parallel to the Picture, are represented by Figures similar to their Originals, according to the 10th Theorem.

† Cor. to Theo. 6.

First, the Step, *A*B, is represented; then, the Plinths, X, Y, and Z, are equal, and have equal spaces between them; their places are obtained by setting off their true measures from *A* towards B, as a, b, c, &c. on the first Step; and after the same manner, the Piers are obtained, and the Plane of the Arches, whose Centers are got by bisecting b c and d e, at *a* and *b*; and *a c* being made equal to the Distance of the Plane they are in from the Picture. Draw *c* E, cuting *a* C at f; and a Line f g (parallel to AB) cuts *b* C, in g; f and g are the Seats of the Centers of the front Arches, on the Plane of the first Step.

Draw FG parallel to AB, and equal to the height of the foot of the Arches; from *a* and *b* draw perpendiculars, *a* F and *b* G, and from F and G, draw to the Center, C; then, Perpendiculars, from f and g, cuting FC, and GC, in h and i, give the Centers of those Arches; which, being parallel to the Picture, are Semicircles. (Cor. 5, Theo. 10.) The Centers of the inner Curves, are in the same Lines, FC, and GC, at k and l.

The Mouldings in the Cornice, Pilasters, &c. may be proportioned by the general method (Ex. 15) as the Doric Entablature in the 16th.

The true Profile of the whole Order being described geometrical, at AD, and the Picture being supposed close to the Plinth, at A, draw AD perpendicular; which is considered as a vertical Section of the Picture, and CE its Distance.

Then, E being the Eye, and EA, ED, &c. Visual Rays from every Part to the Eye, it is obvious, that the parallel Mouldings have their places and proportions transferred from the Profile, to AD (as in Ex. 15 and 16) and, because the Cornice projects beyond the Picture, as D b, it is projected to the Picture, at d; the difference is only in the Proportion of the whole; for if the Picture was at the greatest extreme of the Cornice, as a b; it is manifest, that the proportion of all the Mouldings, &c. would be less than those on AD. (See Example 15th.)

Pl. XXIV. Fig. 110. The returning Face, *A*HIK being perpendicular to the Picture, the Vertical Line (AD) is the Vaniſhing Line of all Planes in that Front. (Theo. 5.)

The meaſures of the Piers, Arches, &c. being ſet off, at *d*, *e*, &c. are proportioned, by drawing Viſual Rays *d* E, *e* E, &c. as in the foregoing Examples; and the Arches, are managed as in the 21ſt Example (Sect. 8) their Repreſentations are ſemi-Ellipſes; to particularize the whole would be ſuperfluous.

The Mouldings, in all ſuch Caſes, viz. when a true Section of them is parallel to the Picture, may be proportioned, by making a geometrical Section of them, at S, where the Picture is ſuppoſed to cut the returning Mouldings, if they were continued beyond the Picture.

In this Example, the Diſtance is conſiderably too little, for taking in ſo much as is repreſented; in this poſition of the Picture. The deſign, of which, is to ſhew how diſtorted the returning Mouldings of the Pilaſters, &c. are, beyond the firſt Arch; which is neceſſarily the Caſe in all ſuch Views, being parallel, except a ſufficient Diſtance be taken; in which Caſe, the returning Front would be greatly contracted. On the contrary, being both inclined, they may be ſhewn to much greater advantage, at a leſs Diſtance, without any ſenſible diſtortion; as may be ſeen in the next Example.

E X A M P L E XXXV.

Is the Repreſentation of a Doric Arcade inclined to the Picture.

Fig. 111. If the Problems and preceding Examples are underſtood, there is nothing more requiſite to the delineation of this Object; the Situation, Diſtance and Poſition to the Picture being previouſly determined.

In Prob. 21, the Rectangle may be conſidered as the Plan of ſuch an Object, promiſcuouſly ſituated; or the Plan of the Building in Fig. 106, inclined to the Picture, on VY; the foregoing Example ſuppoſes the Picture in the Poſition DE; the Repreſentation, of which, is as Fig. 107.

The height of the Eye being determined, and the Center of the Picture (C) draw the Horizontal Line; and, at a convenient Diſtance, take AB for the Interſection of the Ground Plane, on which ſet off the Meaſures, a, b, c, &c. and, having obtained the Vaniſhing Points of the Sides (both which are, in this Caſe, off the Picture) and their Diſtances, D and E, being fixed (by Prob. 12) compleat the perſpective Plan; as in Ex. 16, of the Doric Entablature.

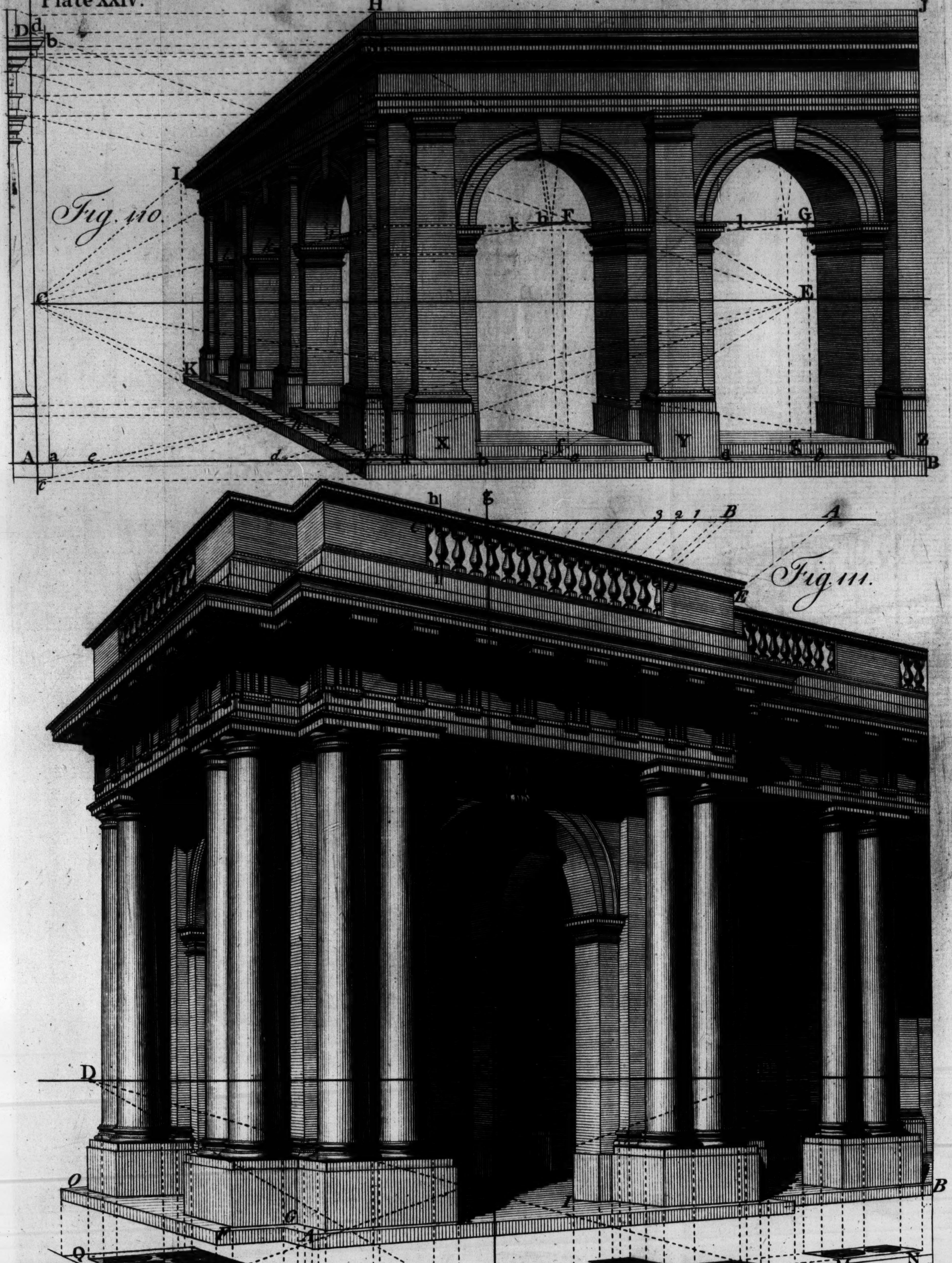
The direction of the Lines, in the Figure, ſhew how it is effected; a repetition of it would be trifling and impertinent.

Let the Reader obſerve, once for all, that, either a true geometrical Plan of the Building, muſt be drawn out, or its meaſures known; which are applied to the Ground Line, or other Interſection, as there is found Occaſion. The Profile, and meaſures of Elevation, muſt alſo be known; for it muſt be obvious, that every Object delineated by the Rules of Perſpective, are projected by the true and real Proportion of one Part to another, which muſt be known, or conceived.

The perſpective Plan being compleated, or ſo much as is neceſſary (for thoſe Parts which cannot be ſeen, it would be uſeleſs to particularize, except for the affinity of thoſe which are) take *A*, at the proper Diſtance from the Vaniſhing Line (perpendicular over A) equal to the determined height of the Eye; and from it draw indefinite Lines to the Vaniſhing Points (as *AB*) and, from the Plan below, cut off ſuch portions as are the repreſentations of the ſeveral Parts, and proceed as in the foregoing Examples.

Here, the neceſſity of a Plan being formed, or, which is the ſame thing, the Lines proportioned, below, by an imaginary interſection, AB, is obvious; on account of the great Diſtance, and the Eye not being far out of the Ground Plane; otherwiſe, that operation would be unneceſſary; for, if the height of the Eye was equal to the Diſtance of AB, from the Vaniſhing Line, then, the Plan formed, by means of that Interſection, would be the real Baſe, or Bottom of the Repreſentation, of the Object, and no other would be wanted; in which, the expediency of Ex. 12 is remarkably obvious.

Plate XXIV.



Published May 14th 1774. by Thos. Malton.

The Step being first drawn, the Sub-Plinths, next, are but so many equal Parallelopipeds of a certain proportion and distance from each other (as in Ex. 4th.) The places and proportions of the Columns are determined from their Plans below, by drawing Perpendiculars from them (or by Ex. 22) the Bases and Capital, being inclined; by Ex. 26; and the Cornice, by Ex. 17, 18 and 19.

The Ballustrade, at the Top, may be planed on the Top of the Cornice, or at some distance from it; the Pedestals, or plane parts, are directly over the Columns, and equal to their Diameters at the Top; and the places of the Ballusters are got by dividing the Line *CD*, perspectively (by Prob. 8) into the number of Ballusters and spaces between them (equal or otherwise.) The Plinths, at their Tops, are perpendicular over the Bases; and central Lines (h i) being drawn, through each, the Ballusters are easily formed, from the known geometrical figure of them.

The Doric Entablature is described at large in Ex. 16 (but parallel to the Picture) how it may be inclined is shewn in the 18th and 19th Examples.

The Plane of the Arches being obtained, as in the last, the Arches are represented by Ex. 21 (Sect. 8) and the Consoles by 20; the rest, it would be needless to describe, as the previous Lessons contain every necessary Example for complex Objects.

E X A M P L E XXXVI.

Is the Representation of a compleat Building, with a Portico in the Ionic Order; having a circular Dome, and Cupola.

At No. 1 is half the geometrical Plan, and, at No. 2 is the Elevation; in proportion to the Representation, as one to four. Pl. XXV. Fig. 112.

The Design is intended as an exterior Building, adapted to St. Stephen's, Wallbrook, but of a different Order; having a Front towards the Mansion House.

This Example sums up all the foregoing, in one compleat Building.

The Horizontal Line being drawn, at discretion, and the Center of the Picture fixed, as usual, let AB be the Ground Line, and D the Place of the nearest Angle of the Building. The Distance of the Picture is 15 Inches.

The Inclination of the Front and End, to the Picture, is determined by the Line a b (No. 1) on which the Picture is supposed to stand.

The Distance of the real Ground Line from the Horizontal, being equal to the height of the Eye, which is elevated above the common height, yet is too low to form a correct Perspective Plan, in its true Place; therefore, at any convenient Distance take another Line *AB*, parallel to AB, and proceed as in the last Example.

Having laid down the Distance of each Vanishing Point, at E and C, as usual; take the known measures of the several parts of the Building and apply them on each Side, from the Angle at *D*. Compleat the exterior Plan, *CDEF*, as in the last Example; from which, the several parts of the Building, above, are proportioned; as the perpendicular dotted Lines shew. F is the Vanishing Point of Diagonals; i. e. of a Line bisecting the Angle of the Building.

It must be obvious, that any Line (AB or *AB*) being considered as the Ground Line, will give the same proportions; which, being properly considered, there is nothing more in it, than a supposition of the Eye being more elevated above the Ground Plane; and consequently it is more seen and better defined.

The exterior Plan being compleated, the Columns are erected and finished as in the last Example; and the Capital, being the antique Ionic, by Ex. 31. The interior part of the Portico has a Pilaster opposite each Column, which are also determined from the Plan, below; some of which are hid behind the Columns; perpendicular Lines from the Plan, shew how much, and where they are seen entire.

The Plane of the Front, of the Pediment, over the Portico, being much inclined to the Picture, its Vanishing Line is very remote; the Distance of the Vanishing Point of horizontal Lines, in that Face, may nevertheless be ascertained, (as E and C) by Ex. 12, and the Lines drawn by the Expedients, in the 13th.

As

Pl. XXV. As the Diſtance C, of the Vaniſhing Point of the horizontal Lines on the left Hand, would fall off the Picture, it is taken half; and conſequently, the meaſures applied on the Interſection or Ground Line (AB, or *AB*) for that Side, are alſo, each half of the real meaſure, agreeable to the 13th Theorem, and Prob. 6 and 7.

The front Pediment is thus determined. If the Cornice projected equal to the Steps, which the Picture is ſuppoſed to touch, a Right Line drawn through D, perpendicular to the Ground Line, would be the Interſection of the Plane of the Pediment; but, as its projecture is leſs, it will fall to the left hand of it; as the Line *ab* (No. 1) which is a continuation of the front Line of the Cornice, cuting a b, at *d*, indicates ſufficiently.

Or, having determined the perſpective Plan of the Cornice, at a, b, c, produce a b, till it cuts the Ground Line at d, and draw d e perpendicular, which is the Interſection, of the Plane of the Cornice in the Pediment.

Then, having obtained the front Line of the upper Moulding, as f g, let it be produced to the Interſection d e, cuting it at e; where, ſet up the height of the Pediment, from e to i, and draw i k, tending to the Vaniſhing Point (by Prob. 13.)

Biſect f g (perſpectively) at h, and draw h l perpendicular, cuting i k, at l, the true pitch, or middle of the Pediment, which may be compleated, by Ex. 20.

The Pediment in the other Front, on the Left hand, may be projected by means of its Vaniſhing Points, by the ſame Example.

To this Front is added a Pedeſtal which is continued around the Building; more for the ſake of diverſifying the Leſſon, than propriety in the Building; the heights of the Pedeſtal, Window, &c. are proportioned on the Interſection DI, of that Front, at *a*, *b*, *c*, &c.

The Body of the Building being compleated, and the Circle of the Dome planed, below, at GH, perpendicular Lines from the extremes of that Ellipſis, give the extreme apparent edges of the cylindrical part, IK, below the Dome; of which Cylinder, the Ellipſis, on the Ground Plane, may be ſuppoſed its Baſe.

The curve Lines, in the Cornice, around the Dome, are deſcribed in their reſpective Planes; which are thus determined.

From the Vaniſhing Point on the Left, draw through the Center, S, of the Plan below, cuting the Ground Line at J; which repreſents a Line paſſing through the middle of the Plan (No. 1) ſuppoſing a Section made by a vertical Plane through the middle of the Building.

Draw JL, perpendicular, the Interſection of ſuch a Plane with the Picture (Prob. 3) on which, all the meaſures of the heights of the Dome, &c. are applied, in the ſame Ratio as on d e, in the Elevation (No. 2) at M, N, &c. From M, the height of the Cornice, draw a Line parallel to the Ground Line, which is the Interſection of the Plane of the Cornice, with the Picture. A Right Line, drawn from M to the Vaniſhing Point on the Left, will cut a perpendicular Line from *S* at ſ, the Repreſentation of the Center of the Circle of the Cornice.

From C, the Center of the Picture, draw CS, through ſ; and through ſ, draw m n, parallel to the Interſection; make SR and ST each equal to half the Diameter of the extreme Circle of the Cornice; and draw RC and TC, cuting m n, at m and n. m n is a Diameter of the Repreſentation, parallel to the Picture; the whole Circle may be compleated, by Prob. 2, Sect. 8th.

After the ſame manner all the other Circles, of the Corona, &c. may be projected, in their reſpective Planes, as the ſeveral parallel Circles in parallel Planes, in the 28th and 29th Examples.

The Cornice, &c. being compleated, make NO equal to the height of the Dome, and between N and O take ſeveral Diviſions *a*, *b*, *c*, &c. anſwering to the ſame Ratio on d e, in No. 2.

Through theſe Diviſions, parallel Lines being drawn, as r s, each may be conſidered as the Interſection of a Plane, cuting the Dome (as *r s*, No. 2) in each of which, an Ellipſis being deſcribed, repreſenting a Circle of the true Diameter, made by that Section; a Curve deſcribed over all their extremes (as in Ex. 25 and 26) is the true exterior Contour of the Dome; which may be conſidered as an Ovolo, inverted; as in the Doric Capital.

By

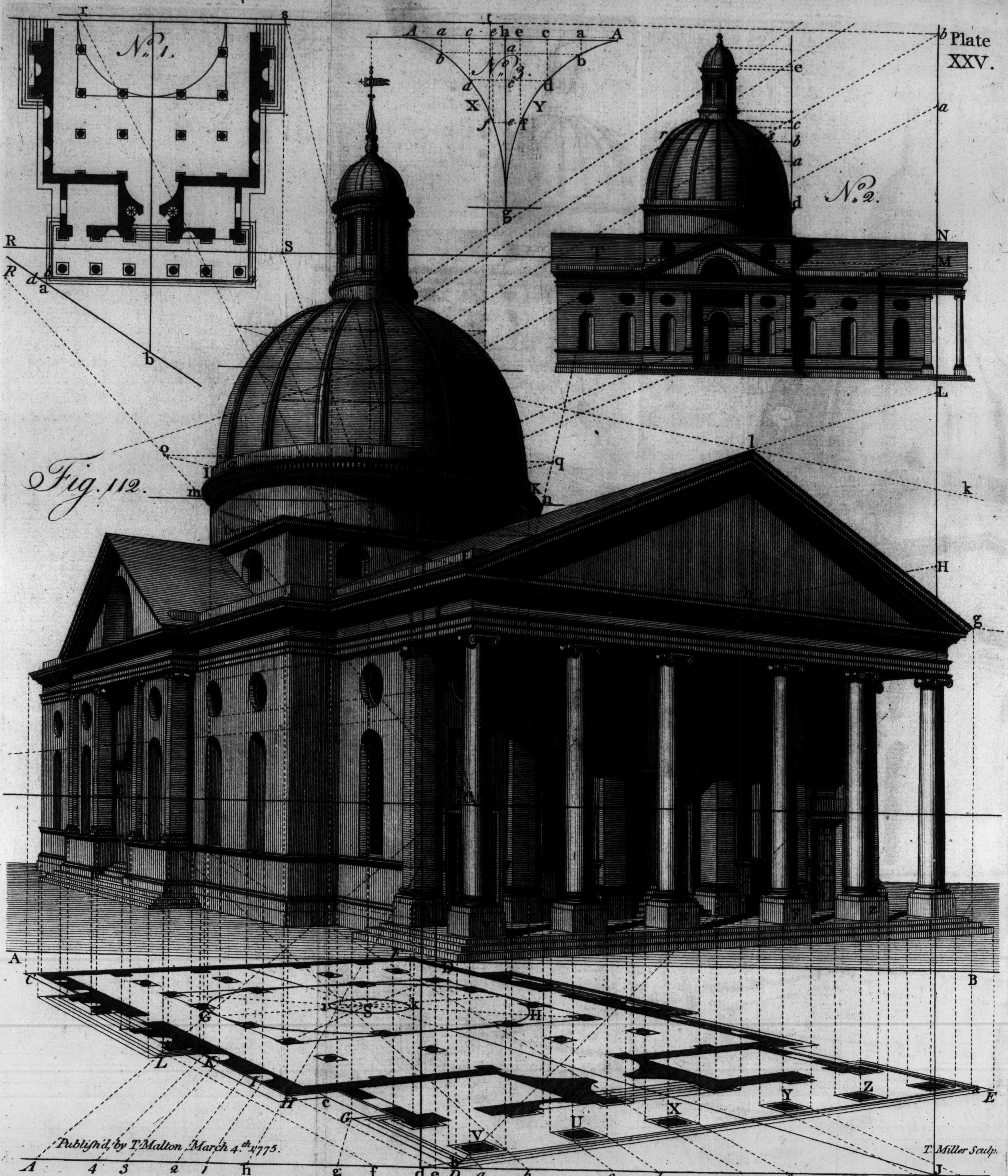


Fig. 112.

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T. Miller Sculp.

Plate XXVI.

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By any other means (the Dome being plain) I cannot conceive it poſſible to obtain the true perſpective Figure of a Dome; which is generally higher than a Hemiſphere; eſpecially in a lofty Building, which would otherwiſe appear flat, below; beſides, it is a more graceful Figure; as the magnificent Dome of St. Paul's ſufficiently evinces, to a judicious Eye. But, being a Hemiſphere, or other Segment of a Sphere, the Eye being below its Center, the Contour of the exterior Curve, on the Picture, will be elliptical. (Th. 3rd of curvilinear Perſpective.)

By this method, it may be truly projected, let the Figure be what it may, whether a Hemiſphere or leſſer Segment. Neverthele ſs, a judicious Eye, and judgment, guided by a thorough knowledge of Perſpective, may direct a ſteady Hand to trace a Curve, ſufficiently correct, in common Caſes; the height and Vertex being truly aſcertained.

The Dome in the Frontiſpiece is octagonal; and, by means of ſeveral Octagons, repreſented perſpectively, in various parallel Planes, the true Figure of it may be projected. Or, by means of ſeveral Points, in an Angle; for, if the true Curve of an Angle be obtained (as at Y, No. 3) by means of Ordinates from the Curve, X, of a vertical Section through the Dome, perpendicular to two oppoſite Faces; then, by means of perpendicular and horizontal Ordinates (*a b*, *c d*, &c.) the true Curve of each Angle may be obtained; being transferred from one to the other, by the Vaniſhing Points of the horizontal Lines in the Cornice around it, at the foot of the Dome.

By the ſame means, the Ribs, in this Dome and Cupola, may be deſcribed.

After this deſcription of the Dome it would be loſs of time to deſcribe particularly the projection of the Cupola. From the Plan below, its place and dimenſions are truly aſcertained, together with the place of each Pilaſter and Window; from which perpendicular Lines are drawn; and, the height of each part is ſet up, on the Vertical Interſection, from O, as the Dome, and other Parts, below; of which, the Cupola is a ſimilar Figure, nearly.

E X A M P L E XXXVII.

Is a Repreſentation of that much famed Building, by Inigo Jones, of St. Paul's, Covent Garden; and the Buildings adjacent. Pl. XXVI.

As the laſt Example was a Leſſon for an entire Building, ſo this is intended for a Leſſon in detached Buildings, or diſtinct and ſeparate Objects; which may be conſidered as a Street-View.

Having, in the laſt, given a compleat perſpective Plan of the whole Building, it would be multiplying Examples to little purpoſe to do the ſame thing here; let it ſuffice, therefore, to ſuppoſe, that the Plan of the whole, in this piece, is truly deſcribed on the Ground Plane, on which the Objects ſtand; their meaſures being applied to the Interſection of the Picture or Ground Line, as in the preceding Leſſons, which it would be quite ſuperfluous to repeat; I ſhall, therefore, only make ſome general Remarks.

It may be obſerved, that the Church is nearly in the ſame Poſition as the laſt Object, but a much ſimpler, and conſequently an eaſier Figure to repreſent.

The Portico is of a Tuſcan Order; which for its ſimplicity and ſingular Stile is much eſteemed by the admirers of the antique Grecian Temples, of which, this is a perfect Model. The great projecture of its Cornice (if it may be ſo called) has a bold and ſtriking appearance; owing to the great effect of Shade it occaſions. The ſmall attached Parts, at Y and Z, which is the ſtaircaſe into the Gallery, ſeem not to belong to the Body, eſpecially the latter, but are adapted merely for conveniency; I am of opinion that it would make a better Figure without them. The other, at V, is the Veſtry Room; which, with a ſimilar Object, on the other Side, adds to the appearance of the back Front.

How theſe ſeveral parts are projected, may be ſeen in the laſt Figure; having formed a Ground Plan of the whole; their ſeveral heights being ſet off, on the vertical Interſection of the front Plane, or proportioned to the Pilaſter, at the corner.

Pl. XXVI. The two Gates, or entrances into the Church-Yard (X) are a great ornament to the Church, and graceful in themſelves. After what has been done, there is nothing ſingular in their Conſtruction, the geometrical Figure being known.

The Houſes, one on each ſide, are equally diſtant from the Church; 'tis pity they are not both in the ſame ſtile, as that on the Left, for the ſake of uniformity. The Hut, adjoining to the Houſe on the Left, at W, is the Pariſh, Watch-Houſe. I could wiſh ſome other place had been deſtined for it, as it blocks up an Avenue, and breaks the regular Order.

The Fronts of the Houſes and the Gates are in a Right Line, much inclined to the Picture; the Portico projects before them. There is no occaſion for a Ground Plan of the whole, for the meaſure of each Part being known, the Line may be proportioned, by the Problems in the 4th and 5th Sections; and, their heights are determined on the Line AB, which may be uſed as an Interſection of the whole, conſidered as one Plane.

The Pediment has been frequently deſcribed; and, the Bell-Turret may be determined by the length of the Roof; or, more correctly, from a Plan formed below; as the Cupola in the laſt Example.

On the Left is ſeen the opening into Henrietta-Street, at P; and one of the Green-Sheds, at Q; the place, of which, is obtained by its known diſtance from the Church, and the line of Direction in which it ſtands, reſpecting the Church, projected if neceſſary. Or, the Ground Line may be ſuppoſed on this ſide of the Shed, parallel to the bottom edge of the Picture. The Horizon is about the natural height of the Eye; the Center of View is at C, in the middle of the Picture, and the Diſtance is about 16 Inches.

E X A M P L E XXXVIII.

Is the Repreſentation of another well-known Building, by the ſame Architect; viz. The Royal Hoſpital, of Invalids, at Chelſea.

Pl. XXVII. The Stile, of this Building, is ſomewhat ſimilar to that of St. Paul's, Covent-Garden. There is, in the Guſto of the whole, ſomething remarkingly pleaſing; the Symetry of the ſeveral Parts, in reſpect of each other and of the whole, is finely preſerved, in a regular and gradual ſubordination, from the Principal to the ſeveral Offices; the agreeable arrangement, of which, is uniform, and perfectly harmonious. Here, the attention is not attracted by the richneſs of the dreſs, or maſterly execution; the mind is not captivated by grandeur and magnificence, but moſt agreeably entertained with an elegance of Deſign; ſimple, yet majeſtic; and comely, though unattired; perfectly agreeable in its native ſimplicity, and almoſt naked Beauty, devoid of every luxuriant Ornament.

In the laſt Example, the Line of the Building was much inclined to the Picture; in this, it deviates little from a Perpendicular; which is far more agreeable than to be perfectly ſo.

In this View, many, who pretend to know Perſpective, would imediately fix on the Point V, in which all the horizontal Lines, of the Front, converge, to be the Center of View; but it is not ſo; for, according to my general maxim, it is at C, in the middle of the length; and, notwithſtanding there is not one Line vaniſhes in it, yet, it governs the whole.

If V had been the Center, or Point of View, the Lines BD and HI, &c. would be parallel amongſt themſelves, and to the Horizon; but, they converge to a Point, at a great Diſtance, on the left Hand.

The Diſtance of the Picture is about 19 Inches; at which Diſtance (the Eye being perpendicularly oppoſite to C) the whole will appear as the Original, at the ſtation intended; and, I am perſuaded no Perſon will ſuppoſe that it would have a better effect, or ſo good, if V had been the Center of View; for, the Optic Angle would then be almoſt double of what it now is; and conſequently, the Repreſentation

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P. Mazell sculp.

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tation would differ more from the true Appearance; which, now, is inconsiderable, the Optic Angle not exceeding 34 Degrees.

The Inclination of the Building to the Picture, and the Distance, being determined, the Vanishing Point, V, is fixed by Prob. 12; and the indefinite Line, GV, being drawn, it is proportioned as usual, by setting off the real Measures, or their Ratio, from G, at K, L, &c. and drawing Lines, which represent Visual Rays, to E, in the Horizontal Line.

The measures are taken half, because EV is but half the Distance of the Vanishing Point, V; as the whole Distance exceeds the bounds of the Picture.

The horizontal Lines on the returning Side, whose Vanishing Point is at a great Distance, may be drawn by the Expedient, No. 5, Prob. 13. Or; having proportioned the Line of the Angle, at AB, into the several Divisions, at a, b, c, &c. for the Windows, &c. and having drawn one Line to its Vanishing Point, as BD (by the 12th) the Distance of the Vanishing Point being determined *; produce BD, and at a convenient Distance, draw FG parallel to AB, and proportion FG in the same Ratio as AB (by Prob. 33, Geom.) draw a*a*, b*b*, &c. which will tend to the same Point.

But, when the Inclination is not very great, the best Expedient is to fix a Lath to the Board and make use of a long Ruler; or, in Large Work, fix a smooth String to the Vanishing Point, and strain it, in a Right Line, to the several Points a, b, c, &c.

The Hip of the Roof, at H, may be determined from a Plan, on the Ground Plane; or by the known Inclination of BH to the Picture, and its Vanishing Point; and, the Line HI by the 13th Problem.

The Window Frames, in the Roof, being drawn, as the other Windows, draw fV, &c. and having found the Vanishing Line of the Roof (by Prob. 5) or, of the front Plane (Prob. 3rd) and the Vanishing Point of gh, &c. which is at X (by the 4th) draw gX, cuting fV at h, which determines the Triangle fgh, &c.

The Cupola is best done by a Ground Plan, as in the 37th Example.

The Columns and Pediments have nothing particular in the delineation, save that, the middle Pediment is more contracted than the hither one; but is done after the same manner; and, if they have the same Inclination to the Horizon, the Lines have the same Vanishing Point; because the Pediments are in parallel Planes†. † Theo. 5. and Cor. 1.

The Trees (on the Right hand) excepting their Distances from each other, are not subject to the Rules of Perspective; because no proportion of the Boughs can be taken. The Rails and Posts have nothing singular in them; as the Rails are parallel to the Front of the Building, they have consequently the same Vanishing Point (V) as the horizontal Lines, in the Front; and the Posts, being equally or otherwise spaced, are determined as in all other similar Cases.

The heights of the Figures, in the Walks, are thus determined.

Take any Point, L or M, in the Ground Line, and make MN equal to the height of a Figure (by the Scale of proportion, of the measures of the Building) and draw MO and NO, to any Point in the Horizontal Line; then, wherever you intend a Figure, as at *a*, *b*, *c*, draw *a*m, *b m*, &c. parallel to the Ground Line cuting MO, at m, or *m*; and draw m n, or, *m n*, parallel to MN, which is the height of the Figure in that place, representing an equal height to MN.

For, MO and NO represent parallel Lines (Cor. 1, Th. 5.) and MN is a vertical Intersection of the Picture; wherefore, every Right Line drawn between MO, and NO, parallel to MN, represents an equal measure, equal MN; for MNnm and mn*nm* represent Rectangles, having equal Altitude, and consequently their opposite Sides are equal (15. 1. El.) therefore mn and *mn*, represent equal heights.

By the same means, the proportion of any other Object may be determined.

EXAMPLE

* This Vanishing Point is about seven Feet Distance, from the Center of the Picture.

Pl. XXVIII.

EXAMPLE XXXIX.

Is a View of the Queen's Palace in St. James's Park, and adjacent Buildings.

This neat and elegant Building was formerly the Town Residence of the Dukes of Buckingham; which Title becoming extinct, it had for some Years been occupied by a distant branch of that ancient Family, without the Title, who made no Figure, suitable to such a noble Mansion; insomuch, that it was gone greatly out of repair when purchased by his Majesty. It has, since, been much enlarged, repaired and beautified, and made the Winter Residence of the Royal Family. The octagon Building, on the Left, was added at the same time, and is the Library.

Since which, there has been great improvements made; the Semicircular Area, in Front, has been enlarged; but I think it would have been more advantageous, had the Pallisadoes stood on a Dwarf Wall; for want of which, they are not distinguishable, at some Distance from it. The Gate leading to Chelsea, and the Lodge are also new; and, several old Trees were cut down, to make a more spacious and open View to the Canal, &c. in the Park.

This Building would, with propriety, admit of a full Front View; but such a one is by no means picturesque; as, one Side, in such Case, is but a duplicate of the other, which is not the Case in this View; the Center, of which, is in the middle of the Picture, as usual.

The Front is inclined, and consequently, the horizontal Lines vanish; but at a great distance, because the Inclination is very great; the Distance of which Vanishing Point is determined by the 12th Prob. The Vanishing Point of the End is at V, in the Pavilion on the Left hand, the Horizon is high, for the conveniency of seeing the Area, more commodiously.

The Distance of the Picture being determined*, and the Vanishing Point V, the Distance of the other is a third Proportional, to CV and the Distance of the Picture, found by squaring the Distance, and dividing by CV. (Pr. 12.) (Pr. 31. Geo.)

As the octagon Building has two Faces parallel to the Fronts of the House, the adjacent Faces vanish in the Diagonals on each Side; i. e. all the horizontal Lines, in one Side of the Octagon, vanish in that Point where a Line bisecting the Angle, made by the Radials of the Front and End of the Building, cut the Horizontal Line; and the other, on the Left, where a Line, making a Right Angle with the Line of bisection, cuts it; as it has been exemplified in various Cases, in Mouldings, &c. both which fall out of the Picture.

On account of the great Distance, and the Horizon being high (between the Mouldings in the upper part) their inclination is not perceptible; as is frequently the Case, in Views taken at a tolerable Distance, and perhaps from an Eminence, which is generally made choice of in Landscape Views; otherwise, the Ground would be too much contracted, unless diversified with rising Grounds, &c. The distant Buildings, in such Cases, are represented nearly geometrical; for, unless they are situated considerably above or below the Eye, the inclination of the Lines is not distinguishable; and, if they are obliquely situated to the Picture, the Fronts and Ends are contracted, almost geometrically; so that, distant Objects, of all kinds, are not cognizable to the Rules of Perspective.

It is unnecessary to say more, in respect of this Object, as Rules have been given for projecting all kinds of Figures of which it is composed, and finding the Vanishing Points. I shall, in the Appendix, give a general Method for contracting the Faces, &c. of each Object, and for obtaining the true place of each, in respect of its Bearings with other Objects; which, in many Cases, is the best method of proceeding, and the least liable to error.

* In this Picture it is about 12 Inches.

SECTION

Publish'd by T. Malton March 18th 1776.

J. Collyer sc.

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SECTION X.

OF INSIDE VIEWS, IN GENERAL.

THE application of the Rules of Perſpective, whether to the interior or exterior parts of Objects, is the ſame, deduced from the ſame Principles, the Theory being general and univerſal; for, Planes and Lines, of which Objects are compoſed, are the ſame however ſituated; whether they form internal or external Angles, of Objects ſeen externally; or, forming Rooms or Concavities of any kind. Neverthelefs, at firſt Sight, there appears a difference; for I believe that, many who have practiſed Perſpective with tolerable ſucceſs, in exterior Objects, are ſomewhat puzled, at the firſt attempting an Inſide View; not knowing, rightly, where to begin or where to leave off; how to place their Picture, or determine its Diſtance.

I have ſeen an attempt to repreſent an Octagon Building, internally, in which Picture were introduced ſeven Sides or Faces, out of the eight; 'twas a misfortune that the other could not be ſeen alſo, quite around, and then it would have been a maſter-piece, indeed. However, from what was ſeen, and the tendency of the Lines, it was manifeſt, that the Diſtance of the Picture was not more than one fourth part of its width, or length; and conſequently, the Optic Angle was above 120 Degrees, which ought not on any account (except when the Eye is confined to the true Point of View, always) to exceed 60.

'Tis the ſame thing, if, in Order to ſee the whole Dome and Cupola, internally, we advance ſo far into the Building, that the Repreſentations of ſuch Subjects are only fit for horizontal Pictures, or Cieling-Pieces; for, when they are repreſented on a vertical Picture, they appear (at a proper Diſtance to take in the whole) as if falling, or not upright. Yet are theſe things to be met with in Pictures and Prints, by Men of ſome diſtinction in the Arts.

That the Perſpective Repreſentation of the inſide of a fine Building is more difficult to manage than the exterior is certain, and is manifeſted in theatrical performances; which, being repreſented on ſeveral detached Planes, it is impoſſible, by the Rules of Art, to make them correſpond in every Point of View; but, they are very rarely connected in any one Point. There are, undoubtedly, many fine Performances of the kind; yet, without attempting to diſparage their Authors, I am confident, that, were they better acquainted with Perſpective, they would produce better and more natural Repreſentations; even that famed Scene in Cymon is a jumble of inconſiſtencies, both in point of Deſign and Execution; although, it was a bold attempt out of the common mode of Repreſentations.

Few Artiſts have made Perſpective ſo much their Study, to know how to proportion one Part to another, on detached Scenes, ſo, as to make the whole unite in the proper Point of View, whether the Repreſentation be internal or external; indeed, from the preſent Conſtruction of the Theatres, it is hardly poſſible to be done; nor are the Rules of Perſpective ſufficient, for the purpoſe, without a tolerable knowledge of Lines, geometrically. It is the leaſt qualification of a Scene Painter to be excellent in Landſcape, in which a ſmall knowledge of Perſpective is requiſite; but, in order to execute Deſigns in Architecture with correctneſs, and a juſt proportion of the ſeveral Parts, requires a thorough knowledge of Perſpective. It is ſomewhat ſurprizing, that all who are concerned, or any way engaged in Scene Painting, do not make Perſpective their immediate Study; being the Baſis, the very ſoul and exiſtence of their Profeſſion; yet, to my certain knowledge, ſeveral Artiſts, employed in it, are not only totally ignorant of it, in Theory, but they are, almoſt, wholly unacquainted with its Rules, which, to me, is moſt unaccountable.

Pl. XXIX. In inside Views, the Bounds of the Picture limits the whole, every way, which renders the Operation, in some Cases, more difficult than external Views.

In order to shew the inside of a Room, Temple, &c. properly, a Section is supposed to be made; that is, the hither Wall (or inclosure of any kind) is supposed to be removed, and the whole Inside laid open to View; so that, a proper Station and Distance may be taken, from which, the whole Inside, or as much as is required, may be taken into the Optic Angle, without being too large; for, to suppose that we can be within a Room and exhibit the whole, or nearly, would be truly absurd, yet it has been attempted, by such as have not a true notion of Perspective.

In respect of proportioning Doors, Windows, &c. it must be obvious, there can be no difference whether they are interior or exterior; the whole of which is contained in Prob. 8th, and has been universally applied throughout the Work; particularly in Ex. 2nd and 4th, also in the 16th, 19th, and 21st, in finding the places of the Mutules, Modilions, Blocks, &c.

Fig. 113. EXAMPLE. Let AV be the indefinite Representation of the Side of a Building in any horizontal or vertical Plane; which, from the point A, is required to be perspectively divided into certain finite parts, representing Windows, Piers, &c.

By the 8th Problem, A b and VE are drawn parallel, between themselves (VE may be considered as the Vanishing Line, and A b, the Intersection, of whatever Plane the Original Line is in, either horizontal or vertical, or in any other position.)

Make VE equal to the Distance of the Vanishing Point (V) of the Line AV; and, on A b, take A a, a b, &c. equal to the known proportions of the Piers and Windows, or Apertures of any kind.

Draw a E, b E, &c. cuting AV in the Points *a*, *b*, *c*, &c. which are the representations of the Original Points, a, b, &c. Wherefore, if a b be supposed the geometrical proportion of a Window, then *ab* represents the same perspectively, and so of any other division.

By the same means, all other divisions are acquired, as *b c* of a Pier, &c.

N. B. Let it be observed, that it does not depend on the real measures being applied on A b, but, that they are in the same Ratio, respectively, as EV is of the Distance of the Vanishing Point; agreeable to Theorem 13th, which is frequently exemplified in the preceding Work; and, whether AV represents a Line perpendicular or inclined to the Picture, there is no difference in the process.

E X A M P L E XL.

How to represent Doors, &c. open, in any given Angle; or otherwise, at pleasure.

Fig. 114. Let AB represent the Aperture of the opening of a Door, in the side of a Room, &c. which is required to be seen open, in any position, at pleasure.

Now, AB, the width of the Door, is the Radius of a Circle, which it would describe if the Door revolved quite around; and consequently, the Door itself (being a Rectangle) would describe a right Cylinder, of which, BC is its Axis†.

By Prob. 10th, Sect. 4th, or, Pr. 2, Sect. 8th, describe the representation of a Semicircle, A a *a* F, whose Radius (given or found) is AB.

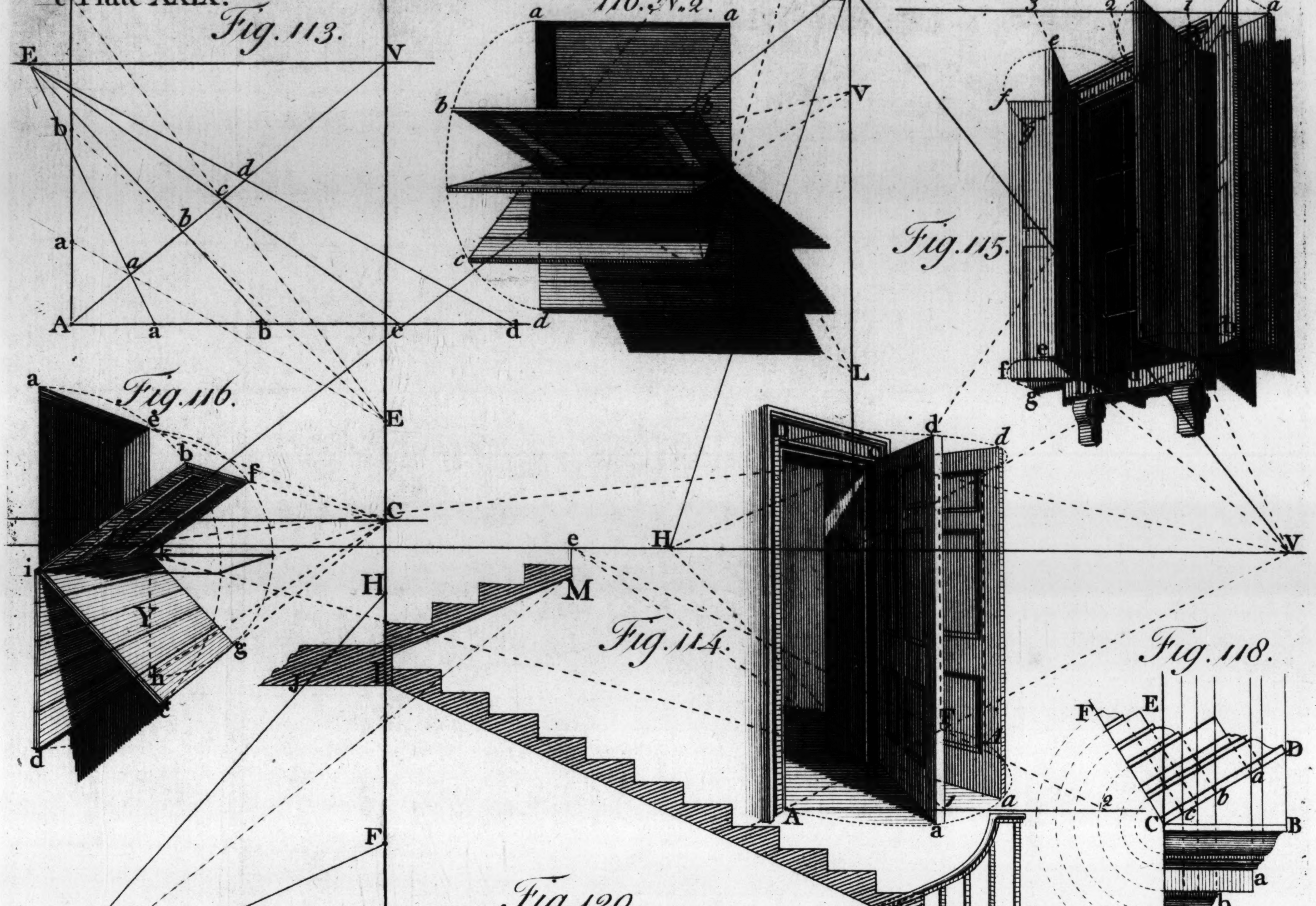
It is manifest, that AB the perspective width of the Door, would, in its semi-revolution (from AB to BF) describe the semi-Ellipsis A a *a* F; that is, the point A (B being fixed) would describe the semi-circumference of a Circle: and consequently, in whatever position the Door is open, the point A will be somewhere in that Semicircumference, as at a or *a*, wherefore, a B, *a* B, &c. are also the Representations of Radii, of the same Circle.

Take the Point a or *a*, in the Circumference, at pleasure (according as you require the Door more or less open) and draw a B or *a* B, which produce to its Vanishing Point, H or *H*.

Draw the Perpendicular a d, or *a d*, and through C, draw HC, or *H*C till it cuts a d or *a d*, at d or *d*; then is a BC d, or *a* BC *d*, the representation of the Door open in the position required.

† Def. 4, 8 Geom.

Plate XXIX.



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Or, if the Angle of the opening, on either side, be known or determined; produce AB to its Vanishing Point, V; S being the Center of the Picture, draw SE perpendicular, equal to its Distance, and join EV; make the Angle VEH, or VE*H*, equal to the Angle determined; which will give the Vanishing Point H, or *H*, of the top and bottom edge of the Door, in the Position required.

If the Door be required more open, as *A*B, making an acute Angle, *A*BF, on the other side; then, the Vanishing Point will be on the other side of V.

The process is the same, in every Position.

After the same manner, the Window Shutters, being vertical, are determined; the Radius, of each, being half the width of the Window. Fig. 115.

Also, the Shutters Y and Z, which describe Semicircles in vertical Planes, as abcd, or ***abcd***; or, more properly, they describe half horizontal Cylinders. Fig. 116.

The first, being parallel to the Picture, is a Semicircle †; the other is perpendicular to the Picture; or it may be inclined, in any Angle whatever; the Vanishing Points are in the vertical Vanishing Line, VL, of the Plane of the Circle; as at V and L, &c.

† Theo. 10. Cor. 5.

EXAMPLE XLI.

To represent an elegant Room, having a large, circular, Bow Window, in the Side, and a Cove-Cieling.

In representing the Inside of a Room, Church, &c. it is usual to take the Station in a Line drawn through the middle of the Building, which indeed appears the most rational; but, in such Case, in a regular Building (one Side being a duplicate of the other) it is not so picturesque, as when the Station is towards either Side, as at E; from which, the inside of the Room AHIB, is to be viewed; by which means, either Side may be shewn to greater advantage, being more inclined to the Picture than the other. Fig. 117.

From the Station E, if Right Lines are drawn to F and B, the extreme on each Side of the Room, intended to be delineated, FEB is the Optic Angle under which it is seen; and, if AB be the Intersection of the Picture, EC, perpendicular to AB, determines its Center, at C, and EC is its Distance; in which Case, the Center of View is not in the middle of the Picture. But, if the Optic Angle be bisected by the Right Line ES, then, FG, perpendicular to ES, is the true Position of the Picture, S is its Center, and ES its Distance; and the farther End, HI, is consequently inclined to the Picture; as it is represented in the next Plate. See Figure the first.

Let AB be the Ground Line, which is not necessarily the bottom of the Picture; but, whatever falls on this Side is supposed projected to the Picture. Pl. XXX. Fig. 1.

Draw AD and BI, perpendicular (the vertical Intersections of the Sides) on which, set up all the measures of the heights (by the Scale of Proportion) for the Dado and Mouldings, the Windows, Chimney Piece, the Entablature, Cove, &c.

On AD, describe, geometrically, the true Profile-Section of all the Mouldings; as DEF for the Cornice, &c. and DG, the Cove, is the fourth part of the Circumference of a Circle, generally.

The Lines in the Cornice (in this Case) not being perpendicular to the Picture, the Section of it, with the Picture, is not the true Profile; the deviation, in this, is inconsiderable. But, when the Lines are more inclined to the Picture, an Expedient for truly proportioning the Mouldings may be necessary.

Let ABC (Pl. 29, Fig. 118) be the Profile of the Cornice to be represented in the Picture. Fig. 118.

The Section with the Picture, being vertical, makes no variation in the heights of the Mouldings; but according to the Inclination of the Picture; their projectures are varied considerably.

From each projecture a, b, c, &c. draw Lines perpendicular to BC, consequently parallel amongst themselves; make the Angle BCD equal to the complement of the inclination of the Cornice to the Picture; CD will be the extreme projecture of the Section of the Cornice, whose Inclination to the Picture is ECD; and, FDC is the true Section, in that position of the Picture; the projecture of each Moulding being taken from CD, where the parallel Lines cut it, at *a*, *b*, *c*, &c.

The Cove must project in the same Ratio to its height; viz. as CD to CB.

On

Pl. XXX. Fig. 1. On the other Side, the heights of the Windows, &c. are set up from B to I.

Having drawn the Horizontal Line, VL, and fixed the Center of the Picture, the Vanishing Point V being determined as usual; which, on account of the inclination of the Sides of the Room, is not in the Center, as is customary in Inside Views; the Room, or Building of any kind, being right angled.

In this Case, these Preliminaries are best determined by a geometrical Plan.

If a Ground Plan of any Building which we intend to delineate be drawn, truly geometrical, the Station may be determined, so, as to see such parts of the Objects as we require, whether internal or external.

Fig. 117. If it be required to see part of the second Window in the Bow; draw a Right Line, from J, through the Angle at L, allowing so much of the recess of the Window, &c. as you wish to represent. It is manifest, that the Station must be somewhere in that Right Line, produced; but if the View was to be central, it would be too near, as at O, for, the Optic Angle, A, or FOB, is too large for the Eye to take in, at one View.

At a further Distance, in the same central Line, at P, if PL be drawn, it is plain; that the Window cannot be seen at all, as it cuts the Pier at M; whereas, any where in the direction of JL, as at E, if the Station be fixed so, that, drawing EF and EB, the Optic Angle does not exceed 50 Degrees, it may be represented from that Station, without much distortion. Then, making EG equal EF, FG is the position of the Picture; and ES, bisecting the Angle FEG, is its Distance;

† 9. 1. El. consequently S is its Center (Def. E) for ES is perpendicular to FG†.

Now, E, is the Station, or Point of View, and FG is the position of the Picture.

Draw the perpendicular ES, and ED parallel to AH and BI; consequently, D, and not S, is the Vanishing Point of those Sides. (Def. L.)

The End, HI, vanishes in a Point where EK, parallel to HI, would cut the Picture, FG, produced; its Distance, from the Center S, will be to SE, as SE

‡ 7. 6. El. is to DS‡. (Prob. 12.) And, if the Angle DEK be bisected, by the Line EN, then, N is the Vanishing Point of a Diagonal, or Mitre Angle.

Now, if AB (Fig. 1, Pl. 30) be equal to FG, then, make VC equal to DS; or, in whatever Ratio AB (Fig. 1) is to FG, so make VC to DS, and V will be the Vanishing Point of the Sides of the Room; that is, of all the horizontal Lines in those Sides, and of all other Lines parallel to them, whether in the Cieling, or on the Floor, or elsewhere. (Theorem 5, Cor. 1.)

Being thus prepared, we now proceed to the delineation.

Pl. XXX. Fig. 1. Draw the indefinite Lines AV and BV, and proportion them, by seting off the measures of the several parts from A or B, and drawing Lines to the Distance Point of the Eye, placed on either Side (at *E*, or E^2) *E*V being half, and VE^2 one third part of the Distance of the Vanishing Point V, there not being room, on the Picture, for the full measure.

Make A a, a b, &c. each one third of the measures of the Originals, and draw aE^2, &c. cuting AV at *a*, &c. which give the Angle of the Chimney, &c.

For its projecture, make A a equal to, or somewhat more than the real measure (because, AB being inclined, it cannot be equal, but is the Diagonal of the Inclination; i. e. as CD to CB, Fig. 118) and draw aV, cuting a Line drawn from *a* to the other Vanishing Point, at *b*. Make a*d* one third part of the front of the Chimney, and draw *d*V, cuting a parallel Line from *b* at *c*.

Divide *bc* in the proportion of the Trusses, &c. and draw Lines to E^2, cuting *b*V in their perspective Proportions.

Draw perpendiculars from *a*, *b*, and *e*; and, from all the Angles of the Cornice, &c. draw Lines to V; and return the Mouldings, at the several Angles of the Chimney, internally, at *gh*, and externally, at *ik* and *lm*, as described in Example 19; also, at *no*, the length being obtained by a Perpendicular from *f*.

To describe every particular would be superfluous and unnecessary, previous Lessons having been frequently and particularly explained.

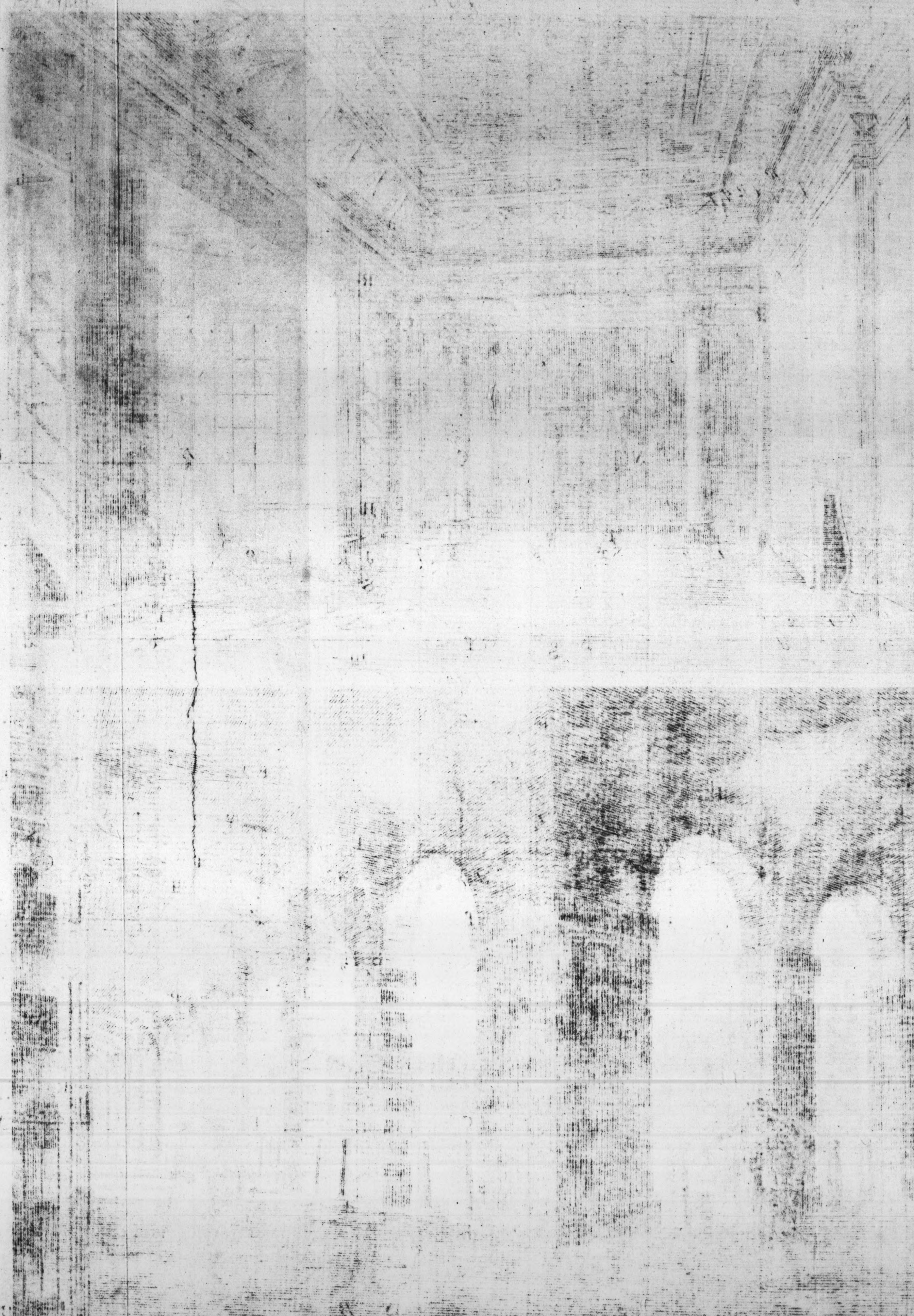
Having

Plate XXX

Fig. 1.

Fig. 2.

Publish'd by T. Malton March 16th 1775.



Having obtained the mitre Angle at *n o*, return the Mouldings of the Entablature, at the End, by means of the Vanishing Point of the End; or by the Expedient in Prob. 13, No. 5.

The Angle of the Cove may be thus obtained, with accuracy.

DG is its geometrical Section with the Picture, and M is the height of the Room. Draw MV, cuting the Perpendicular *f n* at *p*; draw the Diagonal *(p q)* of the Angle, indefinite, and, from G, draw GV, cuting it at *q*.

But, because the Vanishing Point of that Diagonal is not determined, and is out of the Picture; from *p*, draw a Line tending to the Vanishing Point of the End; and, where it is cut by GV, at *r*, make *r s* equal *p r* *, and draw a Line from L, the Vanishing Point of the other Diagonal; which will give the Point *q*. The other Angle is determined by the same Point, L, the Vanishing Point of that Diagonal, found by bisecting the Angle DEK, by EG. (Fig. 117, Pl. 29.)

After the same manner, by drawing several Ordinates to the Curve DG, at *t*, *u*, &c. the perspective Curve at the Angles may be described, and transferred from one Angle to the other.

On the other Side, V*E* being half the distance of the Eye, the measures on the Ground Line, AB, are applied half the real measures, for proportioning the Piers, and the opening of the Bow Window, &c. from B, to e, f, &c. BI is the height of the Windows. Draw I h tending to the Vanishing Point of the end of the Room (as the projecture of the Chimney) and B d being made equal to the recess of the Window, draw dh; regarding the projectures of the Mouldings.

Having obtained the opening of the Bow Window at i k, which is a Segment of a Circle (B e, e f, &c. being in proportion to the distance of the Vanishing Point V, i. e. half) find S the representation of its Center, and draw S i, and S k, which represent Radii; by means of which, other Radii may be obtained, as S l, S m, &c. making certain Angles with S k perspectively (Prob. 10) and thus, not only the Curve (k l m n) but also, the true place of each Window, &c. is acquired.

The Curve at the Top may be described by the same means.

The recedings of the Windows, if they tend to the Center S, are readily determined; by dividing the Perpendicular RS, geometrically, into the several Divisions for the Mouldings, &c. and producing S l, &c. to the Vanishing Line; then, Lines drawn from each Division, on RS, to the respective Vanishing Point of each Jamb, will divide each Window into its several divisions of Mouldings, Sash Squares, &c. and, by the same means, all the Curves, in the Mouldings, &c. at the Top, may be described with the greatest accuracy.

If the Jambs of each Window be parallel between themselves, then, a Right Line, S o, bisecting the Angle l S m, &c. will produce the Vanishing Point of each respectively, and each Jamb must, in that Case, be divided geometrically, on the Angle, into the several Divisions required.

The Entablature may also be proportioned on BK, the same as on AD, on the other Side; which will be more accurate, than to depend entirely on the proportions being carried around, from the other Side; which, as they diminish so much, are liable to error.

In the Columns, at the farther End, there is nothing particular, their places being obtained; which, as the Line they are in is inclined to the Picture, are determined by Prob. 8th, thus.

The Angle *f* at the foot of the Pilaster being obtained, draw *f j* parallel to the Ground Line, which divide, geometrically, in the Proportion required; that is, *f* 1, equal *j* 4, is the Distance of the Column, at the Plinth, from the Wall; and 1 2, equal 3 4, is the width of the Plinth of the Column; from all which, draw Lines to V, cuting the inclined Line, in which the Columns stand, in the perspective proportions of the Columns and Spaces; which may be compleated from the known proportions of the Order, as in former Examples.

* This is not strictly true, *p r s* not being parallel to the Vanishing Line; but, being so little inclined, the deviation is inconsiderable.

Pl. XXX. Fig. 1. If r s be taken, geometrically, the width of the Door, &c. in proportion to the Columns and ſpaces, Lines drawn to V will give the place of the Door, at the farther End, which may be compleated, by Example 20.

The Figure of the Cieling has nothing of difficulty in it, being compoſed of Right Lines, and circular ones, regularly diſpoſed; which, from the geometrical Figure, may eaſily be determined.

If MN be conſidered as the Interſection of the Picture with the Cieling, the Diviſions on MN are geometrically diſpoſed; from which, Lines are drawn to V; and the ſeveral Diviſions, in the length, being found perſpectively, on GV or NV, Lines drawn to the other Vaniſhing Point will cut each Line, drawn to V, in the ratio required; by which, the perſpective Figure is formed on the Picture.

E X A M P L E XLII.

Is the Repreſentation of the inſide of the Piazza, Covent Garden, from the farther corner of the entrance into the Playhouſe.

The Poſition of the Picture being determined, and conſequently, the Inclination of the Lines, in which the Piers ſtand, is known.

Fig. 2. Let C be the Center of the Picture, EV is the Horizontal Line, and V is the Vaniſhing Point of one Side of the Piers, found, or determined at pleaſure; the other is out of the Picture, on the Left, found as uſual; the Diſtance of the Picture is ſix Inches and a half, nearly*.

Let AB be the Interſection of the Picture, i. e. the Ground Line, and, let S be the determined Seat (on the Picture) of the corner of the Pier, on the Ground, its Diſtance from the Picture being known.

Becauſe there is not room, on the Picture, to ſet off its whole Diſtance, take CE half the Diſtance; and make SD half the diſtance of the corner of the Pier; draw DE cuting SC in a, the true place of that Corner, on the Picture.

Draw a V, the indefinite Repreſentation of one Side of the Piers, and a Y, to the other Vaniſhing Point, whoſe Diſtance, from C, is nine inches and three fourths.

Draw a *d* parallel to AB; and having transferred the full meaſure of the Piers and Arches from AB, the Ground Line, to a *d* (by means of the Vaniſhing Point C, or any other) at *b*, *c*, and *d*, draw Lines to E, the Diſtance Point, of V, giving their places, at a b, and c d, &c. the reſt may be obtained to any length, required, by Example 4th. a *d* may be the Ground Line, by a leſs Scale.

On the returning Angle, they are obtained by the ſame meaſures, applied on the other Side, by the Point F the diſtance Point of Y.

Produce V a to the Ground Line, cuting it at A, and draw AH perpendicular, on which, ſet up, from A, the meaſures of the heights of the Piers, &c. at G and I; from which, draw Lines to V. Perpendiculars, from a, b, c, &c. cuting them at e, f, g, &c. give the Piers.

From, a e, the common Interſection, they are returned on the other Side, to the Left.

The circular Arches are all conſtructed by Exam. 21, as the one on the Left; and the Curves over them, by means of Ordinates *a b*, *c d*, &c. their heights being ſet off from e to i and k, &c. the true Curve being determined (ſee Fig. 119) and as many Ordinates drawn as are requiſite, they are returned on both Sides.

The flat Arches and Borders, from the Piers to the Wall may be thus deſcribed.

Produce Y a and Y e, and having found the Point J (Prob. 8.) in the middle of the whole width; from J draw a Perpendicular, cuting Y e and Y k, produced, at l and m; m is the middle Point of the Arch; which might be determined as J, below, by the ſame Problem.

* Let it be obſerved, that the Diſtance, here uſed, is too little; but, being a true Portrait of ſo public a Place, I thought proper to diſpenſe with it; by repreſenting it as it appears, from the Station determined above. See Preliminary Obſervations, Page 117.

Draw

Draw l V and m V; and, from the Vanishing Point Y, produce Lines through g, j, &c. from each Pier at the foot of the Arches, cuting l V at n and p, &c. from which, draw Perpendiculars, cuting m V at o and q, &c. the height of each Arch, respectively; as l m, n o, and p q.

Then, by means of several Ordinates, as *a b*, *c d*, &c. perspectively found, on the Base Line of the Arch, at *a*, *c*, *e*, &c. (see Fig. 119, Pl. 29, for the geometrical construction) their several heights are set off on e k, and projected, by the Vanishing Point Y, cuting Perpendiculars from *a*, *c*, *e*; by which means, half the Curve of the first Arch is described, as e *b d f* m; the other half is determined by the same means, and all the other Arches after the same manner, by drawing Lines from the Seats, *a*, *c*, *e*, &c. of the Ordinates, to V, cuting the Base Line of each Arch, as g n, at 1, 2, 3, &c; from which, Ordinates being drawn, and Right Lines from *b*, *d f*, &c. to V, cuting them at 2, 4, 6, &c. then, through their Intersections, the Curve g 4 o v u is described.

Thus, having obtained all the Arches, or Borders, the Groins are easily determined. The middle Points, r, s, t, &c. are in the Line m V (allowing somewhat for the thickness of the Border) and by means of several horizontal Ordinates, whose Vanishing Point is V, drawn from *b*, *d*, *f*, &c. their lengths, from those Points, as *b g*, *d h*, *f i*, &c. are determined, by the Point *E*, as in all other similar Cases whatever.

Or, their Seats may be easily obtained on the Floor; or at the foot of the Arch, in the Diagonals f u and g x intersecting at z; x V being the indefinite Line, at the foot of the Arches, against the Wall, which is hid by the Column.

The Plan of the Plinth of the Column, being determined, from its known measure and place (by its Seat on the Ground Line, or otherwise) there is nothing singular in its construction; save the Blocks or Rustics (W) which are each equal to a Diameter, in height; the space between is the same; they project equal to the Plinth of the Base. Y and V are the Vanishing Points of the horizontal Lines, in the Cornice, &c. The Picture is supposed to cut the Column, consequently its full measures are applied, on BL or somewhat more, seeing it is projected.

The Piazza, on the Right, is seen to the End; and, through the Arch, at K, is seen the front of the next, on the other Side of James Street.

The distant View of the Church and adjacent Buildings, being so little seen, are best sketched from the place; as it would be attended with unnecessary trouble to find their places, &c. perspectively, from their true geometrical proportions and positions. The Rustics, in the Piers, are a Scale for proportioning them.

E X A M P L E XLIII.

Is the Representation of a Staircase, internal; shewing the descending Stairs, direct. Pl. XXXI.

This Lesson, is calculated, not merely for an Inside View, but in order to shew that a descent is represented by the Rules of Perspective, on the same Principles as any other Subject whatever.

In this Example the Window Side is parallel to the Picture, and consequently, the Sides of the Staircase are perpendicular to it; therefore, the Center of the Picture is the Vanishing Point of horizontal Lines, in the Sides.

C is the Center, and HL the Horizontal Vanishing Line; and VL, the Vertical Line, is the Vanishing Line of the Sides of the Staircase.

The Vanishing Lines of the ascending and descending Planes, i. e. of the Stairs, are determined by Prob. 2, making CE equal to the Distance of the Picture, and the Angle CEF equal to the inclination of the Stairs to the Horizon. Make CG equal CF, and through F and G, draw Lines parallel to the Horizon; one is the Vanishing Line of the ascent, the other of the descent; and because they are, in this Case, parallel, the Vertical Line, VL, is common to them all†, and consequently, F and G are the Centers of those Vanishing Lines, respectively, and their Distance is EF, equal EG. (Theo. 4, and Def. 20.)

† Theo. 7, Cor. 2.

As

Pl. XXIX. Fig. 120. As this is a circumſtance which has occaſioned ſome diſputes amongſt Artiſts, I ſhall diſplay it in the beſt light I can, and doubt not I ſhall do it ſatisfactorily.

In order to which, let Fig. 120 repreſent a Section of the Staircaſe, geometrical, by half the Scale of the Picture; AB is the deſcent, from the landing, at AD, on which the Spectator is ſuppoſed to ſtand, at ED; E is the Eye or Point of View, and EC is the Diſtance of the Picture, of which AH is a ſection. AK is the flight of Stairs, immediately aſcending, as AB is deſcending, and KI is the under ſide of the next flight, over them, parallel to AB.

Now, becauſe the Vaniſhing Point of every original Right Line is where a parallel from the Eye cuts the Picture†, and EC the Direct Radial, produces the Center, C is the Vaniſhing Point of the ſides of the Half pace BL, and of the extremes of the Steps, as may be ſeen in the Picture; alſo, EF, and EG, being parallel reſpectively to the Aſcent, AK, and Deſcent, AB, conſequently, F and G are the Vaniſhing Points of all Lines, on the Picture, parallel to them (as the Hand-Rails, the Wainſcoting at the Sides, &c.) and conſequently, Planes paſſing through the Eye and thoſe Lines, reſpectively, parallel to the Stairs, muſt cut the Picture in the Lines IK and MN paſſing through F and G, reſpectively, and therefore they are their Vaniſhing Lines. (Def. S.)

† Def. 22.

This I preſume is ſufficiently intelligible; and, if Viſual Rays are drawn to the ſeveral Parts of the Staircaſe, they will determine what can be ſeen and what cannot; as EB determines how much the Deſcent riſes on the Picture, which it cuts at b; A b is, therefore, its whole Appearance. The aſcending Flight, parallel to AK, it is evident, is ſeen on the under Side, as the Viſual Ray EM evinces; and conſequently, it deſcends on the Picture.

CE, the Diſtance of the Picture, it muſt be obſerved, is too little, for taking in ſo much as is here repreſented; but it is manifeſt, that, if the Diſtance was a fourth part more, we ſhould not ſee the deſcending Stairs at all, as the Eye would be either in the Plane of the Stairs, or on the other Side.

Pl. XXXI. Let AB be the Interſection of the Picture with the landing of the Stairs, which in this Caſe, is the Ground Line, the Picture being ſuppoſed cloſe to the Stairs; all that lies on this Side is projected to the Picture; as the Door on the Landing, and Cieling, as well as the Floor; and it is obvious, that, whatever appears to deſcend, below the Landing, has its place on the Picture above the Ground Line; conſequently, it riſes on the Picture.

Make BD equal to the known width of the Stairs, the width of the whole, being repreſented by AB; the Station is determined by the Vertical Line, VL.

Draw BG and DG, to the center of the Vaniſhing Line MN; the Originals of thoſe Lines being perpendicular to the Interſection, AB; in which Lines are all the upper edges, or noſeings of the Steps; whoſe places are obtained by Prob. 8th.

Make G*E* equal EG, the Diſtance of the Vaniſhing Line MN, or IK (Def. 20) Make B a, a b, &c. double A a, a b, &c. (Fig 120) and draw a*E*, b*E*, &c. by which means their places *a*, *b*, *c*, &c. are obtained on the Picture; and parallel Lines, *a d*, &c. being drawn, give the edges of the Steps; the Ends, being at right angles with the Picture, vaniſh in its Center; therefore, draw C*a*, C*b*, &c. and produce them to the next Step; and thus the deſcending Stairs are compleated.

The Half paces below, at X, and above, at Y, being horizontal; their Sides, conſequently, vaniſh in the Center, C; alſo, the ends of all the Steps, every where. The width of the Half pace being equal to the Steps, draw *e*C and *f*E cuting *e*C at *g*, which gives the Angle *g b*, at the farther end of the Staircaſe.

The height may be determined on a Perpendicular from B, as BJ, and drawing JC.

The aſcending Flight is managed by Example 9th in every reſpect. The Vaniſhing Point of the Hand Rails, Wainſcoting, &c. of that Flight is at F, as well as of the Lines d e and f g, of the Plane d e f g, being parallel to thoſe below; the other, with the Lines in the Plane Z, is at G.

The height of the Hand Rails, &c. are ſet up on the Interſection BJ, as B h; and from D, as D i, allowing for the ramping Curve, equal i k.

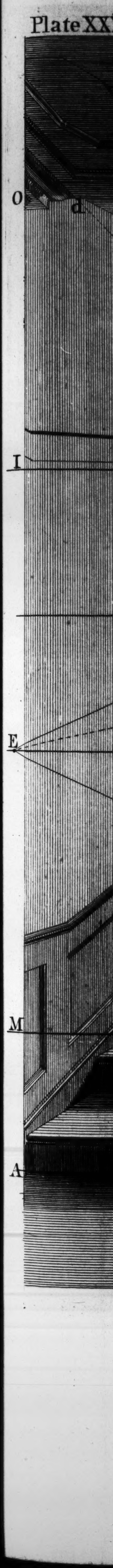


Plate XXXI.

Publish'd by T. Malton March 1st 1775.

Mazell Sculp.

The panneling of the Wainscot is divided in the ratio required, as in any other Case, by Prob. 8th, making use of the Distance EG, not EC, for the inclined part.

The Windows may be proportioned on *g h*, at the farther end, geometrical.

The Cornices, and other Mouldings, are managed by the Lessons in Sect. 7.

The Architrave around the Door, being projected, is thus done.

Make B l, l m, equal, respectively, to its distance from the Steps, and width of the Architrave, and draw E l, E m, projecting those measures to CB, produced, cuting it at *i* and *k*, and giving *i k* for the Architrave.

The Cieling is projected in the same manner; and all beyond BI.

AOPB is the Section of the whole by the Picture; OP is the Intersection with the Ceiling, and AO and BP with the sides of the Staircase; on which Intersections, all the measures are applied, for each Plane respectively.

OF HORIZONTAL PICTURES.

Horizontal Pieces are considered, by many, as performed by Rules different from vertical Pictures; whereas, they are the same, in every respect, being founded on an universal Theory, which regards no particular position of the Picture; but only, the positions of the Planes and Lines in Objects, to the Picture, and to the Eye. For, however the Picture be situated, the Eye being considered as a Point, can have but one position to a Plane; and consequently, its Center and Distance, being determined by a Perpendicular from the Eye to the Plane, are the same in all positions of the Plane, respecting the Horizon, to which we are naturally partial; but, being properly considered, we shall find it of no consequence in the Theory of Perspective.

Let the Picture be situated as it may, all Original Lines perpendicular to the Picture vanish in its Center (Cor. to Th. 6.) Planes, or Lines, being parallel to the Picture, have no Vanishing Line, or Vanishing Point; but, in such Case, the representations of all Figures, in Objects so situated, are similar to the Originals (Th. 10, Cor. 5) also, all Original Lines, however inclined to the Picture, vanish in that Point, where a parallel Line from the Eye would cut the Picture; and consequently, all Lines, which are parallel, amongst themselves, have the same Vanishing Point (Th. 5, Cor. 1.) with every other circumstance in the Theory given, which is quite general, and cannot possibly regard any particular position of the Picture, to the Horizon.

The Representations of most Objects (in common Cases) on a vertical Picture, are managed by means of the Horizontal Vanishing Line, only; but, in many Cases, other Vanishing Lines are requisite.

The Horizontal Plane, which, in other Cases, produces the Horizontal Vanishing Line, is, in this Case, parallel to the Picture, and conseqnently cannot cut it; for it is the Directing Plane, of Pictures in such position. (Def. 4.) The theoretic construction of the elementary Planes is the same, however the Picture be situated. All the difficulty seems to be in fixing the Vanishing Lines, in this Case, having no particular bias, in respect of the Horizon.

As, in all common Cases, the Horizontal Line and Vertical Line cut each other, at right angles, in the Center; so, in this, two Lines cuting each other, perpendicularly, in the Center, answer the same purpose, either of which may be considered as the Horizontal Vanishing Line; for, if the Picture be changed to a vertical position, and, the Objects being supposed changed with it, keeping their position to the Picture, it is then a common Case, in every circumstance; the placing of those Lines, most conveniently, is therefore the principal business.

Now, if the Piece be viewed centrally, it is immaterial how they are drawn, for representing a Dome only; but, if we would represent the Inside of any prismatic Object, whatever, it is most eligible to draw the Vanishing Lines parallel to its Sides, as AB and DE; because they are, then, the Vanishing Lines of those Planes, respectively. But, if the View be not central, then, one of the Lines (DE) Fig. 121.

 generally

Pl XXXII. Fig. 121. generally passes through the center (C) of its Base and divides the Dome equally; which may be considered as the Vertical Line of the Picture; the other (FG) as the Horizontal Line; S being the Center of the Picture, whose Distance is known.

The chief difficulty, in Cieling pieces, is the representing Objects in such unnatural Positions, they being seen on the under sides, instead of looking direct at them, as standing before us, upright, in common Cases; and are represented as if lying horizontal, in respect of a vertical or upright Picture; in which, the difference consists, wholly. The Cielings, Soffits of Doors and Windows, Plancers of Cornices, &c. are, in this Case, parallel to the Picture, as, in other Cases, they are perpendicular to it; being thus reversed, and seldom practised, makes the difficulty greater.

There is one particular circumstance attending Cieling pieces; the Center of the Picture is the only Vanishing Point, generally; not owing to the position of the Picture, but of the Objects to be represented. For the Base of the Dome, or whatever else is represented, is always parallel to the Picture; and consequently, all Lines perpendicular to the Base (being also perpendicular to the Picture) vanish in its Center; and, since the Objects to be represented are always upright, there is no occasion for any other Vanishing Point; as, all Right Lines, in the Objects represented, are either parallel or perpendicular to the Picture. Nor can it be otherwise, unless represented on an inclined Cieling; in which Case, there may be as much variety of Vanishing Lines and Points, as in any other.

These Preliminaries being well considered and digested the difficulty will vanish, and leave the whole, subject to the same common Rules already laid down; for there are no other used, or necessary.

EXAMPLE XLIV.

How to represent on a Cieling, or horizontal Picture, the representation of a Gallery of Communication; with a Dome and Cupola, as seen from below.

Fig. 122. ABCD is a geometrical half Plan of the Gallery, &c. to be represented, which is a Square, suppose of 30 feet; and BFGHC is a Section of the same. The height, BF, to the Cove, is 18 feet, the Cove 3 feet, and the Base of the Dome (which is a Hemisphere) is 24 feet. Through the Arches, in the middle of three Sides, Galleries are supposed to communicate to various Appartments, above.

The Base of the Gallery being a Square, and the Picture horizontal, consequently parallel to its Base, its representation is therefore a Square.

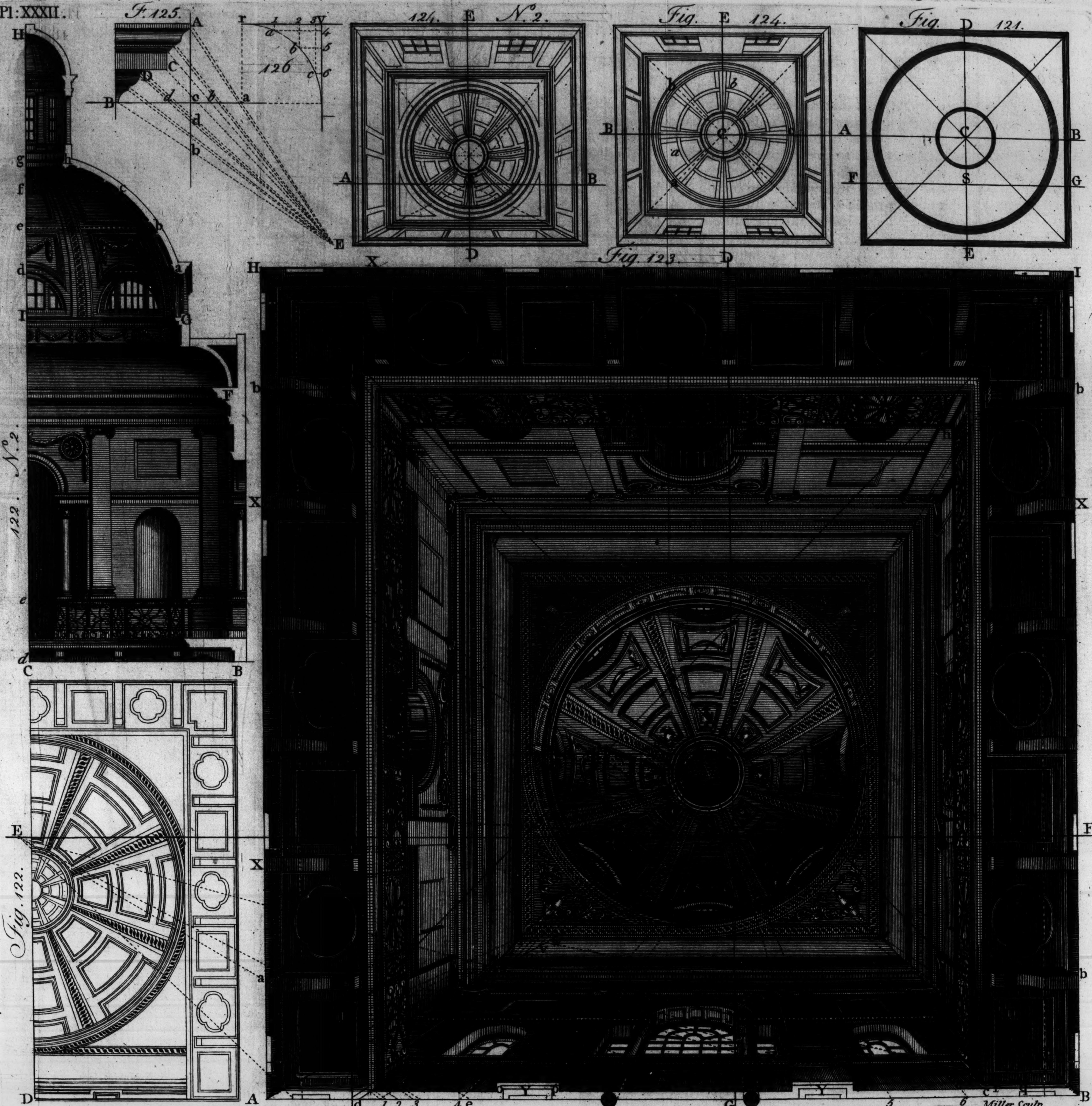
Fig. 123. Describe a Square, AHIB, of the dimensions you intend the Picture, which is here, double that of the Plan.

The Station being determined, and the Distance of the Picture, which, in this Case, cannot admit of much variation, from the true, to any Eye which views it. For, I suppose, every Cieling Piece is calculated to be seen from some particular Station, below; and every Person who would see the Piece, truly, must stand in the same place; consequently, the difference, in Distance, is equal to the different heights of the Spectators.

Suppose the Hall, or Saloon, to be a Cube of 30 feet; then, allowing 5 feet for the height of the Eye, its Distance from the Cieling is 25 feet, which is the Distance of the Picture; upon which, is to be represented, a continuation of the Walls, &c. with the Entablature, from B to F, near 20 feet, with the Cove, Dome, and Cupola, covering the whole.

Now, the Distance of the Picture is 25 feet, which is not equal to its width, but cannot, here, be more, unless the Spectator be supposed to sit or lie down. Then, as, in common Cases, the height of the Eye is next to be determined, so, in this, the place of the Eye in respect of the Picture; which, I would never advise to be in the middle of the Piece, as it cannot produce an agreeable Representation; seeing that, the longitudinal Lines in the Dome (which in that Case are represented by Right Lines) will be but so many Radii of a Circle tending to its Center,

Pl: XXXII



Publish'd by T. Malton March 1st 1776. Miller Sculp.

Center, as a C, b C, &c. (Fig. 124) and, the latitudinal Circles will be so many concentric Circles, as a b c, *a b c*, &c. also, every Side of the Room will advance equally towards the Center.

Therefore, let the Station be determined towards one Side or another, where the Cieling may be seen most advantageously; nor is it necessary to be situated. centrally, in that Side; although it is a deviation from my general Maxim, of fixing the Center of View in the middle of the Picture; but, since the position of the Picture cannot, in this Case, be varied, I shall rather dispense with it than have duplicate Representations.

Let C, then, be the Center of View; somewhat to the Right hand. Let AB be considered as the Ground Line; and if, through C, a Right Line, EF, be drawn, parallel to AB, it may be considered as the Horizontal Line of the Picture; and DG, perpendicular to AB, also passing through C, is the Vertical Line, i. e. they are the Vanishing Lines of the Sides of the Square Room respectively.

The Galleries, on three Sides, which are supported by Trusses from the Walls, are first to be represented; which I have made to project but three feet, as they would hide the Walls too much if they projected more.

Now, since the Picture is parallel to the Trusses, and they are spaced equally, divide the three Sides AH, HI, and BI, geometrically, into as many spaces and Trusses as are to be represented, at X, X, and describe their Seats on the Picture, truly geometrical. Then, the Lines which measure their thickness or height, being perpendicular to their Bases, and to the Picture, vanish in its Center, as in all common Cases whatever.

Make CE equal to the Distance of the Picture, in the Vanishing Line EF; draw their indefinite Representations, as a C, b C, &c. and proportion them in height, making a *a* equal to the height, and draw *a* E, cuting a C at *b*; which, by drawing parallel Lines, to AH, HI, &c. will determine them all around.

The Cieling of the Gallery, between each Truss, is consequently determined. The under side of the Truss is like an Ionic Modilion, having, also, a Cimma reversa, around the top; which, the Figure sufficiently describes.

The Cornice of the Gallery is described as any other; save only, that the Facias, or upright Fillets, are contracted, in this Case, as the under Sides, or Planceers, in common; the difference is owing to the position of the Picture only, the appearance is the same; as it is illustrated in Fig. 125.

Let AB be the Profile of a Cornice, seen from the Point E, by means of the Visual Rays AE, BE, &c. If the Rays are cut by a vertical Plane (Ab) the representations of the several Angles, at C, D, &c. are where the Rays are cut by the Plane, at c, d, &c. and a horizontal Plane, B a, cuts the same Rays, at a, *c*, *d*, &c. It is manifest, that the proportions of the Members, on those Planes or Pictures, differ considerably, yet it is obvious, that each appears the same, in the Point of View, E; the difference, in the Representations, arising from the different Sections of the Rays, as in any other Case.

For the Iron work, which is in the Plane of the Facia of the Cornice, from each internal Angle, k, draw k C. Make A d equal to its width, and d e equal to its known height, from the Cieling (equal twice *d e*, Fig. 122) and draw e E, cuting dC at f; draw f g parallel to AH, and g h parallel to HI; also f i, parallel to AB, cuting c C, where it joins the Wall, on the other Side.

The divisions and proportions of the Railing are determined, geometrically, on d k, &c. at *c d*, &c. from which, Lines are drawn to C.

The Gallery, with the Ballustrade, being compleated, we next proceed with the decorations of the Walls, seen beyond or over it.

Having drawn, geometrically, the Plans, or Seats of the Pilasters, with all the projectures of the mouldings of the Pedestal, &c. at Y, Y; from each exterior

* d k is the Intersection of the Plane d f g k of the Ballustrade, distant from A H, equal A d, i. e. equal to its known distance from the Walls; consequently, d is the Intersecting Point of that Angle. For, because the Picture is parallel to the Base of the whole, the section of every Plane with it, being perpendicular, is geometrically determined; therefore, the Intersecting Point of every Perpendicular Line in the Objects, as d or k of the Angles of the Ballustrade, is readily found, its distance from each Wall being known, or determined; the whole being a true geometrical Plan.

Angle, draw Lines to C, as *j* C, &c. and from l, set off, first, the height to the Plinth; then, the measures of the Plinth, Mouldings, Dado, &c. of the Pedestal, at m, n, &c. and draw m E, n E, &c. cuting l C in their several perspective Proportions, at *e*, *f*, &c. from each of which draw Lines parallel to their Seat, l j, cuting j C, and return them at right angles. From *f*, draw parallel to the Ground Line, cuting p C, which gives the upper Line of the Dado; the Mouldings are determined as usual.

The Base Moulding is hid by the projecture of the Plinth, and also the Bases of the Pilasters, on this Side, being greatly foreshortened; which, where they can be seen, are managed after the same manner.

From o, p, &c. draw Lines to C; make p q equal to the height of the Capital, from the Cieling, and draw q E, cuting p C, at *i*, and compleat the square of the Abacus; which being parallel to the Picture, the curviture of it is geometrical.

The Mouldings of the Capital are managed as usual; their measures of height, being obtained, from p q, at r, &c. and their projectures from the Plans below, drawing Lines from each, to C.

The Volute may be determined, from its Plan, below, in respect of its projectures; but to describe it, particularly, would baffle all description.

The Windows, in this Side, and Niches, &c. in the others, are determined geometrically, on AB, &c. the perpendicular Lines of which, vanish in C, and the circular heads are semi-Ellipses, described, by Prob. 3, Sect. 8.

The other Sides are managed after the same manner, from Plans below.

For the Entablature. Let M, N, O, and P, be the Seats of its projecture, on the Picture. Draw MC, NC, &c. Make MQ equal to its height from the Picture, and draw QE, cuting MC at *n*; draw *no*, *op*, &c. parallel, respectively, to the Sides of the Picture, *n o p q* is the extreme projecture of the Cornice, around.

Make QR, &c. equal to the measures of the Cornice, Frieze, and Architrave, with their several Mouldings; and draw RE, &c. giving them perspectively, on MC. Their projectures are determined on the Diagonal, AM, geometrical; or, on Diagonals from *n*, *o*, &c. parallel to AM, &c. *m n*, &c. being divided in the same ratio, as AM, (Cor 2. Th. 10.)

The Square of the Cieling above the Cove is thus determined.

From T and U, the Seats of the projecture of the Cove (in the Diagonals AI and BH) draw TC, and UC; and, TV being made equal to its height, draw VE, cuting TC at *r*; draw *r s* parallel to the Cornice, and compleat the Square *r s t u*; in which, inscribe a Circle; and, concentric with it, another, leaving a border around the Dome; and also, compleat the same within the Square *r s t u*.

For, because the Cieling is parallel to the Picture, its perspective form is truly geometrical; consequently, one Side being perspectively found, the rest are also determined, in the ratio of *r s* to the Original.

The Angles of the Cove are determined by Ordinates, on the Diagonals *r v*, &c. at 1, 2, &c. geometrically proportioned, alike on each; and others, on AC, &c. perspectively, at 4, 5, &c. from which, Lines drawn parallel to the Diagonals, and others cuting them, from C, drawn through 1, 2, &c. will give Points in the Curve, perspectively, at each Angle. (See the geometrical construction, Fig. 126, which determines three Points, *a*, *b*, and *c*, in the Curve, equally spaced.)

To represent the circular Cornice, at the foot of the Dome.

The Frieze or Facia, below the Cornice, is perfectly cylindrical, its height is determined on AC, which represents a Perpendicular from that Angle; on which, every measure of height may be transferred from AB, geometrical, as s, t, u, &c. of the several divisions in the Dome.

From S, the Center of the Base, draw SC, in which Line are the Centers of every Circle in the Dome, &c. for, the representations of all Circles, in Planes which are parallel to the Picture, are Circles; and, because SC represents a Line perpendicular to the Base, in its Center, SC is the Axis of the Dome, and passes through the Center of every Circle in it; which being determined, on SC, or

transferred

transfered from AC, by lines parallel to the Diagonal AI, as r 1, s 2, &c. draw lines through 1, 2, &c. parallel to AB.

Now v x is the Diameter of the Base of the Dome, cuting the Axis at o; and 1, 2, &c. are the representations of the several Centers, of Circles in the Cornice.

From v, set off the projectures of the Cornice, in proportion to the Radius v o, at *a*, &c. from which draw Lines to C, cuting parallel Lines from 1, 2, &c. which give the Radius of each Circle in the Cornice, as 1 *b*, 2 *c*, &c. by which the Cornice may be compleated; being composed, wholly, of Circles.

The parallel Circles in the Dome are determined after the same manner; which are necessary, for describing the longitudinal Borders.

In Fig. 122, draw as many Ordinates parallel to its Base, GI, as are necessary, from the several points a, b, c, equally spaced in the Curve, to the Axis, or central line CH, cuting it at d, e, f. Then, make o *a*, o *d*, and o *e* in the same ratio to o v, as a d, &c. to the Radius GI; from which draw lines to C, as *a* C, *d* C, *e* C; and, from the perspective Centers, 3, 4, and 5, obtained as above, draw parallel lines cuting them at *f*, *g*, and *h*, which give the Radius of each Circle in its true place, viz. *f* 3, *g* 4, and *h* 5, which, last, is the Base of the Cupola. No. 2.

Divide the Base of the Dome into eight equal parts, at a, b, c, &c. so that, a h and d e are parallel to AB, &c. and set off the widths of the Borders regularly from those Points (half on each Side) draw o a, o b, &c. and, from the Centers 3, 4, 5, draw lines parallel to o a, o b, &c. cuting their respective Circumferences at i, k, l, m, &c. through which, Curves being traced, by a careful hand, they are the central Lines of each Border. Their widths may also be set off at each Circle, in proportion to its Radius, as at the Base, by which they are compleated.

The Compartments are done after the same manner, from those central Lines, respecting the Margins and Mouldings which are longitudinal; the parallel Mouldings are in Circles parallel to the Base.

The semicircular Compartments, at the Bottom, may be truly determined by several parallel Ordinates, geometrically divided at the Base, and parallel Circles cuting them, as at *j i k l m*. The Margin about it may be done by the same means.

The Base of the Cupola being obtained, as above, the Windows in it, are geometrically divided at the Base, and Lines drawn to the Center.

Their apparent height may be obtained on SC or AC, as the other parts.

Thus have I described the whole process of this Cieling Piece, from the beginning to the End; to say more concerning it would be superfluous, as it must be obvious, that the Rules made use of are the same as have been applied in all other Cases, and Subjects whatever.

SECTION XI.

Is applied to Furniture, Wheel Carriages, Machines, &c.

THE foregoing Sections contain the Elements and Rules for the practice of Perspective, in all common Cases; I have also shewn how to apply them, in familiar Lessons and Subjects, adapted to any capacity, in the 6th, 7th, 8th, and 9th Sections; and, in the last, to particular Subjects, and in a particular situation of the Picture; so that, there remains little more to be done in respect of the application of it, to all useful Subjects, almost whatever.

In this Section, which relates to Objects in particular Professions, yet useful to Artists, I shall only make some cursory remarks, in respect of the representations of such Objects as are here treated on. The former Lessons, being well understood, will

Plate XXXIII will be found applicable to whatever Subject the Artist chuses, or has occasion for. To multiply Cases and Subjects, in which the application of the Rules of Perspective may have the appearance of something particular or singular, would be endless, as, in almost every different Object, there is occasion for some variety in the application of its Rules; which, nevertheless, it is obvious, are all founded on the same universal Principles, though the Rules deduced from them are variously applied.

It is a necessary requisite, in many Professions, to be able to give a Design, to a Gentleman, of whatever may be wanted, out of the common run of things; and though a perfect Drawing may not be required, yet to give a slight sketch with propriety is certainly an Accomplishment which cannot well be dispensed with. In an Architect it is absolutely necessary, to give a correct Design, but they are generally contented with geometrical Representations, which does not give the true appearance and effect; that can only be done perspectively. Geometrical Drawings to a Workman are necessary; but to give a true Idea how the Object will appear, when executed from any particular Station, requires somewhat more; to give such an Idea, without knowing something of Perspective, requires a much greater share of genius than falls to the lot of many.

There are few Persons, who have not had a good deal of Practice, in Perspective, know any thing more of it, than the application to Objects, whose Planes are parallel and perpendicular to the Picture, and consequently, have seldom occasion for any other Vanishing Point than the Center of the Picture; they always, therefore, adapt every thing to that position, without thinking about the propriety of it. Some cannot think a Drawing is Perspective, unless an End of the Object is represented; for which reason, we seldom see any Object delineated in Perspective without having an End seen, which, in an Object of a tolerable length, is absurd to represent, the Front being parallel.

E X A M P L E XLV.

Suppose it be required to make a Drawing of a large Library Book-Case.

Fig. 127. In order to give an Idea of the real proportion and figure of such an Object, to a Person, who, perhaps, knows not what Perspective means, and who has no other Idea of its proportion and form, than what the Figure really exhibits, it is most proper to give it parallel, as it is here represented, but by no means to shew an End. The Station is central, and consequently, the Point of View is in the middle. The receding parts, and breaking of the Mouldings around them, sufficiently indicate that it is perspectively delineated; and by reason of its regular Position, I am of opinion that it has a much more natural Appearance, than when viewed oblique, from either End, which always occasions Distortion; as may be seen in Plate 23, Fig. 108.

In respect of the Delineation of it, I shall only observe, that the Plinths of the middle part and the two Ends are in one Plane, which are geometrically proportioned, by the Scale. The parts which recede are determined as usual; for the upper part, proceed thus.

At either Corner, as A, of the Plinth, draw the perpendicular AG; on which, set up all the measures of its height, as AB for the Dado part, with its Base and Sirbase; BD the height of the upper Doors, &c. and DF, FG, of the Cornice, and other decorations, at the Top.

To the Center, C, draw Lines from each division; make B c equal to the receding of the upper part; or, because the whole Distance cannot be on the Picture, take CE half its Distance, and B *a* half B c, and draw *a* E, cuting BC at b.

Draw b d perpendicular, cuting the Lines drawn from B, D, &c. to C, which gives the height of the Doors, &c. perspectively, according to their distance from the Front. For the widths, make B *a*, equal to the projecture of the Mouldings, and *ab* to the width of the Door, and draw *a* C and *b* C cuting a Line drawn from b, parallel to the Base.

The

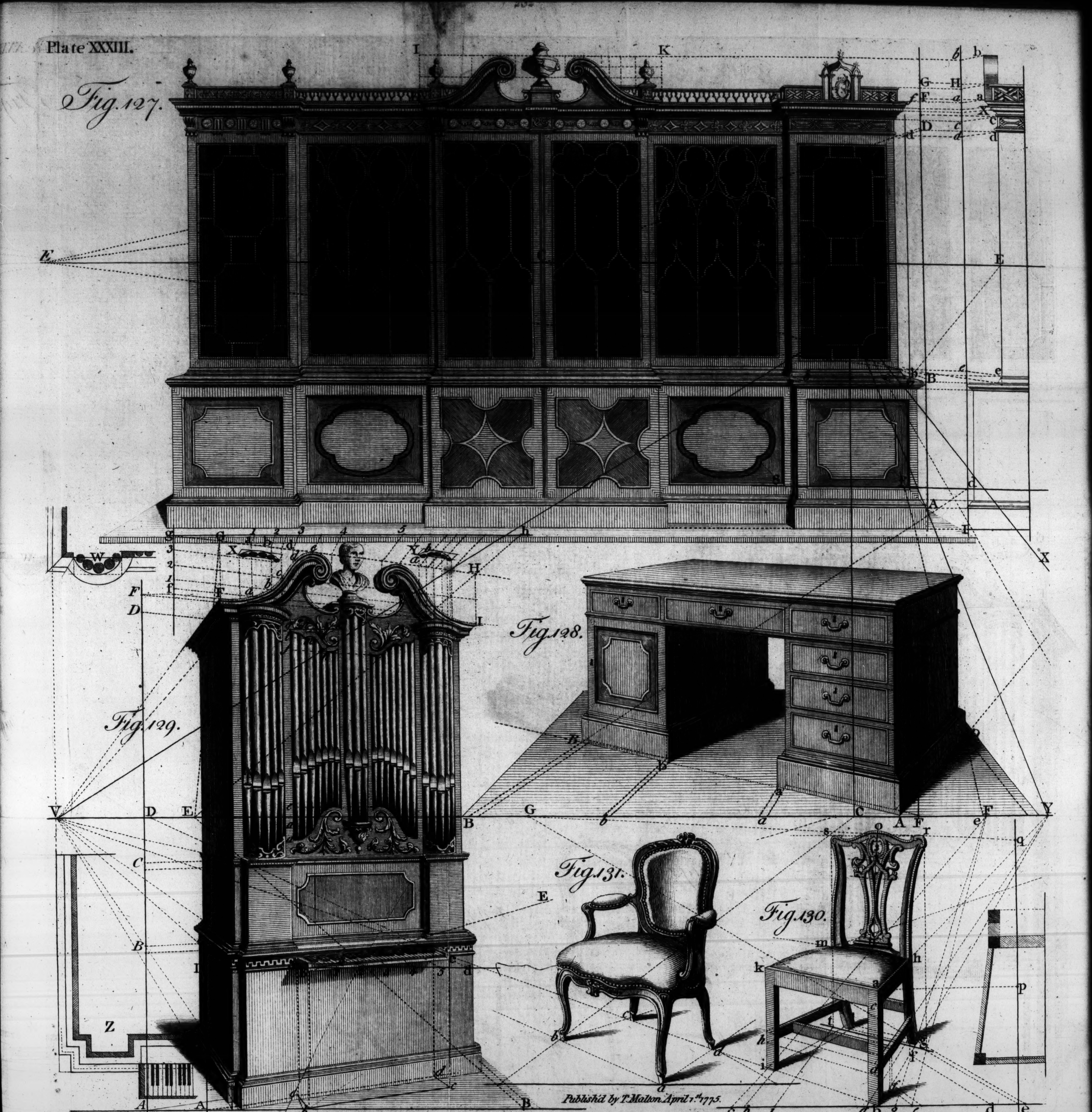


Plate XXXIII.
Fig. 127.
Fig. 128.
Fig. 129.
Fig. 130.
Fig. 131.
Publish'd by T. Malton, April 1st 1775.

The other End and middle Door are obtained by the same means. Or, being all in the same Plane, if either the height or width of one be obtained in its place, the whole is determined; for, being parallel to the Picture, the Doors are all represented similar to their Originals; that is, geometrical.

For the Cornice; if CE, on each Side, be made equal to the Distance of the Picture; they are the Vanishing Points of the Diagonals, which are drawn through the Angles at the Top, *e*, *f*, &c. obtained from the Perpendicular at F, transfered to f. The general method, in Example 15, for Mouldings parallel to the Picture, is applicable here; X, being the Profile, and E, the Point of View.

The Figure of the Outline of the Pediment is geometrical; which certainly gives the best Idea of its true Figure.

If it be required to see an End of any Object of which a Design is to be made, the Front should then be inclined to the Picture, as in Fig. 128; which exhibits a representation of a Library Table, by a larger Scale than the foregoing Figure; for however prepossesed many Artists are, in the parallel position of the Picture, I am confident, that, were they practised in inclined Positions, and saw clearly into Perspective, they would not retain their prejudice long. Whenever a single Object is the subject of a Picture, being right angled, and seen oblique, it is most palpably absurd to be placed parallel to either Side; or in any Case, to have the Center of View out of the middle of the Picture. Fig. 128.

The delineating this Object has nothing in it particular. The Inclination of it to the Picture and the Distance being determined, the Vanishing Points are determined (Prob. 2 and 4) both which are, in this Piece, out of the Picture.

S is the Center, and d the Distance Point, by which the Front is proportioned; the measures being applied on the Ground Line, AB, as usual, at *a* and *b*, for the Front; which give a and b, on A*B*, the line of the Plinth, on the Floor, tending to its Vanishing Point.

As there is not room on the Ground Line to set off the full measure of the End, take Ae a third part, and let f be in proportion, to the Distance of the Vanishing Point of the Lines in the End; then, ef cuts A*D* in the perspective length required.

From each Angle, A, *B*, and *D*, Perpendiculars are drawn, and the real measures of the heights of the Drawers, &c. being applied on EF, where the Seat of the Plane they are in, on the Floor, cuts the Intersection, lines drawn to the Vanishing Point of the front lines, give their perspective heights.

Perpendiculars from a and b determine the opening; and if, from b, a line be drawn to the Vanishing Point of the End, and another from *D*, to the other, of the Front, cuting the former, at c, the perspective width of the inside End is determined. The Mouldings are done as usual.

E X A M P L E XLVI.

Is the representation of a large Chamber-Organ. Fig. 129.

It would be downright tautology to repeat over again the method of proceeding, in obtaining the general proportions, which are subject to one general Rule (Prob. 8) the measures, or their ratio to each other in proportion to the Distance used, being always applied on the Intersection of whatever Plane the Lines are in, or on a Line parallel to it; as on AB, for the Piers, &c. I shall, therefore, pass over that description, and only explain the Parts, where there is any apparent difficulty.

The Finger-Board, ac, is usually made to draw out, the length of the Keys, about 6 Inches. Its place being acquired, and V being the Vanishing Point of the End, draw Va, and produce it; draw ae parallel to the Ground Line, make ae represent 6 Inches, in proportion to the length, and draw Ee, till it cuts Va produced, at b; and, from b, draw bc to the other Vanishing Point, of the Front, by the Expedients, Problem 13.

Plate XXXIII Fig. 129. Draw b d parallel; and, on b d, set off, from b, as many divisions as there are Keys; join d c and produce it to the Vanishing Line cuting it at D; to which Point, draw Lines from all the divisions 1, 2, &c. which give the Keys.

The Mouldings, breaking about the Piers, &c. have nothing singular in them. I shall next shew how the Pipes, &c. are delineated, which are the principal business.

The Column of Pipes, at each End, are Segments of Circles containing five Pipes each, which are unequal; let their Perspective Plans be formed above, at X and Y; from their extreme Edges, *a*, *b*, &c. draw perpendiculars, as shewn by the dotted Lines, giving the places, below, of the cylindrical Part.

The Mouth of each Pipe, at f, &c. is at about one fourth part of their height, from which it is conical, inverted. The Ornaments, at the tops and bottoms, are regular, from the middle Pipe, which it would be impossible to describe; their Figure being known, such minutias are drawn by the Eye.

The Pediment, and middle compartments of Pipes, being inclined, it may be necessary to shew how they are described perspectively.

Let the geometrical Figure of the Pediment be the same as in the Book Case (Fig. 128) which, inscribe in a Rectangle, and draw Ordinates parallel to both Sides, from various points in the Curve, as in that Figure.

The Cornice, at the End, being described, as usual, and the Angle F determined, draw FI to the Vanishing Point of the Front; and make FI represent the Distance of the two extreme Angles; at each of which, draw Perpendiculars.

Make FG equal to the height of the Pediment in proportion to the whole height, which divide in the same Ratio as HI (Fig. 128) from all which, draw lines to the Vanishing Point of the Front; and having divided GH perspectively, as IK (Fig. 128) perpendiculars drawn from each division, cuting the other, give the Points *a*, *b*, *c*, &c. through which the perspective curve is described.

The middle Column, or Pilaster of Pipes, projects somewhat from the Face.

If the geometrical figure of the two Sides be also determined; then, on *d g*, take the divisions *e*, *f*, &c. geometrical, and draw lines to the Vanishing Point; let the horizontal Line *g k* be divided perspectively, at *h*, *i*, &c. from which, perpendiculars being drawn, cuting the other at *m*, *n*, &c. give several Points in the Figure. The rest may be drawn, accurately enough, by hand.

The proportion of the Pipes may be determined on any horizontal Line crossing them, as *g k*, either above or below as is most convenient; dividing it perspectively, in the ratio of the Pipes (by Prob. 8) as they are all in the same Plane.

The mitre Angle, of the streight Mouldings with the circular, must be determined by a geometrical Plan, which varies according to what portion of the Circle is taken; also, where it falls against the Plane, their projectures may vary considerably; in a Semicircle they have their true projecture.

It is unnecessary to give more examples of Furniture, seeing that, if the Rules of Perspective, here used, be clearly understood, the Examples, given, are sufficient for any Person who is clear in the Principles on which they are founded; and may, by such be applied in any Case, whatever, in right lined Objects. Nevertheless, as Chairs, from the various directions of the Lines in them, are somewhat difficult to delineate, I shall give one specimen, how far the Rules of Perspective are applicable to such Objects.

EXAMPLE XLVII.

Fig. 130. *How to represent a Chair, perspectively.*

The position, and inclination of its Front to the Picture being determined, and the representation of the hither Angle, a b, of its Foot, given at discretion, to the scale of the Design you intend.

Let V be the Vanishing Point of horizontal Lines, in the front of the Chair. C is the Center of the Picture, and CE its Distance.

Let

Draw EV, and make the angle VEY equal to the inclination of the Side to the Front, giving Y for the Vanishing Point of the Rails, in that Side; also, make the Angle YEX, equal to the inclination of one Side to the other; giving the Vanishing Point of the other side Rails, &c.

Then, draw a V, and b V, a Y, and b Y; and c d, through b, parallel to the Vanishing Line.

Make b c, equal to the width of the Front, in proportion to the height.

Make VF equal VE, and draw c F, giving the place of the other front foot, at i; also, the breadth of each are set off at b *a* and *b* c.

Let b d be equal to the side of the Chair, at its Seat, and d e to the splay of the back foot; make YG equal YE, and draw d G and e G, cuting b Y at f and g; g is the place of the foot on the Floor; and if a perpendicular be drawn, at f, it will cut a Y in the true width at the Seat.

Make a *c* equal to the width of the seat Rails, and b *d* to the height and width of the low Rail; draw *c* V; and *c* Y, cuting the Perpendicular from f; also, draw *d* Y for the low Rail, to the back foot.

From g and h draw Lines to V, and from i and k to Y, cuting them at l and m, which gives the place of the other back foot, and compleats the Seat.

Make b *e*, equal to the recess of the inner Rail from the Front; draw *e* G cuting b Y at *f*; from which, draw a perpendicular, giving its place on the low side Rail, at *g*; and from *g*, draw to V, cuting the other Side Rail, the measure of which was transfered from *d* to the other foot.

The height of the low back Rail is obtained on the front Foot; and transfered to the Back, by means of the Vanishing Point Y.

Thus much is practicable, with the greatest accuracy; but beyond this, little can be done, the whole Back being curved every way.

The height may be truly determined, by means of perpendiculars, from the floor; and the various proportions of the Banister, may, in some degree, be obtained, by dividing a line (n o) drawn through its middle, into the several heights, 1, 2, 3, &c. through which, Lines drawn to the Vanishing Point, V, determine the raking of the several parts, as in the Figure. For, whether the Back of the Chair be streight or curved, it deviates so little from a Right Line, and from a Perpendicular, that the measures may be applied, geometrically, without any sensible error.

In respect of the modern Chairs, now in use, there is no such thing as applying the rules of Perspective to them, except to ascertain the place of the Feet, on the Floor; also, by inscribing the Seat in a Square, or other Rectangle, its Figure may be nearly determined in its true place. Fig. 131.

The height of the Back, and of the Elbows may also be had, but, as the whole Chair is composed of irregular curved Lines, it is absolutely impossible to give its true figure by the Rules of Perspective; such Objects, must therefore depend on the Hand and Eye, for a representation of them; in which, a Person who has judgment in Drawing can only succeed; but, being well versed in Perspective, he will find great assistance from it.

WHEEL CARRIAGES are the next Subject to be handled; on which, as they are frequently necessary to introduce into a Picture, or for the sake of giving a Design to a Gentleman, by those whose profession it is to make them, I shall make a few Observa ions; but, like many other Objects, not composed of Planes and Right Lines, little can be ascertained or done by the rules of Perspective, further than to give the proportion of one part to another, which may be done with tolerable accuracy. Nevertheless, I am conscious, that few Persons who have occasion to delineate them (except the makers) will take the necessary pains to project them by rule; for such as will, I shall lie down some which are practical.

N. B. It must have been observed, that to delineate any Object by the Rules of Perspective, its true geometrical form and proportion of one part to another must be known, or determined on; consequently, the more complicated the Object is, being composed of irregular curved Surfaces, the greater is the difficulty in applying the measures on the Picture; which, as the several parts to be delineated are not Planes, or plane Figures, their places can only be obtained, on the Picture, from their Seats on some Plane or other, to which they are contiguous. By which means, a sufficient number of Points, in any curved Line in the Object, being perspectively found, and joined carefully by hand, the perspective Figure is determined.

Plate XXXIV. Fig. 132.

EXAMPLE XLVIII.

How to represent a Coach in perspective.

No. 1. Let ABCD be a geometrical Plan of the Carriage of a Coach; AB is the Axle of the fore Wheels, and CD of the hind Wheels, whose Diameters are a b and d e. EFGH is the Plan of the Top of the Coach, all the other Parts are in proportion to them; as in the Figure.

In order to see the fore part better, a little inclination is given to the position of it, to the Picture; of which, IK may be supposed the Section, or Ground Line.

The Vanishing Point, v, of the Axles, &c. being determined, according to the Distance and Station, which is at s; the other is out of the Picture, its distance from c (the Center) is to c s, as c s to c v. (Prob. 12.)

These Preliminaries being settled, as usual, let C be the Center of the Picture, and V, the Vanishing Point, by a Scale of 2 to 1; AB is the Ground Line; and D, the intersecting Point of the line of direction of the Wheels, on the Ground, which are, here, supposed to be in the same Plane.

The Plane of the Wheels not being vertical, but somewhat inclined, from the Carriage, let DG be the Intersection of the Wheels, with the Picture, on which, make DF equal to the Diameter of the fore Wheel, and DG of the hind Wheel.

Describe the representations of Circles in that Plane, a b c d and e f g h, on the Diameters DF and DG, according to their distance from each other, making DH equal to the Distance of the Center of the hind Wheel, from the Intersection, and drawing EH, cuting D e at e; e g parallel to DG, is the Diameter of the Wheel, perspectively obtained, in its true place; from which, the circumference of the Wheel, e f g h, is described (by Prob. 3, Sect. 8.)

The end of the Nave is not in the plane of the Wheel, but somewhat beyond it, at i and k; also, the Spokes dish inwardly, from the front of the Wheel. Find the circumference of the Naves, *a b c d* and *e f g h*, in their true places, perspectively, at the part where the spokes are fixed in it, and the place of each; also let each Rim be divided, perspectively, into equal parts, the number of Spokes, at 1, 2, 3, &c. (by inscribing a Poligon) from which they are drawn to their corresponding parts in the Nave; for, as they are not in the plane of the Rim, their Vanishing Points are not easily determined.

The hither Wheels being compleated, the Body of the Coach is next to be determined; which, as the Sides are not in one Plane each, their Intersections are not easily determined. Therefore, from the Seat, on the Ground, of the hither Angle of the Coach, perspectively determined, at i (from the Plan) draw i k perpendicular; in which, by means of the Vanishing Point, V, of horizontal Lines in the Front, determine the Angle k; the true height being IK, in the Intersection of the Front. The other Angles, m and n, may also be determined from their Seats; or, by their known distances from k, as in other common Cases.

If the corners of the Coach are perpendicular, as in some Coaches, the measures may be determined on i k, geometrical; but if they are inclined, the point l may be obtained as k, from its Seat; and from l lines are drawn to the Vanishing Points, and proportioned as usual.

The Door is not in the same Plane, with the Line k n, but may be determined from the Seat of the hither Line, o p, where it opens; and, on any perpendicular line, cuting the Ground Line, as IK, set up from I, its true measures, of height, at *a*, *b*, and *c*; then, from I, draw a Line through its Seat (s) to the Vanishing Line, cuting it at L; draw *a* L, *b* L, and *c* L, cuting the Perpendicular, from s, at o, p, and q; from which, draw Lines to the Vanishing Point of the Side.

The Curve of the Bottom being geometrically determined, it may be done perspectively, by means of Ordinates, as in foregoing Examples; and, from the Vanishing Point V, draw a Tangent to the hither Curve, at r, which also touches the other.

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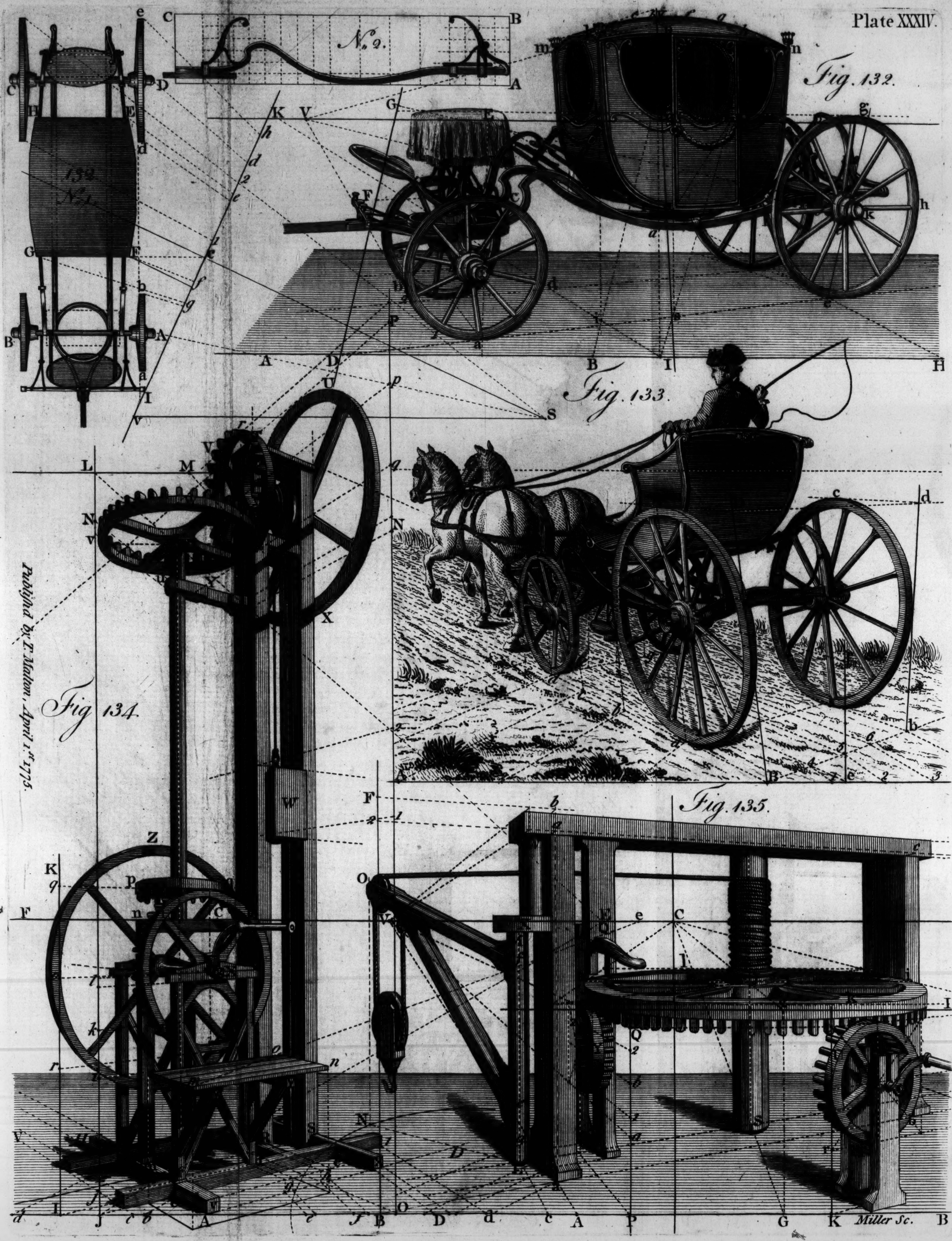
Plate XXXIV.
No. 2.
Fig. 132.
Fig. 133.
Fig 134.
Fig. 135.
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The contour of the Top may be determined, by several vertical Sections described, perspectively (as *dep*, and *fgh* at the Door) then, a Curve described over them all would be the true Contour.

The Axles being parallel to the front of the Coach, their Vanishing Point is V; their thickness may be proportioned to the diameter of the Wheel, and their Shape by means of Ordinates, at 1, 2, 3, &c.

The true geometrical Figure of the Crane Neck being described (No. 2) then, by means of squares (the whole being inscribed in a Rectangle, ABCD) it may be accurately drawn in Perspective; with the other appendages of Springs, &c. by the same means.

The height and true place of the Seat may be easily determined from its Plan on the Ground Plane, perpendicular over the fore Axle.

The other smaller Appendages, such as Iron Stays to the Seat and Springs, Rings, Straps and Buckles, &c. it would be trifling to describe.

The true Figure and proportion of the Carriage being obtained, the Ornaments of carving, &c. will depend on the Hand and Eye, wholly.

Figure 133. represents a light Phaeton, more foreshortened, and going from the Picture; by which the hind part is more seen, as the fore part in the former Figure. A description of the method of delineating it, it is obvious, would be, in a great measure, a repetition, consequently tiresome. The Position to the Picture being fixed and its Proportions known, the method of proceeding is as in the last. Fig. 133.

E X A M P L E XLIX.

To represent a Machine, for driving Piles; which is in the Repository of the Society, for the encouragement of Arts, Science and Commerce, in the Adelphi, Strand.

C is the Center of the Picture, whose Distance is about 8 Inches; the Vanishing Points are determined, as usual, according to the Inclination given. Fig. 134.

Let AB be the Intersection of the Picture, with the Plane of the Frame at the Bottom; and A, the Intersecting Point of the nearest Angle of one of the Timbers; also, let E and F be the Distance Points, for the Vanishing Point of each Side, respectively.

The Vanishing Points being ascertained, draw A*D* and AH, tending to them.

Make A*b* equal to the length of that End beyond the other cross Timber; and *bd* equal to the width between the Timbers, allowing the thickness of one, and draw *b*E and *d*E, giving the Points c and d; through which, the other Timbers are drawn.

Make A*c* equal to the Diagonal of the Inclination, and draw *c*V, to the Vanishing Point; through c and d, draw Lines to the Vanishing Point of the Side, and produce e c, to the Intersection, AB, cuting it at J.

Make A b equal to their thickness, and compleat the Ends (v and x); from the two upper Angles of each, draw lines to the Vanishing Points.

How that Frame is finished, the Figure describes sufficiently.

The places of the other Timbers are obtained by seting the Distances, &c. on the Intersection AB, at *e* and *f*, and thence transfered to A*D*.

From J, the Intersection of the long Timbers, draw a Perpendicular; on which, set up all the measures of the Frame for the Wheels, j, *i*, *k*, and *l*, from each of which draw lines to the Vanishing Point of the Side, cuting Perpendiculars from c and f, giving the height of the Platform, &c. the length of which, *mn*, may be obtained from the Timber, below; and the breadth, *no*, by the Vanishing Point of a Diagonal (*mo*) beyond E.

The feet of the Supporters, (f) being got, and the Bearers, or head pieces (X) drawn to the Vanishing Point of the Front, giving what thickness is necessary, at *l*; draw the Supporters, as in the Figure; also, the Bearers (y) and Stays (z) of the Platform. Their Vanishing Points may easily be obtained, was it necessary, in the vertical Vanishing Line of the End; by Prob. 4.

The

Plate XXXIV. Fig. 134.

The top of the cross Braces being got, by a line from *k*, the Figure shews how they are drawn, so as to shew their thickness, properly; first drawing the Front.

Produce the inside of the long Timber to the Ground Line, and somewhat beyond it, at I, draw IK, perpendicular, the Intersection of the vertical toothed Wheel, which begins the motion.

Make I*p* equal to the height of its Center, and, let *pq* be its Radius, and *qr* its Diameter. Draw from *p* to the Vanishing Point of the Side; and, having bisected the head of the Frame, at g, draw, from g, to the Vanishing Point of the End, which will pass through the Center of the Wheel, at *s*, through which, draw h k its Diameter, and compleat the Rim h i k l (by Prob. 3, Sect. 8.) Its Vanishing Line passes through the Vanishing Point of the side Timbers, perpendicular.

The cross Bars (h k and i l) have their Vanishing Points in it; making right angles, perspectively, with each other, at discretion.

The Teeth, where they are seen, may be done by hand; for although the Rim may be divided into equal parts, yet, to divide it into so many would be unnecessary trouble; observe, as the Wheel is contracted so are the Teeth.

The other vertical Wheel, (Z) of a larger Diameter, on the same Axle, is managed by the same means; having the same Vanishing Line, and Vanishing Points. It's use is to accelerate the motion, by its weight.

In such Objects, great regard should be had to describe the hither parts first; taking particular care that they do not cross each other, improperly.

The middle, upright Timber, m n, and the Bearer, n o, being drawn, the place of the long Axle is opposite the middle of the vertical Wheel; its place may be found in a Line drawn through the foot of the upright Supporter, at r.

The small horizontal Wheel (p q) being nearly on a level with the Eye, appears almost in Right Lines. Its Diameter is, in proportion to the vertical Wheel, nearly geometrical.

The height of the upper horizontal Wheel may be had from any perpendicular Line, cuting the Ground Line, as IL; drawing a Line through r, the Seat of its Center, on the Ground, to the Horizontal Line, at Q.

Make IL, equal to its height, and draw LQ, cuting a perpendicular from r at s, and gives s for the Center of the Wheel.

Draw LM parallel to the Ground Line, make LM equal to the radius of the Wheel, and draw MQ, cuting a Line drawn through s parallel to LM, at t; s t is the Radius, in its true place; by which, the Circumference, t u v w, may be described. The thickness of the Rim, and height of the Teeth, are set up from L; and the cross Bars tend to the Vanishing Points of the Timbers; or at discretion.

The place of the Stay (Y) is also determined on IL, at N.

The large upright Timbers may be determined, in height, by the same means.

Produce the bottom Line of the hither cross Timber (at its foot) to the Ground Line, at O; draw OP perpendicular, and equal to the height. Then, having obtained the place of the upright Timbers, in the middle of the Rails, below, at R and S; and the perpendicular Lines being drawn, a line drawn from P to the Vanishing Point, on the Right, determines the height of the hither one, at T; and, from T to the other Vanishing Point, they may be compleated.

The Center and Diameter of the Wheel V may also be obtained on OP, at *p* and *q*, and transfered to its place, giving *s* for its Center, and *r s* for its Radius; by which it may be compleated. (Prob. 3, Sect. 8.)

Observe, it is the circle of the back part of this Wheel that is first got.

The place of the Weight, or Ram, W, is at pleasure; its height is in proportion to the upright Timbers; also the short Stay above.

The Roller (Y) for winding up the Ram, is a Cylinder, on the Axis of the vertical Wheel; and it is so contrived, as to revolve on the Axis freely, while the Ram descends, without retrograding the motion of the Wheels. When the Ram is down, it catches, and when the Ram arrives at the Top, the Cylinder is set at liberty again; so that, the motion of the Wheels is continual, the same way. Behind the Frame is another heavy vertical Wheel (UX) on the Axis of the Cylinder, intended to accelerate the motion, whilst the Ram descends.

E X A M P L E L.

Is the representation of a Crane, from the same Repository.

Having been minutely particular in the description of the foregoing Machine, less will suffice in this. It is worked after the same manner, by means of Winches; the Model has three, here is but one; which, is sufficient for the purpose of describing the manner of delineating it.

A geometrical Plan, of the place of each upright piece of Timber, &c. in the Frame, being drawn, to any Scale, and the position of the Picture determined, let Perpendiculars be drawn, from each, to the Picture, giving their Seats. Or, being so prepared, let the Lines, of some of the principal, be produced to the Picture; which, where it can be conveniently done, facilitates the process.

The Center of the Picture is at C, in the same Horizontal Line; the Distance is about 6 Inches and a half. V is the Vanishing Point of horizontal Lines at the Ends, determined at discretion; the other runs out, on the Right, distant from the Center almost 14 Inches (a third Proportional to CV and the Distance of the Picture, always, when the Lines are at right angles with each other) BD, on the Floor, is the Ground Line of the Picture, on which it stands. Fig. 135.

From A, the intersecting Point of the end Line, at the foot of the hither Post, draw AV; and in it, find a, the nearest Angle (Prob. 7) from which draw a Line to the other Vanishing Point, and produce it to the Ground Line, at B; and draw BF, perpendicular; on which, the several measures, of the heights, are set up, from B, and Lines drawn to the other Vanishing Point cuting Perpendiculars from a and b.

The Center of the Axle of the large Wheel is in the middle of the Supporters; a line drawn from its seat G, on the Ground Line, to the Center, determines its place, at S; and its Diameter is got by the same means.

The height of the Wheel from the Floor being known, draw its Intersection HI, parallel to the Ground Line; and from G, draw G g, perpendicular, and g C, cuting the Axle in the Center of the Wheel. Make g H and g I each equal to the Radius of the Wheel, and draw HC and IC, cuting a line drawn through the Center J, parallel to HI, at h and i; by which Diameter, h i, the Circumference may be compleated. (Prob. 3. Sect. 8.)

Then, if V be made the Vanishing Point of one of the Spokes, k l, the Vanishing Points of the other are found by making Angles at the Eye, with the Radial of that, equal to the Angles they make with each other; which, in this, are equilateral, having six Spokes, forming a regular Hexagon. (Prob. 24.)

The Winch Wheel (U) touches the Picture; its place being obtained, at K, from the Plan, draw its Intersection K t, and set up the height of its Center K s; make r s, and s t, each equal to its Radius; and, on the Diameter r t, describe the Circumference.

Observe, that the hither upright piece, of the Frame, must be first drawn.

On the Intersection K t, of the Face of the Wheel, its height, and also the place of the cross piece (T) may be determined.

The other vertical Wheel, at Y, is got after the same manner, on PQ.

Its use is to stop the Crane from going backward, by means of the Catch, at Z.

The upper Gudgeon-piece (X) being drawn; find the foot of the Crane, at L, in the central line; its dimensions being obtained, draw the upright Piece, LM, and set up the measures of the Brace and Headpiece, at *d*, *e*, and *f*.

On the Floor, describe a portion of the circumference of a Circle, perspectively, on the Center L, (Radius, the projecture of the Crane) and, according to the direction of the head of the Crane, its Seat being somewhere in that Curve, as at N, draw NO perpendicular, and make NO represent its height from the Floor.

The Rollers, which direct the Cord, are Cylinders; their upper Bases are determined at W, from which, perpendicular Lines are drawn to the head Gudgeon-piece at X, which is level with the Eye, so that, its Plane is not seen.

The Cord, the Pullies, Teeth of the Wheels, the canting of the Timbers, and all such minutias, I shall pass over, as the Figure describes those parts better than the Pen. For those who would be particular, in large Drawings, various Lessons may be found in the Work, for truly projecting every part.

Plate XXXV.

SECTION XII.

On INCLINED PICTURES and PLANES, in general.

AS the Plan which I have hitherto purſued, has been ſolely confined to the uſeful ſtudy of Perſpective, ſo, this Section is adapted more for the curious Artiſt than the uſeful; yet, the inclined Picture may, in ſome Caſes, be neceſſary. However the univerſality of the Principles are, here, more fully evinced; it being demonſtrated that the poſition of the Picture to the Horizon, is not of the leaſt conſequence, nor has any meaning in the Theory of Perſpective, as it has already been ſhewn, in reſpect of horizontal Pictures, and is, in this Section, made general.

For, whether the Picture be vertical or horizontal, and the Planes or Lines, in Objects repreſented on it, be inclined to the Horizon or to the Picture, or both, 'tis the ſame thing if the Picture be inclined, and the Planes or Lines be either parallel or perpendicular to the Horizon; ſeeing they are, or may be, nevertheleſs, inclined to the Picture. Wherefore, however either the one or the other be ſituated to the Horizon, it matters not, if the inclination of one to the other be known, and the Interſecting Points of any two Lines in either, with the other, be determined; the ſituation of the Eye being known, in reſpect of either, as ſhall be made manifeſt, to conviction.

PROBLEM I.

The Interſecting Points of any two Lines in a Plane being given, and the Inclination of the Plane, to the Picture, known; to find the Vaniſhing Line of that Plane, its Center and Diſtance; the Center and Diſtance of the Picture being given.

Fig. 136. The Interſecting Points of two Lines, in any Plane with another, being given, the common Interſection of thoſe Planes is determined. (Theo. 11.) Therefore, if A and B are the interſecting Points given, or found, A B is the Interſection of the Plane they are in, with the Picture. (Cor. 3.)

Draw AB, and CE parallel to AB, equal to the Diſtance of the Eye; through C draw DF perpendicular to AB; make the Angle CEF equal to the complement of the inclination of the Plane to the Picture, cuting DC, produced, at F. Through F draw GH, parallel to AB, which is the Vaniſhing Line required; its Center is F, and its Diſtance is EF.

Turn up the Triangle CEF, on CF, till EC be perpendicular to the Picture; then, E, is the true Place of the Eye, and EC the Direct Radial.

DEM. Now, becauſe DF is perpendicular to AB (the common Interſection) and CEF is a Plane, perpendicular to the Picture, paſſing through DF, it is perpendicular to the other Plane,† for, it is perpendicular to their common Section; therefore, DF is the vertical Line of that Plane, (Def. 11.)

† 9. 7. El.

Then, becauſe CEF is the complement of the Angle of Inclination of the Plane to the Picture, EFD is the real Angle, (Cor. 3. 10. 1. El.) for, ECF is a Right Angle, by Conſtruction.

Conſequently, ſince the Vaniſhing Line of any Plane is parallel to the Interſection of that Plane, (Th. 3) if a Plane be ſuppoſed to paſs through the Eye, at E, parallel to the Original Plane, it is inclined to the Picture in the Angle, EFD; and the Line GH, in which it muſt cut the Picture, is the Vaniſhing Line of that Plane, (Def. 8.) for, the Sections of parallel Planes, by another Plane, are parallel between themſelves. - - - - - - - 8. 7. El.

But, EF is perpendicular to GH; therefore F is its Center (Def. 19.) and conſequently, EF is its Diſtance (Def. 20.) Alſo, CF is perpendicular to GH, (Th. 4.) for, it is a Line drawn in the Plane CEF, from the Point F, in which a perpendicular (GH) to that Plane, cuts it. - 2. 7. El.

COR. Hence it is manifeſt, that, as the Center of the Picture is the Vaniſhing Point of all Lines which are perpendicular to the Picture; ſo, the Center of every Vaniſhing Line is the Vaniſhing Point, of all Lines which are perpendicular to the Interſection of any Plane, of which it is the Vaniſhing Line; and of all Lines parallel to them.

Becauſe, EF, producing the Center, (Def. 19.) is parallel to all ſuch Lines. It

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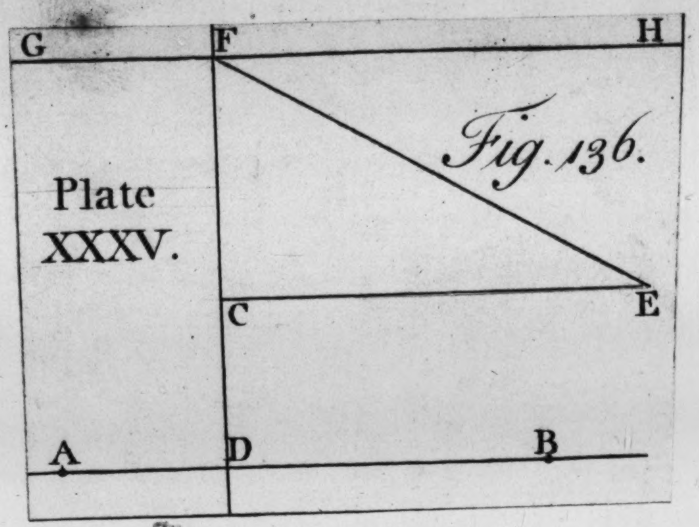
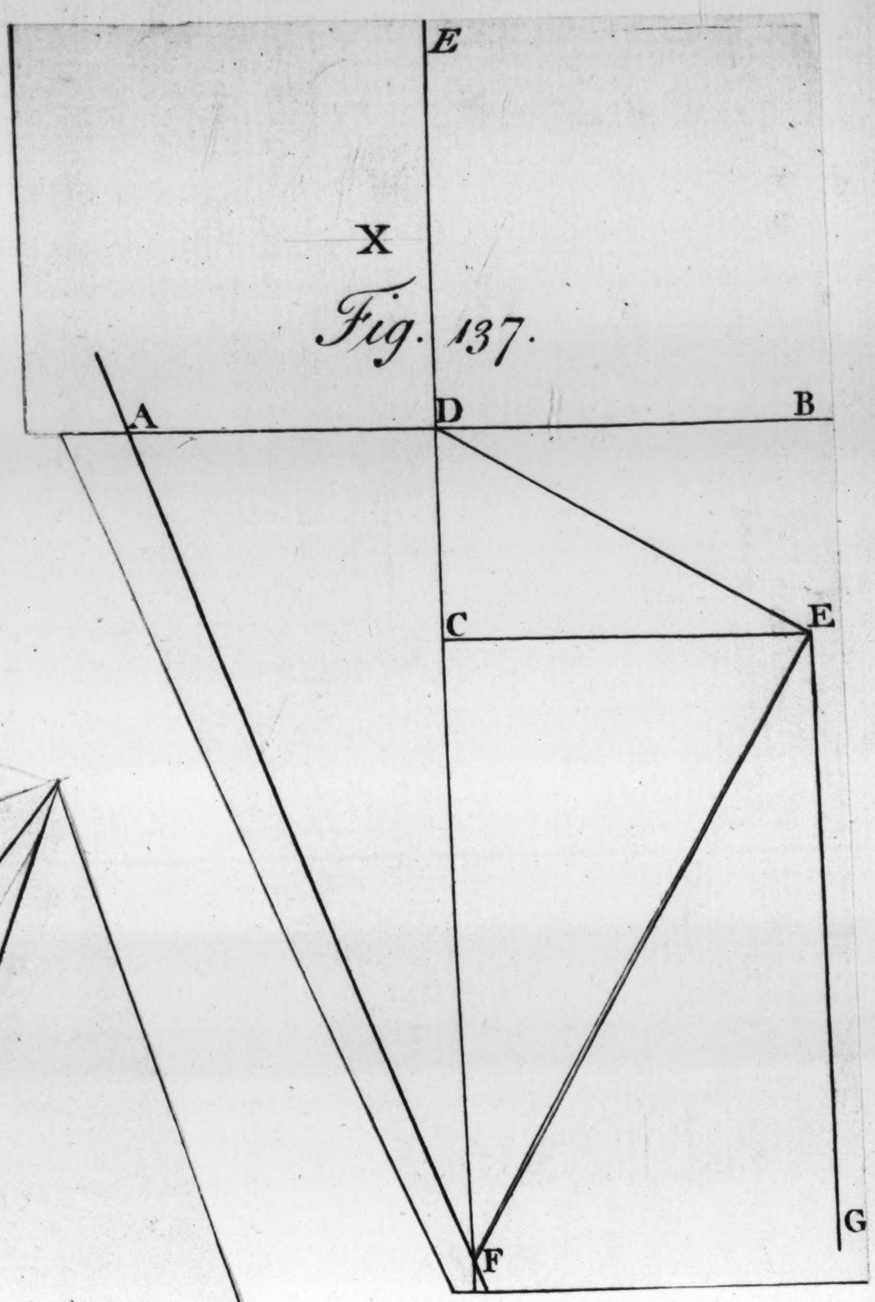
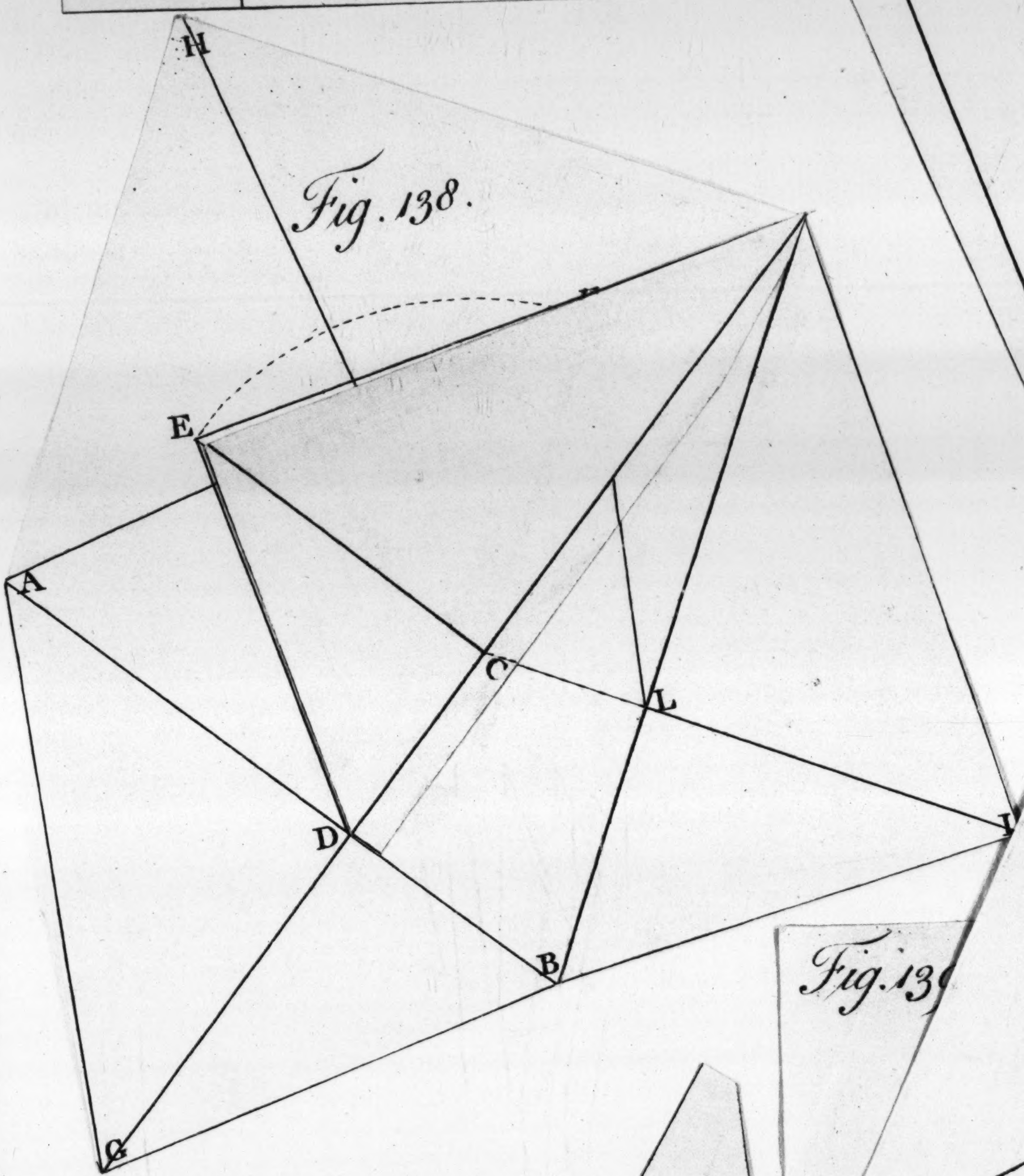
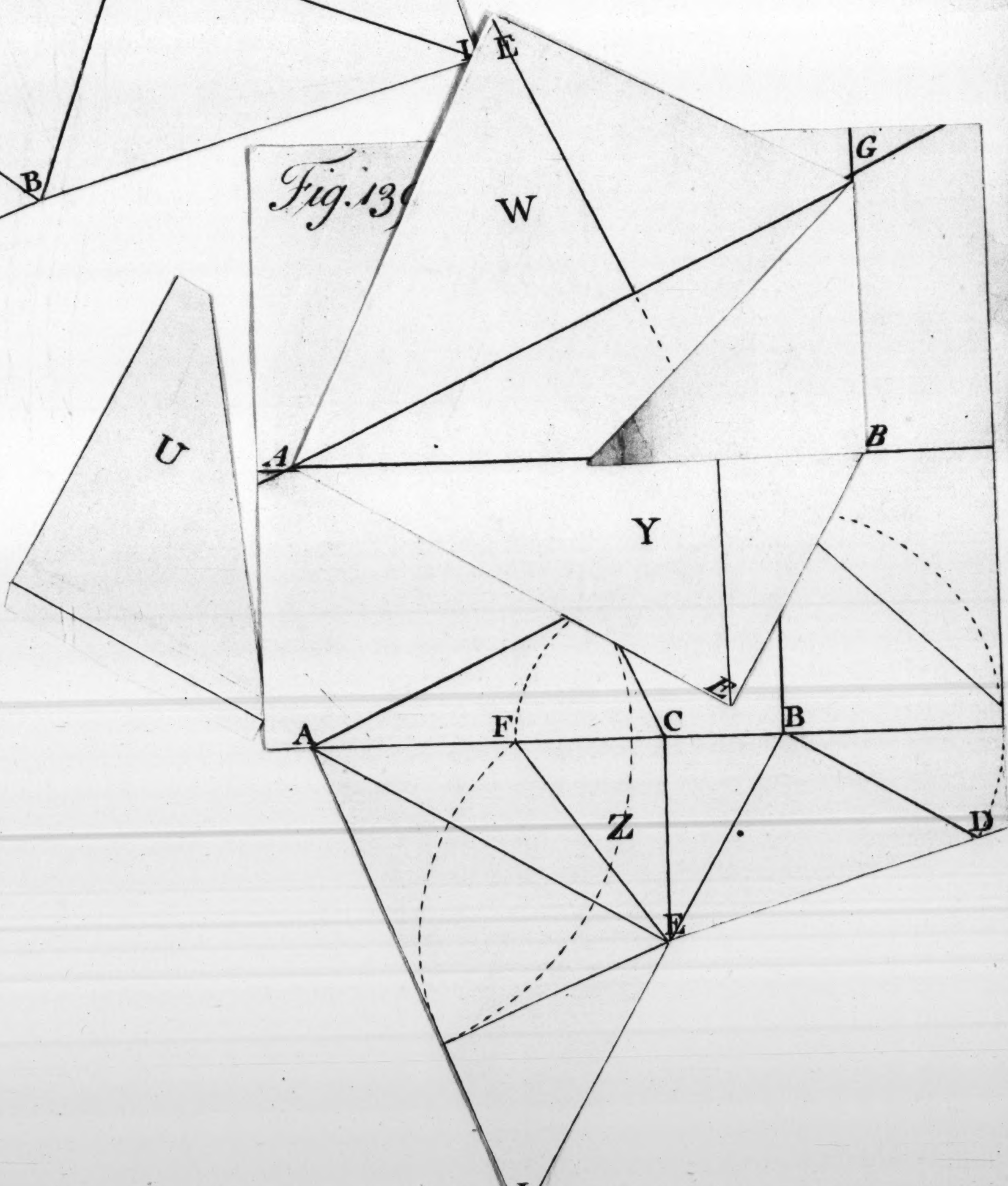
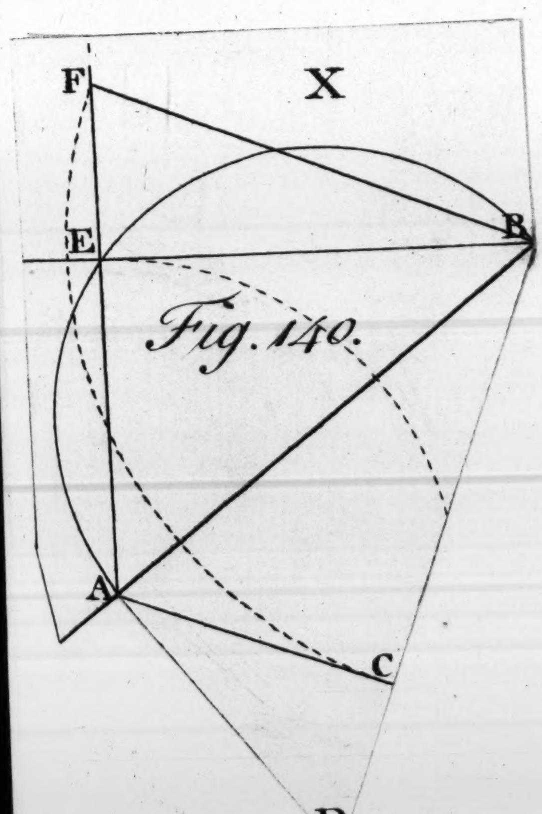
Plate XXXV.











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It must be obvious that this Problem is universal, and cannot possibly regard any position either of the Picture, or of the Original Plane to the Horizon; for, let them be any how situated in that respect, every thing remains the same. But, nevertheless, in many Cases, the position they have to the Horizon is necessary, in practice, to be known; seeing that, all Plain Objects, as Buildings &c, are determined from the position of their Planes to the Horizon, as well as to the Picture; nor is it possible to determine any thing, in respect of their Projections on the Picture, without the previous determination of some Plane or other in respect of them and the Picture, and which is best done, by means of a Plane perpendicular to the Picture, either horizontal or vertical.

PROBLEM II.

The Center and Distance of the Picture being given, and the Vanishing Line of some Plane which is inclined to the Picture; to determine the Inclination of that Plane, and to find the Vanishing Point of Lines perpendicular to that Plane.

Let A B be the Vanishing Line given, and C the Center of the Picture. Fig. 137.

Through C, draw D F perpendicular to A B, indefinite; draw C E perpendicular to D F, i. e. parallel to A B; make C E equal to the Distance of the Picture, and draw E D; also, E F, perpendicular to E D.

E D C is the angle of the Inclination of the Original Plane, to the Picture; and F is the Vanishing Point, of Lines perpendicular to the Plane.

Turn up the Triangle D E F perpendicular, also turn over the Plane X on A B.

DEM. Because C E is perpendicular to the Picture, and equal to its Distance, E C is the direct Radial, and E is the Point of View (Def. 15 and 16) and, because the Plane A E B passes through the Eye, E, and the Vanishing Line, A B, it is parallel to the Original Plane, (Def. 8.) and its Inclination to the Picture is the same, equal E D C.

But, the Plane D E F is also perpendicular to A E B, and consequently to the original Plane; therefore, it is the Vertical Plane, (Def. 5.) and D F is the Vertical Line, (Def. 11.) which, being in a Plane cuting two parallel Planes, perpendicularly, makes equal Angles with them both.

Therefore, the Angle E D F is equal to the inclination of the Original Planes.

But, E F is in the Vertical Plane, and it is perpendicular to E D (by construction) consequently, E F is perpendicular to the Plane A E B, and consequently it is parallel to all Lines perpendicular to the Original Plane; and therefore, F is their Vanishing Point. (Def. 22.) Q. E. D.

PROBLEM III.

The Center and Distance of the Picture being given, and the Vanishing Point of Lines, which are perpendicular to some Plane, to find the Vanishing Line of that Plane.

C is the Center of the Picture, and F is the Vanishing Point given. Fig. 137.

Join F C, and produce it; draw C E perpendicular to D F, and equal to the Distance given; draw EF, and ED perpendicular to EF, cuting FC, produced, at D; and through D, draw A B perpendicular to D F.

A B is the Vanishing Line required; which is manifest, from the foregoing.

PROBLEM IV.

The Vanishing Line of a Plane being given, and the Vanishing Point of the Intersection, of that Plane, with another Plane, perpendicular to each other; with the Center and Distance of the Picture; to find the Vanishing Line of the other Plane.

Every thing remaining as in the last Figure; let A B be the Vanishing Line given, and A the given Vanishing Point.

Find the Vanishing Point, F, of Lines perpendicular to any Plane, whose Vanishing Line is A B (by Prob. 2.) and draw A F the Vanishing Line required.

DEM. For, since the Plane, whose Vanishing Line is required, is perpendicular to that Plane whose Vanishing Line is A B, F will be the Vanishing Point of some Line in that Plane, viz. of Lines perpendicular to the other Plane; and A is the Vanishing Point of one Line in it, by Supposition; consequently, A F is the Vanishing Line sought. - - - - - - - - Th. 11.

N. B. If the Vanishing Line given was of a Plane perpendicular to the Picture, seeing it would pass through the Center of the Picture. (Theo. 6.) Wherefore, Lines perpendicular to it, would have no Vanishing Point; for E G, perpendicular to E C, being parallel to D F (a Line in the Picture) can never cut the Picture and produce a Vanishing Point; consequently, A F, the Vanishing Line (in that Case) would be perpendicular to A B.

Plate XXXV.

PROBLEM V.

The Vanishing Line of a Plane being given, and the Vanishing Point of the common Intersection of another Plane with that Plane, whose Inclination to the former is known; to determine the Vanishing Line of the other Plane; the Center and Distance of the Picture being given.

Fig. 138. Let AF be the given Vanishing Line; and F the given Vanishing Point.

Find AB, the Vanishing Line of a Plane, to which, Lines whose Vanishing Point is F are perpendicular (by the third.)

Make DG equal to DE (the Distance of AB) and draw AG; also draw GB, making the Angle AGB equal to the inclination of the two Planes, cuting AB at B, and draw BF, the Vanishing Line required.

Dem. If the Triangle DEF be turned up, as before, and the Plane AGB be also turned over on AB, DG will coincide with DE. Then, if the Plane AHF be turned over, on AF, AH will coincide with AG, and AHF will be parallel to the Original Plane, whose Vanishing Line is AF (Def. 8.) seeing it passes through the Eye, E, and the Vanishing Line.

But, AGB is perpendicular to AHF (9. 7. El.) for EF is perpendicular to AGB; and conseq. the Angle of inclination of any other Plane, with that Plane, may be determined in the Plane AGB.†

But, AGB is the Angle given, of the Inclination of the Planes; wherefore, if BIF be turned over on BF, till BI coincides with BG; then BIF, passing through EF, is also perpendicular to AGB; consequently, it inclines to AHF in the Angle AGB.

But, Planes producing the Vanishing Lines of Original Planes are inclined to each other as the Originals; and, their common Intersection passes through the Eye, parallel to the common Intersection of the Original Planes. - - - - - - - - - - Theo. 8.

Wherefore, EF is parallel to the common Intersection of the Original Planes; consequently, F is its Vanishing Point, (Cor. 2.) for, it is the common Intersection of the Vanishing Lines; and BF, the Line in which BIF cuts the Picture, is the Vanishing Line sought; (Def. 8.) for, BIF is parallel to the Original Plane, and passes through the Eye, at E. Q. E. D.

† See Art. 4. Page 44.

N. B. If AGB was a Right Angle; then, AF, AB, and BF would be the Vanishing Lines of a solid Right Angle; each Plane, of which, being inclined to the Picture, respectively, as the Planes AGB, AHF, and BIF; all which pass through the Eye, at E.

The Center of each Vanishing Line is where a Perpendicular from C cuts it; as D, K, and L, (Theo. 4.) and DG, KH, and IL are their Distances respectively; G, H and I, are considered as the Eye for each Vanishing Line, in the application of them to practice; each Distance being the Hypothenuse of a right angled Triangle on CE; with CD, CK and CL, respectively. (Def. 20.)

Note. When the given Vanishing Line passes through the Center of the Picture, it is the Vanishing Line of a Plane perpendicular to the Picture, which also passes through F, as DF; in which Case, A and D coincide; and, the Angle is made with DF, as DGA or DGB; according to which Side of that Plane the other Plane inclines.

PROBLEM VI.

The Intersection of any Plane, which is perpendicular to the Picture, being given, and the Intersecting Point of the common Intersection of that Plane with another Plane, whose Inclination to it is known, and, the Angle which their common Intersection makes with the Picture; to determine its Intersection and Inclination to the Picture.

Turn over the Plane Y out of the way, it being of no use in this Problem.

Fig. 139. Let AB be the Intersection given, and A the Intersecting Point.

Make BAE equal the Angle which the common Intersection of the two Planes makes with the Picture; and, at any point (E) in AE, make a right Angle, AEB.

Make BED equal to the angle of Inclination of the two Planes, and draw BD perpendicular to BE, cuting ED.

Draw BG perpendicular to AB; make BG equal to BD, and draw AG.

AG is the Intersection of the inclined Plane, with the Picture.

Turn up the Picture (X) on AB perpendicular, and the Triangle BDE on BE, also perpendicular; then, BD will coincide with BG; also, turn over the Triangle AIE; EI being equal ED, and AI equal AG, they will form a solid Angle, at the Point E.

DEM. BAE is the given Angle, which the common Intersection, AE, makes with the Picture; and, the inclination of the Plane AIE to AEB (which is perpendicular to the Picture, cuting it in AB) is BED; consequently, AG is its Intersection with the Picture.

For, if the Plane AEI be supposed continued; beyond the Picture, it will cut the perpendicular Plane in a continuation of EA, making equal Angles on the other Side; wherefore, A is the Intersecting Point of their common Intersection, and the Intersection AG remains the same.

Secondly. From any Point, E, in AE, draw EC perpendicular to AB, and CH perpendicular to AG, cuting it in H; make CF equal CH and draw EF. CFE is the Angle of inclination of the inclined Plane to the Picture.

DEM. For, if the Picture be turned up, and the Plane AIE, meeting it in AG; then EH is the Hypothenuse of a right Angled Triangle CHE, congruous with CFE; consequently, those Angles are equal.

But, CH and HE are both perpendicular to the common Intersection of the Plane AIE with the Picture, therefore, CHE (equal CFE) is the Angle of its Inclination to the Picture. (See Inclined Planes, Art. 4. Page 44.

This Problem, is general; for, the Intersection given is not, necessarily, either horizontal or vertical; nor is the Picture necessarily either, all that's required is, that the Plane be perpendicular to the Picture, whose Intersection is given.

But if it was inclined to the Picture, that inclination being known, and which Side it inclined on, the rest is determinable; whereas, the given Intersection being of a Plane perpendicular to the Picture, it cannot possibly be misunderstood and applied.

In Prob. 5. Sect. 3. is shewn how to find the Vanishing Line of a Plane which is inclined to the Horizon, and to the Picture, when its Intersection with the Horizon is also inclined to the Picture, as in the last Problem. The horizontal Plane, being considered, simply, as a Plane perpendicular to the Picture, the Problem becomes general, and universally applicable; however that Plane or the Picture be situated in respect of the Horizon.

I shall here, as it is there proposed, give a brief Demonstration of that Problem, which will now, I presume, be better understood, the foregoing being previously necessary; and the assistance of moveable Planes will render it far more intelligible and satisfactory, which, to have used there, would be improper for several reasons.

Let the Picture be turned up perpendicular, and the Plane Y perpendicular to it; also, turn over the Plane V, till *FB* coincides with *EB*; and W being turned over, AE will coincide with *AE*, &c.

The Planes being thus constructed, let the former be also placed as was directed.

Then, if the Original Plane, Z, be horizontal, Y, i. e. *AEB* being parallel to it, is the Horizontal Plane; or, Z being considered, simply, as a Plane perpendicular to the Picture, Y is its Vanishing Plane; for *AB* the Vanishing Line, produced by it, passes through C, the Center of the Picture (Th. 6.)

The Plane V passes through the Eye, parallel to AGE, and therefore produce its Vanishing Line, *AG*; and, *EB* being parallel to EB, also, the Angle *BEG* being equal to BEG, *EG* is parallel to EG; and consequently, *G* is the Vanishing Eoint of EG (Def. 22.) also, *EA* being parallel to EA, *A* is the Vanishing Point of EA. Wherefore, since *A* and *G* are the Vanishing Points of two Lines in the Plane, EA, and EG, consequently, *AG* is the Vanishing Line of the Plane those Lines are in. (Theo. 11.)

Also, because *E* is the Eye, and *EA*, *EG*, are parallel to EA and EG, respectively; therefore, the Plane *AEG* is parallel to AEG (7. 7. El.) and it passes through the Eye, at *E*; therefore, *AG* is the Vanishing Line of the Plane. (Def. 8.)

Now, if the Plane *BEG*, be turned on *BG* into the Plane of the Picture, on either Side, *BF* being equal to *BE*, and the Angle *BFG*, equal *BEG*; consequently, the Point *G* is the same, however the Plane V be situated to the Picture, in its revolution on *BG*; therefore, the Vanishing Point *G*, is truly ascertained; and consequently, the Vanishing Line *AG*; being parallel to the Intersection AG of the Original Plane, also *BG* to BG. (Theo. 3.)

For, the Original Planes, being situated any where on the other Side of the Picture, being parallel, respectively, to their present Station, their Vanishing Lines are the same. (Theo. 5.)

And if the Intersection EA of the inclined Plane AEG be the same, produced on the other Side, as AH, and the Plane be inclined on the same side, its Intersection, AG, with the Picture is the same; as the Plane U, being raised up to the same inclination with the Horizon (equal BEG) evinces.

PROBLEM VII.

The Inclination of a Line to the Picture being given, and the Angle of inclination of any Plane that Line is in, to the Picture; to determine the inclination of the Line, to the Intersection of the Plane it is in, with the Picture.

Let AB be the given Line, and ABD the Angle of its inclination to the Picture. Fig. 140.

 From

Plate XXXVI. From any Point, A, draw AC perpendicular to BD; make the Angle CAD equal to the Complement of the inclination of the Plane to the Picture, and ADC, is the Angle of Inclination.

On AB describe a Semicircle; and, with the Radius AD, describe the Ark DE, cuting the other at E; i. e. make AE equal AD; draw EB, and ABE is the Angle required.

Draw AE and produce it; make EF equal CD, and draw BF.

Dem. Turn up the Triangle ADC, on AC, and CAB, on AB, till AD coincides with AE; then, turn up the Plane X, till BC coincides with BF.

Now X being considered as the Plane of the Picture, and ACB as a Plane passing through the Line AB perpendicular to the Picture, ABC is the angle of its inclination to the Picture, given. But, ADC is the inclination of the Plane it is in, to the Picture, and EB is the Intersection of that Plane with the Picture; consequently, ABE is the inclination of the given Line, AB, to the Intersection, EB.

From these Problems it must be obvious, that the Position of the Picture, to the Horizon, is of no consequence in the Theory of Perspective; but is very much so in common Practice; because, all perpendicular Lines, in Objects, are parallel to the Picture, being vertical; and horizontal Lines are easily determined, whether perpendicular or inclined, their inclination being known. To an inclined Picture, their inclination is the Angle they make with their Seats, on the Picture.

N. B. The Vanishing Line and Intersection of some Plane in the Object, must be given, to determine others, if necessary; for which end, the Center and Distance of the Picture are absolutely necessary. The Vanishing Line of horizontal Planes, being perpendicular to the Picture, is therefore first determined, and the Intersection of the Ground Plane; which of all other are fittest. Without them there would be great difficulty in proceeding. For want of that consideration, the work of Dr. Brook Taylor is almost useless to a Practitioner; he not giving, properly, one Specimen, how to find the representation of a Line, from its known length, situation, and place, in respect of the Picture; but only, by means of the Intersecting Points of other Lines, or drawing Visual Rays, from the Eye to the Original Points, in their true places, which is not practical in many Cases; or, to determine the Figure from some Line given in it, on the Picture; so that, the Student knows not how or where to begin the process, from the geometrical proportions of the Object, and its known or determined position to the Picture, and to the Horizon.

EXAMPLE I.

To find the representation of a right angled Parallelopiped, whose Sides are known, situated at some distance from the Picture; whose Faces are all inclined to the Picture, the inclination of one being determined, and the inclination of one Side in that Face, to the Picture. The Center and Distance of the Picture, with the Seat, and distance of the nearest Angle of the Object, to the Picture, being given.

Fig. 141. Let C be the Center of the Picture, and S the given Seat of the hither Angle.

Find a, the representation of that Angle. (Prob. 6. Sect. 4.) Draw DF, at pleasure, through C, and CE, perpendicular to DF; make CE equal to the Distance of the Picture, and the angle CED equal to the given inclination of a Face.

Through D, draw AB perpendicular to DF (the Vanishing Line of that Face, Prob. 1.) and, perpendicular to DE draw EF; F is the Vanishing Point of Lines perpendicular to that Plane. (Prob. 2.)

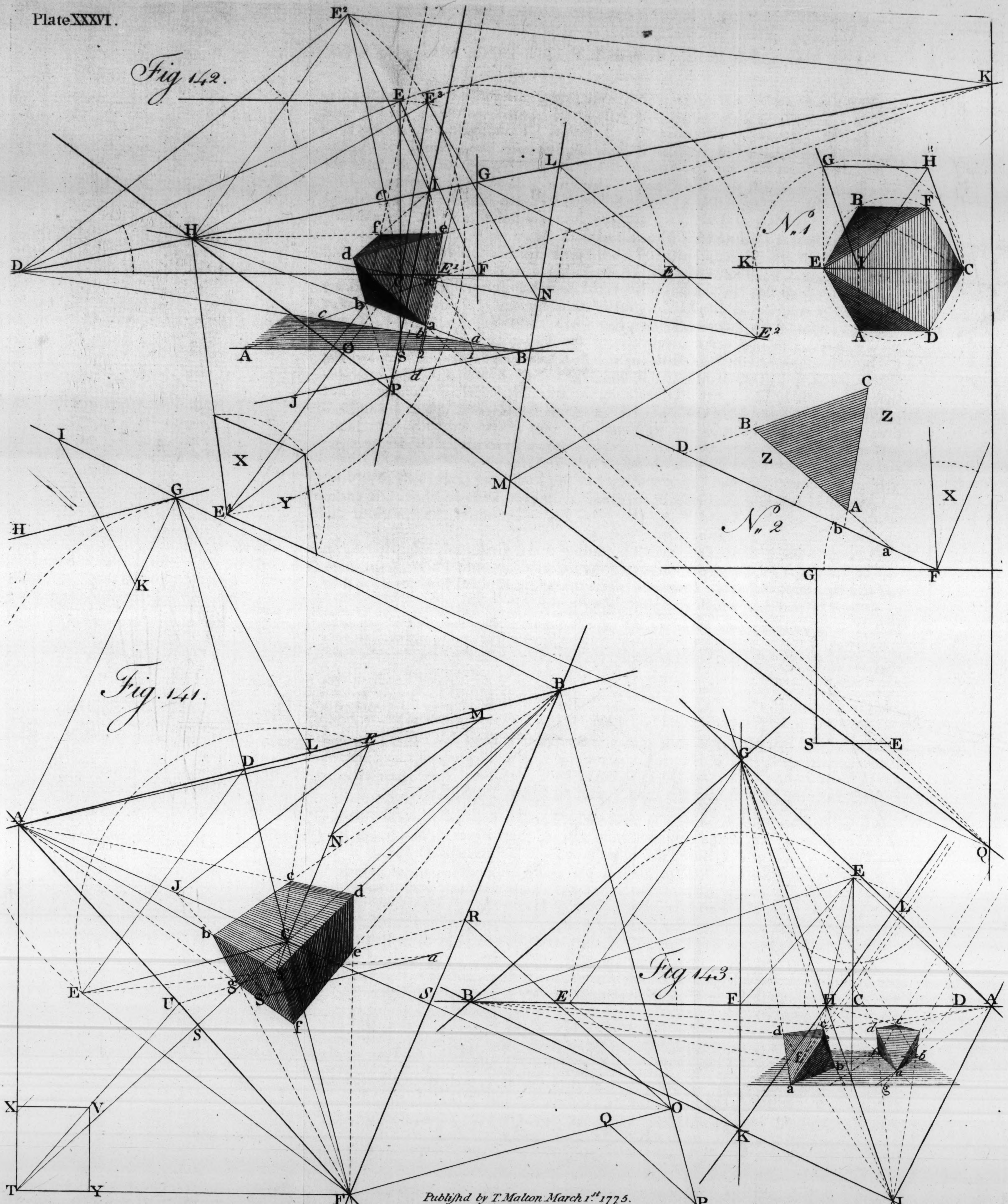
Find the representation of that Face whose Vanishing Line is AB, D is its Center, and DE its Distance. (Prob. 21.)

Produce FD; make DG equal DE, and draw GH parallel to AB.

Make the Angle HGA equal to the inclination of the given Line to the Intersection of that Face, whose Vanishing Line is AB (Prob. 7.) and, make AGB a right Angle; A and B are the Vanishing Points of its Sides; and, F being the Vanishing Point of Lines perpendicular to that Face, AF, and BF are the Vanishing Lines of the other Faces; as it is manifest by Problem 4.

Draw aA, aB, and aF, the indefinite Representations of three Sides, forming the hither Angle; how to proportion them, I shall shew, as follows.

Plate XXXVI.



Publishd by T. Malton March 1st 1775.

The measures of those Sides are known, and the distance of the Angle a, from the Picture; which was found by its given Seat, S, and its Distance S *a*.

Now, A, B, and F, are the Vanishing Points of the Sides of the Object, any one of which may be in the same Plane with SC (Ax. 6.) wherefore, ACS is the Vanishing Line of such a Plane. (Theo. 11.)

Make a b to represent a Line in proportion to that which S a represents, as the Original of a b is to the Distance of the Angle a (Prob. 10.) thus.

Make CE equal to CE, and perpendicular to AC; draw AE, and produce it; make EM, to EN, in the ratio of the Side, to the distance of the Angle a; join MN, draw EJ parallel to MN, and draw SJ, cuting a A at b; so shall a b represent a Side of the Object, whose length and inclination to the Picture was given.

Draw b B and b F; and, by means of the Radials AG and GB, make a d, or b c, to represent a Line, in proportion to the Original of a b, as one Side of that Face is to the other (Prob. 10.) viz. as GI to GK; and draw A d (through c) and d F.

The Face a b c d, whose Inclination was given, being compleated; find e or g, so, that d e, or b g, shall represent the proportion of the other given Side, by means of a b, or a d, as before; the Center and Distance of either Vanishing Line being determined, (by Prob. 1.) as S, the Center of BF, by a Perpendicular from C (Th. 4.) and SO the Distance, in AS produced; making BO equal BG, and joining OF. BO and OF are the Radials of the Sides in that Face, forming a right Angle BOF, at the Eye, O.

Make OP to OQ in the ratio of the Originals of a d to d e; draw PQ and OR parallel to PQ. Draw a R which will cut d F at e; and, through, e, draw B f, cuting a F at f; and lastly, draw f A, cuting b F at g, which compleats the Parellelopiped, a b d f, required.

For, because of the Vanishing Points A, B, and F, the Sides a b, a d, and a f, form a solid Right Angle, at a; AGB, ATF, and BOF being Right Angles, which b a d, b a f, and d a f represent, respectively; and the other Sides vanish in those Points, respectively; as d c and d e, &c. which represent Parallels to a b, a f, &c. (Cor. 1. Th. 5.) Therefore b d f represents a right angled Parallelopiped, whose proportion was known, and position to the Picture determined.

a c, a e, and a g, represent Diagonals, in each Face, respectively, which are in proportion to the Sides, as IK, PQ and TV, respectively, to GI and GK, OP and OQ, TX and TY, respectively.

The Parallelopiped, b d f is truly determined, according to its position given, in respect of the Picture, its place in respect of the Eye, and its proportion in respect of its Distance; no regard being had to its position respecting the Horizon. Wherefore it is obvious, that, to determine its Position, in that respect, the horizontal Vanishing Line is essentially necessary, and the Vanishing Lines of its Faces are determined in respect of their position to the Horizon, as well as to the Picture, as shall be exemplified in the next.

EXAMPLE II.

*To represent an Octaedron, * perspectively, situated on a Plane inclined to the Horizon, and to the Picture; the angle of Inclination to the Horizon, being given, and the Angle which its Intersection with horizontal Planes, makes with the Picture; together, with the Seat of the Object on the inclined Plane; and, its situation in respect of the Picture.*

Let ABC (No. 2.) be the Seat of that Face on which the Object rests, on the Plane Z, which is inclined to the Horizon in the Angle X; DF is its Intersection with the Horizon, and DFG the Angle that Inclination makes with the Picture; FG is the Intersection of the Picture with some horizontal Plane. The Station Point is S, the Distance is SG, and SE the Height of the Eye. Fig. 142. No. 2.

These preliminaries being determined, let AB be the Intersection of the horizontal Plane, or Ground Line; and, at the height of the Eye (SE, No. 2.) draw the Horizontal Line, DF, parallel to AB; let C be the Center of the Picture.

* An Octaedron is a regular Solid, one of the five Platonic Bodies, having eight Faces, which are equal, equilateral Triangles; about which Solid, if a Sphere were circumscribed, every Angle of the Solid would be in the Surface of the Sphere.

Its geometrical Construction is necessary to be understood, before it be possible to describe it perspectively. Its geometrical Plan, on the Plane on which it rests, is a regular Hexagon, AEBFCD (No. 1) ABC is the Face, on which it rests. DEF is the upper Face, EAD, EBF, and FCD are inclined Faces, above; and ABE, ACD, and BCF, below, out of sight; IGHC is its geometrical Elevation, or Section through EC, shewing the Inclination of its Faces, to each other; viz. the Angle GIK equal IGH. No. 1.

Plate XXXVI. Fig. 142. Draw CE perpendicular, equal to the Diſtance of the Picture, and find the Vaniſhing Point, D, of the inclination given. (Prob. 2. Sect. 3.) Make DEF a right Angle, and, through F, draw FG perpendicular to DF. FG is the Vaniſhing Line of a Plane, to which, Lines vaniſhing in D are perpendicular. (Prob. 3.)

Make F*E* equal EF, and the Angle F*E*G equal X, (No. 2.) and draw DG, the Vaniſhing Line of the inclined Plane, (as by Prob. 5. Sect. 3.) indefinite.

Produce EC, cuting AB at S; make SB equal GF (No. 2.) draw BD, the indefinite Repreſentation of DF, (No. 2.) and find the Points *a* and *b*, repreſenting a and b, in which, BA and CA (No. 2.) cut DF. (Prob. 8.)

Find the Center, *C*, and Diſtance *C*E². of the Vaniſhing Line DK (Prob. 1.) make *C*E² (in C*C* produced) equal to its Diſtance, and draw DE².

Make the Angle DEH equal BaD (No. 2.) H is the Vaniſhing Point of AB (No. 2.)

Make the Angles HE² I and IE² K each of 60 Degrees; H, I, and K are the Vaniſhing Points, of the Sides of the two Faces ABC and DEF (No. 1.) i. e. of AB, AC, and BC, (No. 2.) and conſequently of the ſides of the oppoſite Face.

Find the repreſentation of that Face, a b c (Prob. 18) having obtained the Points, *a*, *b*, and *c* (repreſenting a, b, and D, No. 2.) by means of the Interſection AB; making D*E*¹, equal DE; or, by the Interſection BJ, of the Face, a b c.

Find the Vaniſhing Line HM, of the contiguous Face, whoſe common Interſection, with that found is a b, and its Vaniſhing Point H. (by Prob. 5.) Its Inclination is the acute Angle GIK (No. 1.) determined by a perpendicular from E, and IG being made equal IC; which Angle is 70 Degrees.

Find the Center (O) and Diſtance (OE⁴) of the Vaniſhing Line HM, (Prob. 1.)

Draw *E*⁴H and find the Vaniſhing Points P and Q; as I and K, above; or make equilateral Triangles, X and Y, at the Eye, E⁴, and produce their Sides to the Vaniſhing Line H Q, cuting it at P and Q.

Through a and b, draw Q a, and P b, cuting at d; giving a b d for that Face.

The Point d is in the upper Face, and the Sides of oppoſite Faces are parallel, reſpectively, two and two; therefore, draw d I, and d K; and P c, cuting d K at e; draw e H and a e, which compleats the Figure.

N. B. This Figure having eight Faces, two and two of which are parallel, there are conſequently (if none are parallel to the Picture) four Vaniſhing Lines; but here are only two found, DL, and HM; a third will paſs through I and P, and the fourth through K and Q, which, being produced, would meet in the Vaniſhing Point of a e. (Theo. 11. and Cor. 2. Th. 8.)

It may be obſerved, that the three Sides of each Face, vaniſh in its reſpective Vaniſhing Line, (Theo. 11.) and the Vaniſhing Point of the common Interſection, of any two Faces, is the interſection of the Vaniſhing Lines of thoſe Faces, (Theo. 8. Cor. 2.) as H, of a b and e f, the interſection of the Vaniſhing Lines DL and HM; and K, of DK and QK, the Vaniſhing Point of d e, &c.

Fig. 143. The next Figure exhibits two of the ſame Objects, on a level Plane (or any Plane perpendicular to the Picture) ſituate alike to the Picture, but on different ſides of the Station Line; the Sides, in both, have conſequently the ſame Vaniſh-points, and, their Faces have the ſame Vaniſhing Lines, reſpectively.

AB is the Vaniſhing Line of the Faces which are perpendicular to the Picture, and C its Center; a g is the Interſection of one of thoſe Faces. The Angle a, of one Object, touches the Picture, the other is at ſome Diſtance beyond it.

FG being the Vaniſhing Line of a Plane perpendicular to the common Interſection a b, and F*E* (equal EF) its Diſtance; the Angle F*E*G is made equal to that of a Diagonal Plane with the Faces, (equal HIC, No. 1) viz. 55 Degrees. AG is the Vaniſhing Line of the Diagonal Plane a b c d, which is a Square.

The Vaniſhing Points, A, H, and B, of the Sides a b, a f, and b f, or d e, being found (as above) and *E*K drawn, perpendicular to *E*G, cuting GF, produced, at K, the whole is determined. BK and GH interſect at I, the Vaniſhing Point of e b and d f; and BG, KH, and IA, being produced, would meet in the Vaniſhing Point of a e and f c.

AB, AI, BG, and GI, are the Vaniſhing Lines of the Faces; and AG, BI, and KL of the three Diagonal Planes. The reſt the Figure deſcribes.

The Vaniſhing Lines of the Faces produce, by their Interſections, the Vaniſhing Points of every Side of the Figure, viz. ſix, A, B, G, H, I, and, if BG and IA be produced, they will meet in the Vaniſhing Point of a e and f c.

These Examples are ſufficient, for finding the Vaniſhing Lines, and applying them to uſe, in projecting Figures in inclined Planes, in ſuch Objects. I do not intend to lead the Reader through all the Mazes, attending the projection of the Dodecaedron and Icoſaedron, becauſe I know no end it can anſwer

answer to any Person; such Objects never coming in the course of their several Studies. But, if any Person be inclined to amuse himself with them, he may find Rules in this Section and the 5th. for his purpose, in every position he can devise. As they require a geometrical construction, previous to the projecting them perspectively, I recommend the Reader, who has curiosity, to the Work of Mr. Hamilton; or, in that, to the ingenious Mr. Highmore, they being rather foreign to my Plan.

EXAMPLE III.

How to represent a piece of regular Fortification, in Perspective.

The foregoing Examples are calculated more for Lessons than real use; in this, the knowledge and application of inclined Planes is necessary.

In Fortification or military Architecture, as the Walls of the Poligons and other Outworks are all inclined to the Horizon, and mostly to each other, it seems to be the fitest Subject of any I know. Indeed, the vertical Planes of other Buildings are, frequently, as much inclined to the Picture, but, their position to the Horizon familiarizes them, on a vertical Picture; and yet, in reality, there is no difference, in Theory, and very little in Practice.

The position of the Poligon (Z) to the Picture being determined, find the perspective Plan of its Seat on the Ground Plane (Sect. 5.) AB being the Ground Line, and A the intersecting Point of one Line in the Face X, find the Intersection, AD, of that Plane (Prob. 6.) and, F being the Vanishing Point of a b, draw FG parallel to AD; FG is the Vanishing Line of that Face (Th. 3.) S is its Center, *E*S its Distance, and *E*SC the inclination of the Plane, X, to the Picture. Plate XXXVII Fig. 144.

Now, if the Figure of the Face X, be known, the inclination of the Sides a c and bd, to the horizontal Lines, a b and c d, which are parallel, their Vanishing Points are determinable (Prob. 4. Sect. 3.) by making the Angles F*E*G and F*E*I, respectively equal to them. If they do not fall within the compass of the Picture, yet, by their means, a c and b d may be drawn, by Prob. 13. Then, J being the distance of the Vanishing Point G, draw Ja to the Intersection AD, cuting it at *a*; make *a b* equal to the length of the Side ac, and draw *b*J, which determines the Angle c; and cF gives the Angle d, b d being drawn indefinite.

Or, if the Intersecting Point, D, of cd be obtained, the rest is unnecessary. Make AK equal to the perpendicular height of ac, and draw KD parallel to AB.

By the same means, any other Face, as Y, may be obtained, and continued around. If the Vanishing Point G be within compass, and the inclination of the Faces, X and Y, to each other be known, the Vanishing Line GH is determined, by Problem 5; or, H, the Vanishing Point of a f, being found, draw GH; by which, the Face Y is described; as X, by means of the Vanishing Line FG.

Of the INCLINED PICTURE.

I presume, the Reader will, ere now, be fully convinced of the universality of the Principles on which the Theory of Perspective is founded; and, that it is the same thing, whether the original Plane or the Picture be inclined, or both, in respect of the Horizon; seeing it is the position of the original Planes and Lines to the Picture, only, that is considered, in projecting them.

In Example 1st. if the Parallelopiped, being right angled, had any of its Faces parallel to the Picture; or, if they were parallel to the Horizon, and the Picture vertical; or, which is the same thing, if any of its Faces are perpendicular to the Picture; then, the Sides which are perpendicular to them, are parallel to the Picture, and consequently, have parallel representations (Theo. 10.) But, the Picture being inclined to the Horizon, whilst some Face of the Object is parallel to it, the Sides may then be all inclined to the Picture; and the perpendicular Lines, which, when the Picture is vertical, are parallel to it, and consequently they have no Vanishing Point; but when the Picture is inclined, they vanish either above or below, according as the Picture is inclined, towards the top or bottom of the Object; of which I shall give some Examples.

Plate XXXVII
Fig. 145.

EXAMPLE IV.

How to represent, an upright Object, on an inclined Picture.

No. 1. Let AB be an object perpendicular to the Horizon, and let DF be a Section of the Picture, inclined towards the top of the Object; E is the Eye of a Spectator, EC, perpendicular to the Picture, produces its Center and measures its Distance; ED, parallel to the Horizon, determines the Horizontal Line, and EF, being perpendicular, that is parallel to AB, determines the Vanishing Point (F) of Lines perpendicular to the Horizon (Prob. 2.)

No. 2. Let the Picture be prepared accordingly, drawing the Horizontal Line as usual, and the Vertical Line, at right Angles, cuting it at D.

Take DC and CF in proportion to CE the Distance taken (as in No. 1.) DI (not EK) is the distance of the Ground Line from the Horizontal.

In this Case, it is obvious, that the Center of the Picture, C, is below the Horizontal Line; as, if the Picture inclined towards B, it would be above it, at S; but, seeing that the Vertical Plane is always perpendicular to the Picture, its Center is always in the Vertical Line, DF.

If the Object be otherwise inclined to the Picture, laterally, that Inclination known, find the Vanishing Points, H, and I, of horizontal Lines in the Object (Prob. 2. and 4. Sect. 3.) and proceed, in every respect, as in Example, 1st.

F, being the Vanishing Point of Lines perpendicular to the Horizon, and, H and I, the Vanishing Points, of horizontal Lines in the Object, at right angles with each other; the Object being right angled; consequently, HF and IF, are the Vanishing Lines of the upright Planes in the Object (Prob. 4.)

The Seat (s) of any Angle (the nearest to the Picture is the most convenient) being determined; or, the intersecting Point (A) of some Line it is in; draw AB, the Intersection of any Plane that Line is in †. Find *a*, the representation of the Angle a, its distance from the intersecting Point, A, being known. (Pr.6,or7. Sect.4.)

† Theo. 3.

By means of the Intersection, AB, of the upper Plane of the cross Arms, is got *a*, *b*, &c. the real measures being applied on AB, as usual; and, on AL, the Intersection of the upright Plane (parallel to IF, Th. 3.) in which is the same line, a b, is applied the measures of height (Ab, ac, &c.) and projected by means of the Point Q, the Distance of the Vanishing Point of those Lines; or the intersecting Points, D, H, and I, may be found, thus. A being determined (by its height above the Eye, the Seat of a, (No. 1.) and distance from its Seat, with the inclination of the Line it is in, to the Picture, or to its Seat) draw a b through A, perpendicular to HI; make Aa, Ac, Ab, in the ratio of Ab, ac, and bB; as CE, to CE, No. 1; draw aK, cJ, and bL, parallel to HI, cuting the Intersection AL, at K, J, and L; from which, draw to the Vanishing Point I, and thro' *a*, *b*, &c. to F, cuting them at *f*, *g*, *h*, &c. From *f*, and through *h*, *g*, &c. draw to H.

Find the Center and Distance of the Vanishing Line HF, and the place of the Eye at O (Prob. 1.) make OM, and ON, in the ratio of a b to ac (No. 1.) draw OP parallel to NM; and, through *g*, draw P*i*, and compleat the end *ik*, by means of the Vanishing Point R (in IF) of a diagonal of a Square.

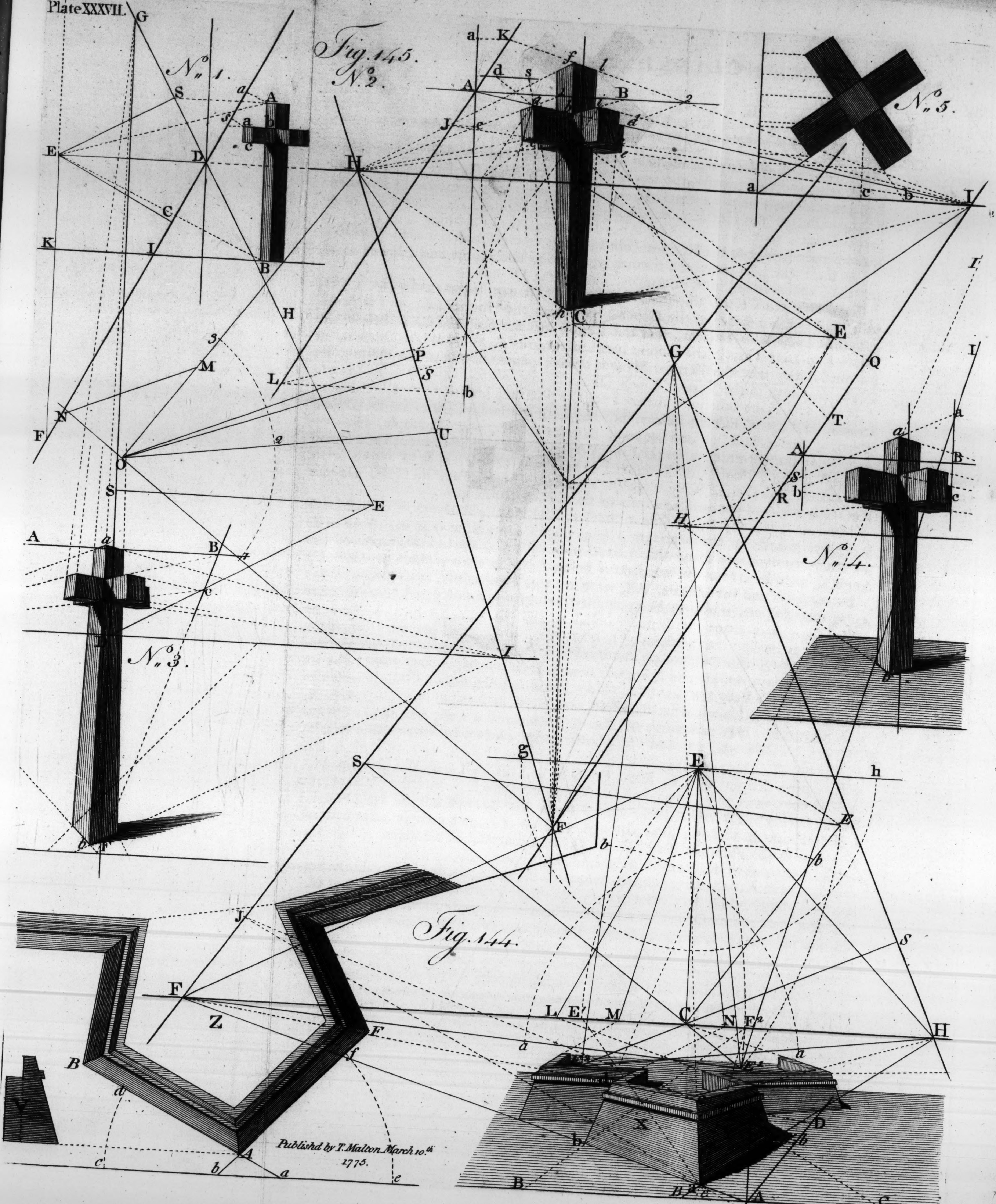
The rest is obvious, from inspection of the Figure.

No. 3. exhibits the same Object, when the Picture is inclined towards the bottom, (as GH, No. 1.) its Center is at S, and the perpendicular Lines vanish at G.

No. 4. is the same Object on a vertical Picture, whose Distance is ED.

In these Examples may be seen the universality, and the superiority of Brook Taylor's Principles to all other; by which is shewn (in this last Section) that any Plane Object, whatever, may be projected, from the known position of one Plane to another, in the Object, and the proportion of the Lines in those Planes, together with their position to each other; without regarding their position to any other Plane, whatever, except the Picture. Whereas, by the Old Authors, it was almost impossible to project them at all; or, with the utmost difficulty, by the Seat of each Angle on the Ground Plane, and its height above it, a troublesome and laborious process; by which was obtained the several Angles, and then joined by Right Lines; without Vanishing Points, of which they had not the least conception, in any other Lines but horizontal; and those they called accidental Points, for they had no certain method of producing them. Having found the two extremes of an inclined Line, then, drawing the Line, and producing it to the Horizon, they found its Vanishing Point; which is now fixed with absolute certainty, in all Positions, and directs the certain position and place of each Line on the Picture, indefinite. And, by means of Visual Rays, drawn on the Picture, certain portions are cut off (truly mathematical) which represent the Originals, as they appear to the Eye, in the true Point of View.

Plate XXXVII.



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BOOK IV.

Of SHADOWS, in general; of LIGHT and SHADE, REFLECTION, KEEPING, &c.

SECTION I.

An introductory PREFACE, concerning LIGHT and SHADE.

IN the foregoing Work is contained the full knowledge and practice of linear Perspective; the last Book containing Rules, necessary for the projection of any regular Object, almost, whatever; with Examples, varying the application of the Rules, frequently, as occasion requires. The linear part being projected, strictly by the Rules there prescribed, will convey a just Idea of the figure of the Object, and the proportion of its parts to each other (the Eye being in the true Point of View.) Nevertheless, there requires somewhat more to be done, in order to give an appearance of solidity, and receding of one part behind another; which indeed is effected, in some measure, by their perspective proportions; and perfected, by a just gradation of Light and Shade, properly distributed to each part of the Object; and which, with the projection of Shadows, &c. is the Subject of this fourth Book.

It must not be expected that I should define what Light is, having already given my Opinion on that Subject (Sect. 1. Book 1.) yet, it may be proper to give some general Idea of what is understood and meant by Light, as it is used by Artists in general, in the application of it to a Picture.

LIGHT, in that respect, means nothing more than the bright parts of Objects; which differs greatly in degree, according as the Surfaces are situated in respect of some luminous Body; which is effected either, directly, from the Luminary, or reflected, from some illumined Object.

SHADE is a deprivation of Light, the dark parts of Objects; occasioned either by the Object itself, on those parts which are not towards the luminous Body, or, by some other opake Substance, interposed between the luminous Body and the Object; depriving it, either wholly or partly, of Light.

To give or prescribe Rules, absolutely, for perfecting a Picture, in respect of Light and Shade, is as impossible as in respect of Colour; yet, by adhering to Reason, and carefully observing Nature, we may arrive at a tolerable degree of perfection. In the first place, it is necessary to consider how the Object is supposed to be situated to the luminous Body, from which it is to receive the Effect.

It has been almost a general Rule, amongst Artists, to suppose Light to flow from the left hand to the Right; but that is entirely arbitrary, and can have no foundation in the nature of things, but merely an habitual custom; it is, however, proper to imagine it to flow from one hand or the other; for, to suppose the Luminary, or luminous Body, directly opposed, either on this or on the other side of the Object, can never produce a pleasing effect; seeing that, in the first case, it is almost wholly illumined, in the other, it is wholly deprived of Light; an agreeable mixture of both, judiciously disposed, is what contrasts one Object, or part of an Object from the other, and renders the whole agreeable to the Eye, as a pleasing imitation of Nature.

Let

Plate XXXVIII Let it be carefully observed, that, that part of an Object, to which any luminous Body is most directly opposed, will be the brightest; but, when, from the situation of the Eye, it is much contracted, or foreshorted, it may appear as dark, or perhaps darker than other parts, on which, the Light is not so direct: although that part in the Object, is considerably brighter, than others which appear so.

Fig. 1. For Example. Let Z be the Plan or Base of some prismatic Object, and AB, &c. Rays of Light, falling thereon. Because those Rays fall more perpendicularly on the Face BC, than on BD, that Face is, consequently, more illumined than BD, and BD than DE, whilst FG is wholly deprived of Light; and GH, being farther removed from the Light, will be darker than FG, from the effect of Light simply, or direct. Now, if a Spectator be so situated, at E, that the Face BC, which is most illumined, is much contracted, it may appear darker than BD, which is more opposite to the Eye; whereas, if the Station was at *E*, where both appear equally contracted, the Face BC will appear the brightest; but, if the Light was in the direction of *E*B they would be, and also appear equally bright, without distinction.

Thus may any direction to the Rays of Light be given, at discretion, as AB; and, by drawing others, parallel to AB, it is easy to know which Face should be the brightest, and which the next &c; but to determine in what degree, positively, is not possible; a careful observation of Nature is the best criterion by which to judge of that; and, even in that case, it is not easy to determine, without experience and sound judgment.

After the same manner, the inclination of the Rays of Light to the Horizon, being taken into consideration, may be determined, whether the upper faces of Objects, below the Eye, or inclined Faces, are more illumined than the Vertical. As Light flows from above, the whole Hemisphere being illumined (when the Luminary itself is not much elevated) the horizontal faces of Objects, or such as are much inclined, are, generally, the brightest; provided they are opposite to that quarter from which the principal Light flows. Notwithstanding it is usual to shade the Roofs of Buildings, Pediments, &c, more than the vertical Planes, it can only be from a supposition of their being of darker materials, as Lead, Slate, &c; for, if they were composed of the same, that is, if they are of one uniform Colour (and otherwise, no positive determination can be made, in respect of Light, from observation) inclined Planes, will be brighter than either vertical or horizontal, being more opposed to the Light.

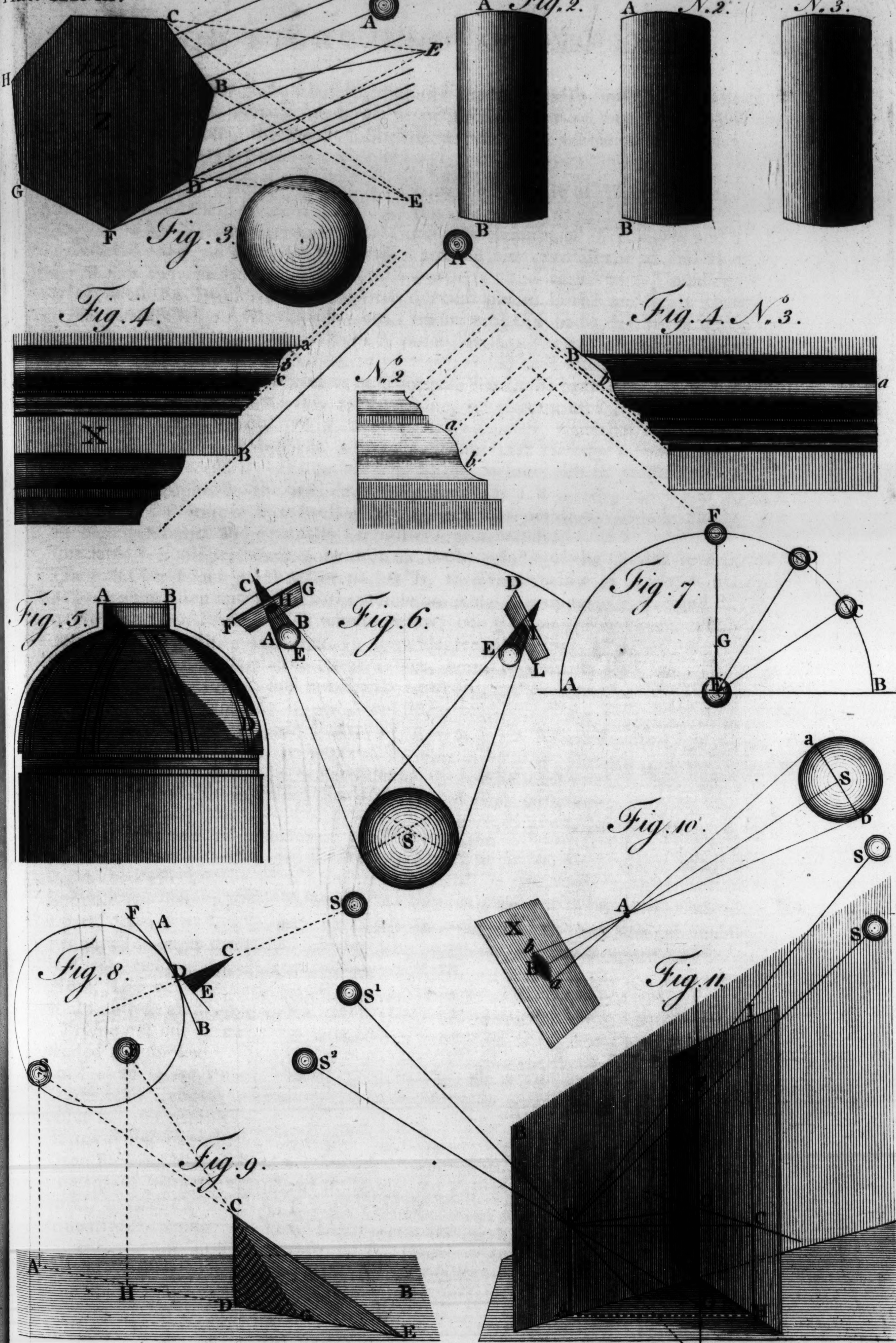
In respect of curved Surfaces, from the effect of Light, simply or directly, those parts which are most prominant, towards the luminous Body, are the brightest; and consequently, those which are farthest removed from the Light will (without reflection) be the darkest.

On cylindrical Surfaces, as Columns, streight Mouldings, &c. the Light is always parallel to the sides, or edges of the Mouldings; the greatest Light being
Fig. 2. on that part which is most perpendicularly opposed to the luminous Body (as AB) from which it graduates regularly, on each Side; consequently, that side which is farthest from the Light will be the darkest, supposing the Luminary to be situated on this Side of the Object, on either hand. But, if it be supposed situate on the other Side, the Edge, AB, towards the luminous Body, will be the brightest; and it will be graduated from that Edge, not to the other, as in No. 2. (which has not the appearance of an entire Column or Cylinder, but of a Segment, cut off parallel to its Axe) but, it will be darkest somewhat from the other Edge (as No. 3.) not owing to reflection from other Objects, but from the luminous Body, being on the other Side; which, as it is more or less direct, will occasion the darkest part to be more or less removed from the middle.

Convex spherical Surfaces are brightest on that part to which the luminous Body is perpendicular, from which it graduates every way equally; insensibly varying, to the extremes, provided no other Object interfered, to reflect Light.

If

Plate XXXVIII.



Publish'd by T. Malton, May 21st 1775.

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If a Hemisphere, or lesser Segment, be properly Shaded, it will represent equally as well a concave as a convex Surface, by supposing the Light on one Side or on the other; that is, when the Eye is so situated as to see the whole circumference of its Base, or nearly; as Fig. 3.

Respecting streight mouldings, which are composed wholly of Planes and cylindrical Surfaces, their different situations, towards the flow of Light, occasion variety of Shades. As for example; the Cima-recta (composed of a convex and a concave cylindrical Surface) in its proper position, in a Cornice, &c. has two strong Shades and one Light, in the middle; whereas the very same Moulding, reversed, as for a Base, has two Lights, and one Shade, in the middle, with a faint one at each Edge. The Cima-reversa, in its proper position, has two Lights and one strong Shade, in the middle; the same Moulding reversed, has two faint Shades and one Light, in the middle.

Let X be the Profile of a Cornice, and suppose the Light to flow in the direction AB, at discretion. It is easy to determine, what parts of the Cornice will be Light, and which shaded. For, drawing several Lines, touching the projectures and prominances, parallel to AB, it is obvious that the edge a, obstructing the Light, must necessarily occasion a strong Shade, below; which gradually dies away into the Light, in the middle, where the Rays fall directly on it, at *b*; and where, the prominance or swell of the convex part occasions another Shade, at c; begining faint, and gradually strengthening, reversely. Fig. 4.

The same reasoning accounts for the Shade in the middle of the Cimma-reversa.

The great projecture of the Corona, at B, throws all below in Shade, which gives great expression and force to the whole, making the upper part to stand off, from the Canvas or Paper. The whole, below, being immersed in Shade, would be totally lost to sight, was it not, in some degree, illumined by reflection; on which I shall speak in its place; let it suffice, here, to observe, that the effects are almost wholly reversed, and but faintly expressed.

Let No. 2. represent the upper Mouldings reversed, for a Base Moulding. No. 2.

Here, it is obvious, that, the Rays, falling on it in the same direction, illumines the whole; insomuch that, no part can be said, with propriety, to be in Shade; nevertheless, the parts *a*, *b*, &c. which are most directly opposed, will be brighter than the other, which are faintly shaded, as the Surface falls off.

I shall just make an observation on the prevalent Custom of shading Mouldings, in architectural Designs, as strongly illumined by the Sun; which entirely destroys their effect.

Suppose AB (No. 3.) to represent a Ray from the Sun. It is obvious, that all the part, from B to *b*, will be in Shade; and, being a Shadow, projected by the upper edge, strongly defined by a Right Line (as *a*a) has not a very agreeable effect. The Shade, below, is also more sudden and hard. No. 3.

Now, if by means of Light and Shade, it is intended to give an Idea of Mouldings, I would ask, seriously, which has the most natural appearance and effect? Suppose the Profile cut off, or covered; would any Person conceive what this Moulding is intended to represent? But, where is the necessity to suppose the Sun to shine on them? as there are strong Shades when it does not; nor are they intended as Pictures, but Designs, which ought to exhibit what is intended, in the most expressive manner possible.

If my opinion might be allowed to have any weight, I should suppose that a Section shewing the inside of a Building, geometrical, would be more expressive, by means of penumbral, instead of strong edgy Shades, which are by no means natural, in such Cases. Nay, so fond are some Architects of forced Effects, that we frequently see a strong right line of Light introduced into a Section of a Dome, through an opening at the Top (AB) as strongly defined on the part at C, which is nearly horizontal (where it is absolutely impossible for Light to come) as elsewhere, at D; whilst the concave Cylinder, below, is shaded without any edge at all. Fig. 5.

Plate XXXVIII Similar to this are the forced and unnatural Effects, which may frequently be seen, in Prints and Paintings, of a strong Light being introduced, striking in a Right Line, from the upper edge of the Cornice, across the Side of a Room, as if it was laid open. Also, on exterior Objects, a strong diagonal Shade, frequently crosses a Building, without the least apparent Cause for such an Effect.

There is (to me) another great impropriety; which, though it does not come imediately within my cognizance, I shall beg leave just to mention; which is, the introducing Landscape, and some appearance of Perspective, in a geometrical Design. I am well assured that it need but be pointed out, to convince any sensible Person of its inconsistency, as a Picture; exhibiting a geometrical Elevation of a Building, placed upright in a Garden, &c, where every thing else is real.

SECTION II.

The THEORY of SHADOWS, projected by the Sun.

IN treating of Shadows, it is necessary, first, to consider the Situation, Distance, and Magnitude of the luminous Body, by which the Shadows are projected. Secondly, the Situation of the Object to the Luminary, and, of the Lines forming Planes, &c. whose Shadows are to be projected; their Position in respect of some other Plane, on which the Shadow is to be projected; and lastly, the Position of the Picture, in respect of the Luminary.

The Sun, which is the grand fountain of Light, being placed in the wide Expanse, dispenses its light, equally, all around, and illumines a concave Sphere, as far as its influence extends; which differs, in degree of Splendor, according to the Distance from the luminous Body. If an opake Body be interposed, that Body is illumined on its Surface towards the Sun; and consequently, must deprive some part of the Expanse of Light; and, if another opake Body intervene, the Shade, or Shadow of one, is thrown upon, or projected on the other.

A SHADOW, therefore, is the outline projection of one Object on another, Plane or other Superficies, by Right Lines, from some luminous Body to the PLANE of PROJECTION; i. e. the Plane on which the Shadow is projected.

By RAYS of LIGHT is to be understood Right Lines, from the Luminary, passing by the Extremes of an Object, and projecting its Shadow.

Fig. 6. To illustrate what has been advanced, suppose the Body, a Globe, at S, to represent the Sun, and CD a portion of a concave Sphere, illumined by its Light; let E be supposed an opake Body, illumined on the Surface towards the Sun.

Now, if this opake Body be also a Globe, representing the Earth or any other Planet; the Sun being, in proportion to it, immensely great, the Space deprived of Light, in the Expanse, will form a Cone; as ACB.

If a Plane, or any other opake Surface, be placed at FG, within the extent of the Cone, it will receive, thereon, the Shadow of the opake Body, at H; which Projection, if the Plane be direct to the Sun, will be a Circle; but, if the Plane be inclined, as DL, it will form an Ellipsis, at I. These Projections, or Shadows, are less than the Object; because the luminous Body is greater. But, if the luminous Body be equal to the Object, or at an infinite Distance; the Shadow, on the direct Plane, will be equal to the Object, and may be considered as its orthographic Projection.

Hence it is evident, that, from the magnitude of the Sun in proportion to the Earth, the Rays of Light, from its extremes to the Earth, are converging; but, when its immense Distance is considered, they are, to all sense, parallel. Then, seeing that the largest Object, on the Earth, can scarce be said to bear any proportion to the whole, we may consider them as being perfectly parallel.

Next, the Altitude of the Luminary is to be considered, so as to give the best and most pleasing effect to the Picture; this is generally at the discretion of the Artist. It may be necessary to explain what is meant by its Altitude.

Let the Semicircle AFB be supposed an Arch in the Heavens, in which is the Sun's *apparent* diurnal motion; and let E be the Earth, *supposed* at rest, in the Center of the Universe. Fig. 7.

If the Sun be at C, then, the Ark CB is the measure of its Altitude; and, the Angle CEB is the Angle of Elevation; EB, being considered as the Horizon, and CE the direction of the Rays of Light. If the Sun be at D, then, DEB is the Angle of the Inclination of the Rays of Light to the Horizon, and the ark DB the measure of its Altitude; but, when its place is at F, in the middle of the Arch (90 Degrees, each way, from A and B) its Rays have, then, no Inclination to the Horizon, AB; and consequently, a perpendicular Line, GE, will project no Shadow. This can only happen to those parts which lie between the Tropics.

Lastly; the Angle of Inclination which a vertical Plane, passing through the center of the Luminary, makes with the Picture, is necessary to be known, when it is not in the Plane of the Picture.

Now, although the Rays of Light, proceeding from the Sun, are parallel amongst themselves, in respect of the Earth, yet their inclination to its Surface, varies infinitely.

Suppose a Globe illumined by the Sun, at S, whose Center only is considered *. Let AB be the Horizon of the part at D, to which the Sun is directly opposite; the Ray SC is perpendicular to the Horizon, AB; and consequently, a perpendicular Line, as CD, to the Horizon of that place, can have no Shadow, but will be projected towards the Center. But, if any other Tangent, as EF, be drawn; the direct Ray SC, to the Globe, is inclined to it, in the Angle SDE; and consequently, Lines perpendicular to EF will project Shadows. As CE to D. Fig. 8.

To illustrate this more clearly; suppose the Plane AB, horizontal, a portion of the Earths Surface, to which the Rays are inclined, and SE the direction of a Ray of Light. The Angle of Inclination, to the Plane AB, is SED; and, the length of the Shadow of the Perpendicular CD, is DE. Fig. 9.

Hence it is manifest, that the Shadow of a Right Line, on a Plane, is always a Right Line (1. 7. El) for it is projected by a *Plane of Shade*, occasioned by the Line; as CED, which cuts another Plane, whose common Section is the Shadow.

And, it is evident, that the greater the angle of Elevation is, the shorter is the Shadow; for, FE projects the same Point, E, to G.

In projecting Shadows by the Sun, there are three Cases to be considered, or situations of the Luminary; viz. it must be either on this side, or on the other side, or in the Plane of the Picture.

* In projecting Shadows by the Sun, in order to describe the true Contour, or outline of the Shadow, it is necessary to consider it as a luminous Point only, at an infinite Distance; for, if its magnitude was taken into consideration, since every part of its Surface emits Light, the Shadow of a Point, at any Distance, would always be, in proportion to the Sun, as the distance of the same Point from the Sun; as in Fig. 10. S is supposed the Sun, and A, a Point whose Shadow is projected, on the Plane X.

Now, the whole extent of the Shadow of the Point A is a Circle, whose Diameter is *a b*; for, since it is obvious, that every part of the Sun must emit Light; consequently, a Ray of Light emited from a, and passing through the Point A, will project its Shadow to a; and a Ray from b will project the same Point to b &c. while the Center (S) only, being considered, will project it to B, which is the center of the Shade, and its real Shadow. For (supposing the Point to have substance) any other part of the Shadow, as at c or d, is more languid, the farther it is from the Center, B; consequently, at its extremes, it cannot be distinguished from the surrounding Light; seeing that, every Point, save a or b, in the Sun's surface, emits light to *a* or *b*. Wherefore, since the Triangles, aAb, *aAb*, are similar, it will consequently be, as, *Aa* is to Aa, so is *ab* to ab, that is, as AB to AS. Fig. 10.

Hence, it is easy to account for the Penumbra of Shadows; which, at a distance, appear distinctly defined, but on approaching near, we find it otherwise; insomuch that, except where the Lines, in any Object occasioning the Shade, cut the Surface on which the Shadow is projected, we cannot trace a Line at all; and the farther the Shadow is from that Point, the more penumbral it becomes; that is, the less distinctly defined; till, at a considerable distance, it becomes insensibly mixed with the Light.

Plate XXXVIII

In order to determine the Shadows of Objects, delineated on the Picture; having first considered and determined on the situation and altitude of the Luminary, the next thing requisite, if the luminous Body, or Point (for it is always considered as such) be not in the Picture, is to find the Vanishing Point of the Rays of Light; but, when it is in the Plane of the Picture, the Rays are parallel on the Picture, and consequently they have no Vanishing Point; the Angle of Elevation is, then, only necessary to be considered.

Fig. 11.

Let FGHI be the Picture, and EC the Distance of the Picture. Also, let SE be supposed a Ray of Light.

Imagine the Plane ABFG to pass through the Luminary, and through the Eye; at E, cuting the Picture in FG; in which Line, whether the Sun be on this, or on the other side of the Picture, as at S, its apparent, or transprojected place must necessarily be; as at F or G.

For, being on the other Side, at S (supposed at an immense Distance) and being in the Plane ABFG, consequently ES is in that Plane, and will cut the Picture in their common Intersection, FG, at F. Therefore, F represents the Sun on the Picture, and is consequently the Vanishing Point of the Rays of Light.

When the Sun is on this Side, at S^1, or S^2, seeing it is still in the same Plane, ABFG, consequently S^1, or S^2E, being produced, will also cut the Picture in FG, as at G; and because all the Rays are parallel amongst themselves, and all Lines which are parallel have the same Vanishing Point, consequently, G is their Vanishing Point. (Def. 22.) for $S^1.G$ is a Ray of Light, to which all other Rays are parallel; or, it is a Right Line, passing through the Eye, parallel to them.

N. B. In the former Case (when the Sun is on the other Side of the Picture) it is obvious, that its place, in the Picture, must necessarily be above the Horizontal Line, as it cannot be seen till it is above the Horizon; so, in this case (seeing it is transprojected) being above the Horizon, its transprojected place must necessarily be below the Horizontal Line; from which (in either Case) it is farther removed, as the Luminary is more elevated above the Horizon. Also, whether its real place be on the right Hand, or on the left, its apparent place is the same, in the former Case, but reversed in the other. (See TRANSPROJECTION, in the Introduction; Page 52.)

Hitherto I have proceeded introductorily, which, I have called the Theory of Shadows; seeing that, all which has been said is theoretic. I shall now proceed to practice, and lie down such Rules as are necessary, for projecting the Shadows of all regular Objects, particularly such as are right lined; which may be described on plane Surfaces, with all the facility imaginable, by adhering to the following Rules.

FIRST. The indefinite projection of the Shadow of a Right Line, on any Surface whatever, is the Intersection of that Surface by a Plane, passing through the luminous Point and the Line; which, for distinction sake, I shall call the PLANE OF SHADE. This is obvious in itself.

SECOND. The Vanishing Line of the *Plane of Shade*, projecting the Shadow of any Right Line, is a Right Line drawn through the Vanishing Point of the Rays of Light, and the Vanishing Point of the Line whose Shadow is required (Th. 11.)

Because, the Vanishing Point of the Line, whose Shadow is projected, and the Vanishing Point of the Rays, projecting the Shadow, are in the Plane of Shade.

THIRD. The Vanishing Point of the Shadow of any Right Line, on a Plane, is the intersecting Point of the Vanishing Line of that Plane; and the Vanishing Line of the PLANE OF SHADE. (Cor. 2. Theo. 8.)

Because, the common Intersection of those Planes is the Shadow required.

N. B. When the Sun is in the Plane of the Picture, there being no Vanishing Point of its Rays, a Right Line, drawn through the Vanishing Point of any Line whose Shadow is required, parallel to the given Ray, cuts the Vanishing Line of the Plane of projection, in the Vanishing Point of the Shadow.

Also, if the Original Line be parallel to the Picture, and consequently has no Vanishing Point; (the Sun being on either side of the Picture) then, a Right Line drawn through the Vanishing Point of the Rays, parallel to the Line whose Shadow is required, cuts the Vanishing Line of the Plane of projection, in the Vanishing Point of the Shadow.

But, when the Luminary is in the Plane of the Picture, and the Line, whose Shadow is required, parallel to it, the Shadow has no Vanishing Point; for it is, in such Case, necessarily parallel to the Picture; therefore, parallel to the Vanishing Line of the Plane of projection.

SECTION III.

Of the PROJECTION of rectilinear SHADOWS.

Plate XXXIX.

PROBLEM I.

Having the Angle of the Sun's Elevation (X) and the Inclination, to the Picture, of a vertical Plane passing through its Center (equal Z) with the Center and Distance of the Picture given; to find the Sun's place on the Picture, or the Vanishing Point of the Rays of Light.

Let CE be the horizontal Vanishing Line, and C the Center of the Picture. Fig. 12.

Make CB equal to the Distance, perpendicular to CE, and draw AB parallel to CE.

Draw BD, making the Angle ABD equal to Z, cuting CE, at D; and through D, draw FG, perpendicular to CE. FG is the Intersection of a vertical Plane, passing through the Sun, with the Picture, whose inclination to it is equal Z.

Make DE equal to DB, and the Angle DEF, or DEG, equal to X, the Angle of the Sun's Altitude; F or G is the Vanishing Point sought.

DEM. Turn up the Triangle CBD, perpendicular, and turn over the Plane FEG, or FG, till DE coincides with DB.

Then, if the Sun be on the other side of the Picture, EF is parallel to the Suns Rays; because the Luminary is in the Plane EFG, and, the Angle DEF is equal to its Elevation; consequently, since its Distance is, to sense, infinite, EF produced would pass through the Center of the Luminary, wherefore, F is the place of the Sun on the Picture; i. e. F represents the Sun; consequently all the Rays center there, and therefore it is their Vanishing Point. For its Distance is infinite.

CASE 2d. When the Sun is on this side the Picture, being behind the Spectator, it cannot appear in the Picture, but its place is transprojected to G; for, DEG is the Angle of its Elevation, and it is in the Plane FEG; wherefore, EG is parallel to the Rays of Light; consequently, G, the transprojected place of the Luminary, is their Vanishing Point.

Or, as Brook Taylor, very pertinently, calls it, the Shadow of the Spectator's Eye, on the Picture; for E is the Eye, wherefore GE, being produced, would pass through the Sun's Center.

PROBLEM II.

The Vanishing Point of the Sun's Rays being given, and the representation of a Line perpendicular to some Plane, whose Vanishing Line is given; together with the Center and Distance of the Picture; to find the representation of the Shadow of that Line, on the Plane, and its Vanishing Point.

Let S be the Vanishing Point of the Rays, VL the Vanishing Line of the Plane, and A B the given representation of a Line perpendicular to that Plane. C is the Center of the Picture, whose Distance is known. Fig. 13.

Find F, the Vanishing Point of Lines perpendicular to the Plane, whose Vanishing Line is VL (Prob. 2. Sect. 12. B. 3.) consequently, AB vanishes in F; draw SF, cuting VL at V. V is the Vanishing Point of the Shadow.

Draw VB and SA, intersecting at D; BD is the Shadow of AB, required.

DEM. Whether S be considered as the Image of the Sun, on the Picture, or S its transprojected place, SF is the Vanishing Line of the Plane of Shade, for all Lines perpendicular to that Plane; seeing that, F is the Vanishing Point of all such Lines; wherefore, V, its intersection with VL, is the Vanishing Point of the Shadow, of all Lines perpendicular to the Plane whose Vanishing Line is VL.

For, SV and AB represent parallel Lines (Cor. 1. Theo. 5.) wherefore, a Plane may pass through both Lines (Ax. 5.) and consequently, SA, VB, AF, and SF are all in that Plane.

But, F is the Vanishing Point of Lines perpendicular to the Plane; and, because S represents a Point at an infinite Distance in that Plane, SF is the Vanishing Line of a Plane passing through AB.

But, S represents the Sun, the Vanishing Point of the Rays of Light, and V is its Seat on the Plane (whose Distance is supposed infinite) consequently, V is the Vanishing Point of the Shadow, BD; for, the Plane of Shade, SABF, projecting its Shadow, is parallel to a Plane passing through the Eye and the Points S and F, seeing they are at an infinite distance; and the Shadow, BD, is the intersection of that Plane with the Plane of Projection; therefore its Vanishing Point is V, the intersection of their Vanishing Lines, VL, and SF. (Cor. 2. Theo. 8.)

Plate XXXIX. This Problem is univerfal; I fhall, next, apply it to Planes which are perpendicular to the Picture.

Fig. 13. No. 2. SECONDLY. Let AB be a Line perpendicular to the Horizon, whofe Shadow is required, and S the reprefentation of the Sun, on the Picture; or *S* its transprojected Image, the Sun being fuppofed on this Side.

Draw SV perpendicular to the Vanifhing Line; then, V is the Seat of the Luminary, its Diftance being fuppofed infinite.

Draw VB and SA, as before, interfecting at D; BD is the Shadow required.

For, becaufe AB, whofe Shadow is to be projected, is perpendicular to the Horizon, and the Picture is fuppofed vertical, SV is the Vanifhing Line of the Plane of Shade; wherefore, V, the Seat of the Luminary, is the Vanifhing Point of the Shadow; and, SD is a Ray of Light, projecting the Shadow of the Point A, which determines its length, BD.

This is univerfally applicable to all Planes which are perpendicular to the Picture, whether they be horizontal, vertical, or inclined to the Horizon.

THIRDLY. When the Luminary is in the Plane of the Picture.

In this CASE, the Rays having no Vanifhing Point, the inclination of the Rays to the Plane of projection, being determined, they are all parallel on the Picture feeing, the diftance of the Vanifhing Point is infinite.

No. 3. Draw BD parallel to the Vanifhing Line, and SA, cuting BD at D, making the Angle SDB equal to the Angle of the Sun's Elevation; BD is the Shadow required.

For, becaufe the Luminary is fuppofed in the Plane of the Picture, it is confequently in every Plane parallel to the Picture; wherefore, the Plane of Shade, SDB, is parallel to the Picture, feeing the Line AB is parallel; and confequently, BD, its Shadow, is parallel to the Vanifhing Line of the Plane of Projection, feeing it is parallel to the Picture.

If the Plane, to which a Line is perpendicular, be parallel to the Picture, there can be no Shadow of that Line projected on it, but when the Luminary is on this Side.

No. 4. Let AB reprefent a Line perpendicular to the Plane X; and, let S be the transprojected Image of the Sun, on the Picture; C is its Center. Join CS.

Draw BD, parallel to CS, and AS, cuting BD, at D; BD is the Shadow required.

Becaufe the Plane (X) on which the Shadow is to be projected, is parallel to the Picture, it has no Vanifhing Line; wherefore, feeing the diftance of the Luminary is fuppofed infinite, confequently its Seat, on that Plane, is alfo at an infinite Diftance.

But CS, produced, is the Seat of a Ray of Light, projecting its Vanifhing Point S; for it paffes through the Eye, which is perpendicularly oppofite to C; wherefore, the Seat of the Luminary is at an infinite diftance, in the Line SC produced. Confequently, the Shadows BD, *BD*, &c. are all parallel to CS; and S is the Vanifhing Point of the Rays, AS, *AS*, &c. which determine the length of the Shadow BD, &c. which, being of Lines perpendicular to a Plane that is parallel to the Picture, are confequently all parallel amongft themfelves.

PROBLEM III.

To project the Shadows of Right Lines, on a Plane to which they are parallel; and, in all pofitions to the Picture.

FIRST, when the Lines are parallel to the Picture, and the Luminary in the Plane of the Picture.

The Shadows of Lines, on a Plane to which they are parallel, cannot be determined, conveniently, without having their Seats on that Plane; and is the fame as finding the Shadows of Lines perpendicular to the Plane; or, the Shadow of one extreme of the Line being found, the other is eafily determined.

Fig. 14. Let AB be a Line parallel to the Picture, and parallel to the Ground Plane, on which the Shadow is to be projected. Its Seat is *a b*.

If the Sun be in the Plane of the Picture, and if it was in the Zenith of that place, the Shadow of AB is *a b*. But, if the Altitude of the Sun be the Angle SCD; then, drawing AD, parallel to SC, the Shadow of AB is CD; for, the Shadow of a Perpendicular, A*a*, is *a* D; and CD is equal AB. feeing, ABCD, the Plane of the Shadow, is a Parallelogram.

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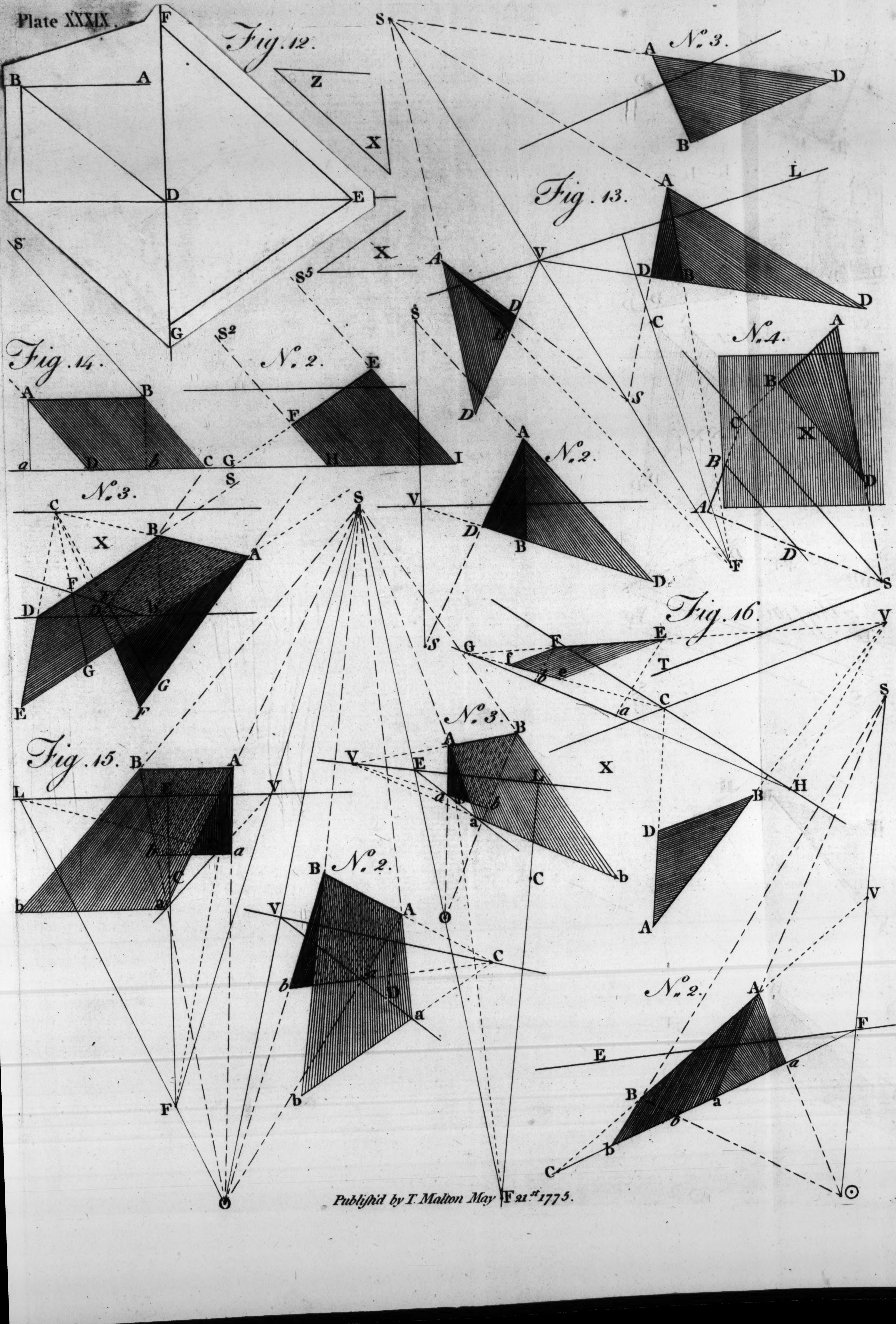
Plate XXXIX.



Publish'd by T. Malton May 21st 1775.

After the same manner, the Shadow of the inclined Line, EF (being parallel to the Picture) is projected; by producing the Line till it cuts the Plane, at G; then, drawing GI, parallel to the Vanishing Line, and S^2H, S^3I, through F and E, parallel to SC, cuting GI, at H and I; HI is its Shadow. No. 2.

SECONDLY. If the Line AB be either perpendicular or inclined to the Picture, and parallel to the Horizon, its Seat, at least of one extreme (B) must be determined; which, as it may be in some vertical Plane, (as X) its Seat on the Ground is in the Intersection of that Plane, at b; and because the Plane X is parallel to the Picture, the Luminary is in that Plane; and consequently, the Shadow of the extreme B will be somewhere in the Intersection of that Plane. No. 3.

Draw SD, through B, making the Angle SDb equal to the Altitude of the Sun, giving the Point D, in the Intersection of the Plane X, for the Shadow of B.

Then, because AB is perpendicular to the Picture, C, is its Vanishing Point; and, because the Shadow is projected on a Plane, to which AB is parallel, the Shadow is necessarily parallel to AB, and consequently it has the same Vanishing Point, C.

Wherefore, draw CD and produce it; draw AE parallel to SD, (the given Ray of Light) cuting CD, produced, at E, the Shadow of A.

DE is the Shadow of AB on the Ground; to which it is parallel and equal.

For the Plane of Shade ABDE is a right angled Parallelogram in Perspective.

FG is its Shadow on an inclined Plane, to which AB is parallel; projected by the same Rays of Light, or Plane of Shade.

SCHOL. Because the Sun is in the Plane X, it cannot be said to be illumined; and consequently, no Shadow can be projected on it; otherwise, if the Luminary was ever so little on this Side, the Shadow of AB will be first projected on it, from B to D, and then projected to E; in which Case, BD would not be a Ray of Light, but a Shadow, and consequently, not parallel to AE.

CASE the Second and Third.

When the Luminary is on this Side, or on the other Side of the Picture.

FIRST. Let AB be parallel to the Picture, and to the Plane, on which its Shadow is to be projected. Let S be the Image of the Sun, on the Picture; or ⊙ its transprojected Image. Let VL be the Vanishing Line of the Plane of projection. Fig. 15.

Now, the Line AB being parallel to the Picture, has no Vanishing Point; consequently, its Shadow, being on a Plane to which it is parallel, is parallel to the Line. Let D be the Seat of A on the Plane; C is the Center of the Picture.

Because the Plane, on which the Shadow is to be projected, is inclined to the Picture, through C, the Center, draw its Vertical Line EF; and because D is the Seat of A, on the Plane, draw AD and produce it, cuting the Vertical Line at F, the Vanishing Point of Lines perpendicular to the Plane.

For the Seat of a Point on any Plane is produced by a perpendicular to that Plane.

Draw SF, or ⊙F, cuting the Vanishing Line at V, or L, the Seat of the Luminary on the Plane, at an infinite Distance.

Draw DV, or DL; and SA, or ⊙A, cuting DV or DL, at a or *a*, the Shadow of the extreme A, on the Plane. Draw a b, or *a b*, parallel to the Van. Line, that is to AB; and, through B, draw SB, or ⊙B, cuting ab, or *a b*, at b, or *b*.

a b is the Shadow of AB by means of S the Image of the Sun, on the other side of the Picture, or, *a b* is its Shadow, projected by the Vanishing Point ⊙, the transprojected Image of the Luminary, on this Side; both which, are Vanishing Points of the Rays of Light. a D, or *a* D, is the Shadow of the Perp. AD.

SECONDLY. When the Line AB is perpendicular to the Picture, and parallel to the Plane of Projection; C is the Center. No. 2.

Because the Line whose Shadow is to be projected is perpendicular to the Picture, and parallel to the Plane of Projection, that Plane is consequently perpendicular to the Picture, and its Vanishing Line VC passes through the Center. (Theo. 4.)

Let

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Plate XXXIX. Fig. 15. No. 2. Let D be the Seat of the extreme A, on the Plane; and let S, or ⊙, be the Image of the Luminary; projected or transprojected.

Draw SV, or ⊙V, perpendicular to the Vanishing Line; and, through D, draw VD indefinite. Draw SA, or ⊙A, till it cuts VD, at a or *a*, the Shadow of the extreme A, on the Plane, in either Case.

For, AD is perpendicular to the Plane, and D a or D*a*, is its Shadow. (Pr. 2.)

Draw C a, or C *a*, indefinite, and SB, or ⊙B, cuting it at b, or *b*.

a b is the Shadow of AB, the Sun being beyond the Picture; or, *a b* is the Shadow of AB, the Sun being on this Side, by means of its transprojected Image, ⊙.

For AB being parallel to the Plane of Projection, its Shadow is parallel to AB; consequently, because AB is perpendicular to the Picture, the Center, C, is their common Vanishing Point. And, because the Rays of Light are parallel amongst themselves, they have the same Vanishing Point, S, or ⊙.

N. B. When the Line, AB, is inclined to the Picture, and the Plane of Projection is perpendicular to it, there is no difference, in the process, but only in its Vanishing Point.

No. 3. Thirdly. When the Line AB is inclined to the Picture, and the Plane of Projection also inclined to the Picture. Let VL be its Vanishing Line.

Find the Vanishing Point (F) of Lines perpendicular to the Plane, (Prob. 2.) and draw SF, or F⊙, cuting the Vanishing Line at E, the Seat of the Luminary.

Thro' D, the Seat of A, draw ED indefinite, and SA, or ⊙A cuting ED, at a or *a*.

Then, V being the Vanishing Point of the Line AB, and because the Shadow is parallel to it, draw V a, or V *a*, indefinite, and SB, or ⊙B, cuting it, at b, or *b*; a b, or *a b*, is the Shadow of AB, projected on the Plane whose Van. Line is VL.

PROBLEM IV.

To determine the Shadows of Lines inclined to the Plane of Projection and to the Picture; in any Angle, whatever.

Fig. 16. Let AB be inclined to the Plane X, on which the Shadow is to be projected; and, let ST be a given Ray of Light, the Luminary being supposed in the Picture.

Through V, the Vanishing Point of AB, draw VC parallel to the given Ray of Light, cuting the Van. Line of the Plane X, at C, the Van. Point of the Shadow.

For, the Luminary being in the Plane of the Picture, its Rays have no Vanishing Point; consequently, a Right Line drawn through V, the Vanishing Point of any Line, parallel to the Rays, is the Vanishing Line of the Plane of Shade, projecting the Shadow of that Line.

Therefore, draw AC, the indefinite Shadow of AB; and BD parallel to the given Ray; that is, parallel to VC, the Vanishing Line of the Plane of Shade.

AD is the Shadow of AB on the Plane X.

If the Line EF, whose Shadow is required, does not cut the Plane, on which it is to be projected, let it be produced till it cuts the Plane, at G.

Or, if the Seat of either extreme be given, as *a* of the Point E, or *b* of the extreme F; then, having obtained the Shadow of either, e or f (Prob. 2.) draw C e, or C f; and Rays, through the extremes, E and F, parallel to the given Ray, ST; by which means the Shadow, e f, of EF is projected.

Case the Second and Third.

When the Luminary is beyond, or on this Side of the Plane of the Picture.

No. 2. Let AB be a Line, whose Shadow is to be projected. V is its Vanishing Point.

Let S and ⊙ be the projected and transprojected Images of the Sun.

If the Line does not cut the Plane of Projection, let it be produced, to C.

Draw SV or ⊙V, cuting the Vanishing Line EF, at F, the Van. Point of the Shadow. Draw CF; and SA, SB, or ⊙A, ⊙B, cuting it at a and b, or *a* and *b*.

a b

a b is the Shadow of AB when the Luminary is beyond the Picture, and *a b*, when it is on this Side; according to its determined place.

Or the Shadow may be found by means of the Seat of either extreme, given or found, on the Plane of Projection; as in the first Case.

SCHOL. Any Line, whose Shadow is to be projected, cuting the Plane of Projection, (whose Vanishing Line is given or found) the process is the same, whether the Plane of Projection be perpendicular or inclined to the Picture; for, if the Vanishing Point of that Line be obtained, the Vanishing Line of the Plane of Shade is also; whose Intersection with the Vanishing Line of the Plane of Projection is the Vanishing Point of the Shadow; which, it must be obvious, always passes through that Point in which the Line cuts or would cut the Plane.

2. When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this Side; in which Case, whether the Line be perpendicular or inclined to the Plane, it is the same to the Picture, and, its Vanishing Point being determined, the process is the same. In this Case, the Plane of projection having no Vanishing Line, the Shadow, of any Line thereon, is parallel to the Vanishing Line of the Plane of Shade.

3. All Lines which are parallel to such Planes are also parallel to the Picture; and their Shadows, being parallel to the Lines, are always so represented; which is not the Case on other Planes, but when the Lines are parallel to the Picture, as well as to the Plane of projection.

4. The Seats of such Lines, on the Plane, must be had, or the Shadow cannot be determined; in which Case, it is but finding the Shadows of two equal Lines, perpendicular to the Plane; and, joining their Extremes, the Shadow of the parallel Line is obtained, as in Prob. 2.

Having, in these Problems, given every Rule which I conceive necessary for the projection of right lined Shadows, I shall, next, give some Examples of their utility, in the application of them to the Shadows of Objects.

As every Object, which we delineate, is composed either of Planes or curved Surfaces, so likewise, those Planes are bounded by right Lines or curved; and, as the outline of the Shadow only is required, we should carefully observe on what part of the Object the Light falls, that we do not give the Shadows of such Lines as can (from their situation to the Light) have no Shadow; for, according to the altitude of the Luminary, or its inclination to the Picture, different Lines in the Object define its Shadow. Wherefore, since it is from the linear perspective representation of the Object, that the Shadow is projected, it is very easy to mistake the Line which casts the Shadow; and, as they are frequently out of sight, they ought to be determined with caution, before we begin the process.

The consideration of the Luminary being beyond, or on this Side, or in the Plane of the Picture, is also a circumstance which should be maturely considered. The best effects, are (in my opinion) produced, by considering the Sun to be on this Side; because it is obvious, that, when it is beyond the Picture, though ever so little, that is, though ever so much inclined, it is also beyond every Object in the Picture; but, if the Sun itself be represented, that is, if it be so little elevated and so little inclined to the Picture, that the Sun's apparent place is within its limits, it must then be entirely beyond the Objects; consequently, the whole of any Object, on that Side, is immersed in Shade; and the Shadow, of each, is projected towards the Eye, greatly distorted, and larger than the Object. Whereas, the Sun being on this Side, towards either hand, as is most suitable to the Object (according to what Faces we would have illumined) the Shadows are projected forwards, from the Eye, diminished, and great part of it hid by the Object; which, with the various effects of Light and Shade, on the Object, cannot fail rendering the whole agreeable, and most pleasing to the Eye, as a Copy of Nature.

EXAMPLE I.

To represent the Shadow of a Plane Figure, on a Plane.

FIRST. Let the Plane ABCDE be perpendicular to the Ground Plane, on which the Shadow is to be projected. Let S and ☉ be the Image and transprojected Image of the Sun; VL is the Vanishing Line of the Plane ABCDE, and FG of the Plane of projection. Fig. 17.

Draw SF, or ☉F, perpendicular to the Horizontal Line, cuting it at F; the Vanishing Point of Lines perpendicular to the Horizon. (Prob. 2.)

 Through

Plate XL. Fig. 17. Through A, draw Fb, indefinite; and, if the Shadow be projected by the Image, S, through B, draw SB, cuting the former, at b. Or, draw B⊙, cuting it at *b*.

Then, for the Shadow of BC (on this Side) join SV; if it be parallel to the Horizon, draw b c alſo parallel; if it be not parallel, then, SV produced will give its Vaniſhing Point. For the ſame Line on the other Side, join V⊙, cuting the Horizontal Line at H, and draw *b*H. SC, or C⊙, cuts it at c or *c*, reſpectively.

CD is parallel to the Plane of projection, and G its Vaniſhing Point; therefore, draw c G, or *c* G, and SDd, or D⊙, giving d, or *d*, for the Shadow of D.

Laſtly; for the Line DE (on this Side) draw SL, cuting the Horizontal Line at I, and draw I d. For the other Side, draw ⊙L; which being produced to the Horizontal Line, will give the Vaniſhing Point of *d* E, and compleats that Shadow. Or, join d E, or *d* E, only.

A b c d E is the Shadow of ABCDE projected on the Ground Plane, by the Image (S) of the Sun, on the other ſide of the Picture; and, A *b c d* E is its Shadow, by the tranſprojected Image, ⊙, being conſidered on this Side.

Fig. 18. Secondly. Let the Plane ABCDE be either perpendicular or inclined to a vertical Plane, whoſe Vaniſhing Line is VL.

Whether the Plane, whoſe Shadow is required, be perpendicular or inclined to the Picture, the proceſs is the ſame. FH is the Van. Line of the Plane ABCDE.

First. Let S be the Image of the Sun, on the other ſide of the Picture.

The Line, AB being parallel to the Picture, draw SV parallel to it; V is the Vaniſhing Point, of its Shadow; wherefore, through A draw V b indefinite, and, through B, draw S b, cuting it at b, which determines its length, A b.

H being the Vaniſhing Point of the contiguous Line BC, draw SH, and produce it to I, the Vaniſhing Point of its Shadow.

Draw b I, and S c, through C, cuting it at c. b c is the Shadow of BC.

Then, becauſe CD is parallel to the Plane of projection, and G is its Vaniſhing Point, draw c G; and SD, cuting it at d, the Shadow of D.

Laſtly, draw SF, giving the Vaniſhing Point J, if it was neceſſary; but it is obvious, that, d E, being joined, compleats the Shadow, and tends to J.

If the Shadow was obſtructed by another Plane, cuting it at MN, the Vaniſhing Line of which is FN; SV, being produced, would cut that Vaniſhing Line in the Vaniſhing Point of the Shadow of AB, on that Plane; from which, draw M *b*. SH produced, cuts the Vaniſhing Line at P; draw *b*P, cuting the Ray SCc at *e*; and, SG produced cuts it at O; draw *e*O; or join *ef* only, which compleats it.

Second. Let ⊙ be the tranſprojected Image of the Sun, on this Side.

AB being parallel to the Picture, draw ⊙K parallel to it, cuting the Vaniſhing Line, VK, at K, the Vaniſhing Point of its Shadow.

Draw AK, and B⊙, cuting it at *b*, the Shadow of B. ⊙H cuts the Vaniſhing Line at L; draw *b* L, and C⊙, cuting it at *c*. Then, draw *c* G, and D⊙, cuting it at *d*. *c d* is the Shadow of CD, being parallel to the Plane of projection.

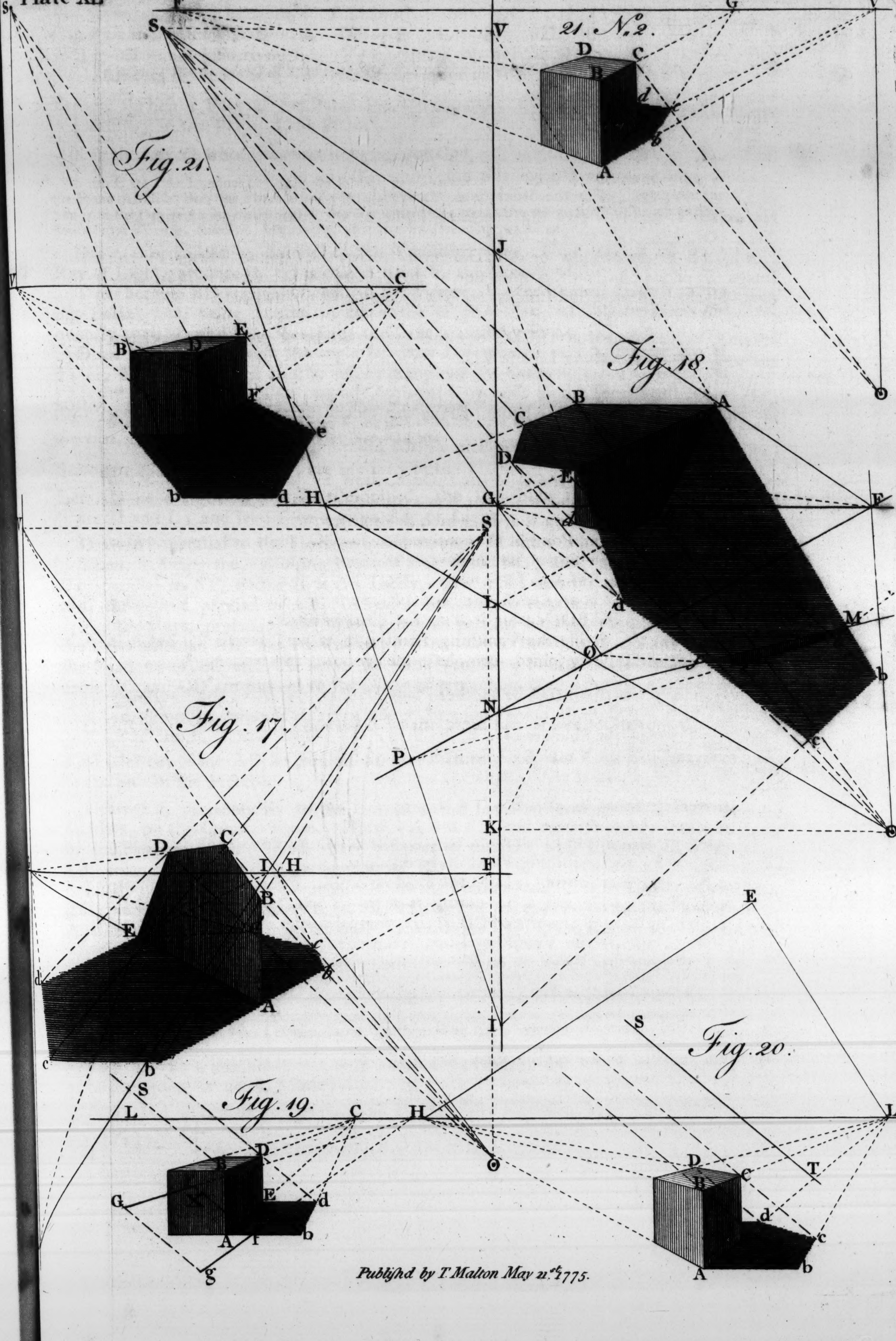
If ⊙F be parallel to the Vaniſhing Line, VK; then, *d* E is alſo parallel to it. Or, it will tend to that Point, in which ⊙F, being produced, would cut VK.

SCHOL. If the Plane of projection be much inclined to the Picture, there can be no Shadow caſt on it, the Sun being beyond the Picture; or they will be drag'd out to an immoderate length. For, if the Image of the Sun appear in the Picture, it will be behind the Plane of projection, being much inclined.

By obſerving the Rules, given above, the Shadow of any plane Object may be projected on any Plane, or on various Planes, of which I ſhall give ſome Examples.

EXAMPLE

Plate XL.



Publish'd by T. Malton May 21st 1775.

EXAMPLE II.

To project the Shadows of right angled Parallelopipeds, on the Ground Plane.

FIRST. When a Face of the Parallelopiped is parallel to the Picture, and, the Luminary in the Plane of the Picture.

X is the Object whose Shadow is to be projected. Fig. 19.

In this CASE, the Luminary being in the Picture, the Shadow of the whole Solid is no more than of the Plane ABDE. For, the Plane X being parallel to the Picture, cannot be illumined; seeing that the Sun is in the Picture, it is, consequently, in every other Plane parallel to the Picture. The whole Shadow of that Plane is, therefore, but a Line, generated by producing the Plane.

Draw A b, indefinite, parallel to the Vanishing Line, HL, and, if SB be a Ray of Light, produce it, till it cuts A b, at b, the Shadow of B.

Then because BD is perpendicular to the Picture, C, the Center, is its Vanishing Point; and, being parallel to the Plane of projection, its Shadow is consequently parallel, and has, therefore, the same Vanishing Point, C.

Draw b C; and through the angle D, draw D d parallel to S b, cuting b c at d; or, draw E d parallel to A b, which compleats the Shadow, A b d E, required.

SCHOL. If a Line, FG, projected from the Plane X, its whole Shadow will be projected on the Ground. For, since the Luminary is in the Plane X, no part of its Shadow can be projected on that Plane; wherefore, f g, its Shadow, is wholly on the Ground Plane.

SECONDLY. When its Planes are inclined to the Picture.

Let AD be a right angled Parallelopiped, the Vanishing Points of its Sides, are H and L; and let ST be a given Ray of Light. Fig. 20.

Draw A b parallel to the Horizon; and through B, draw B b parallel to ST.

Then, L being the Vanishing Point of BC, draw b L; and through C, draw C c, parallel to ST, cuting it at c. Lastly, draw c H; and D d parallel to ST; and, through d parallel to A b. A b c d is the Shadow required.

For the Rays, projecting the Shadow, are all parallel amongst themselves; and the Perpendicular AB, has its Shadow A b parallel to the Vanishing Line; also, the Shadows of BC and CD have the same Vanishing Points, respectively; because BC and CD are parallel to the Plane of projection.

CASE 2d. When the Luminary is on the other side of the Picture.

First; let the Plane AD be parallel to the Picture, and, let S be the Image of the Sun in the Picture. Fig. 21.

Draw SV, perpendicular to the Horizon, V is the Vanishing Point of Perpendiculars, on the Ground Plane. Draw VA and SB, and produce them, cuting at b, the Shadow of B. Then, draw b d parallel to BD; and, through D, draw S d, cuting it at d; b d is the Shadow of BD.

Lastly; because DE is perpendicular to the Picture, and C is its Center, draw d C; and, through F, draw V e, or SE, cuting it at e, and compleats the Shadow, AbdeF; which, as it is projected back, towards the Eye, is larger than the Object.

SCHOL. When the Object is inclined to the Picture, as in No. 2, the process is the same in every respect, having regard to the Vanishing Points of the Lines BC and CD, to which their Shadows, b c and c d, are respectively parallel, and consequently have the same Vanishing Points, V and F.

CASE 3d. When the Luminary is on this Side the Picture. No. 2.

Let AD be a Parallelopiped, right angled, as before; and let ⊙ be the transprojected place of the Luminary.

Draw ⊙ V perpendicular, and, draw AV and B⊙, cuting at *b*, the Shadow of B. Then, draw *b* G and C⊙, cuting at *e*; and lastly, draw *e* F, which compleats the Shadow A*bed*. The Shadow of the Angle D is not seen.

EXAMPLE

Plate XLI.

E X A M P L E III.

To project the Shadow of a plain Building, on the Ground.

Fig. 22. Let ADF represent a Building obliquely situated to the Picture; and let the Sun be in the Plane of the Picture. The direction of its Rays is SC,

Draw A b parallel to the Vanishing Line, and B b to the given Ray, cuting A b, and determining the Shadow of AB.

Through V, the Vanishing Point of BC, draw VG parallel to the given Ray of Light; and draw b G; SC produced, cuts it at c, the Shadow of C.

Because the Vanishing Point of CD is below the Horizontal Line, equal VH, the Vanishing Point (I) of its Shadow, will be on the other side VH, equal GH.

Make HI equal GH, and draw c I; and D d parallel to SC, cuting it at d.

Because of the Sun's altitude, and the situation of the Building, the other side of the Roof is illumined; and consequently, the Shadow of CF (the Ridge) is not projected, which is evident; because, being parallel to the Plane of projection, its Shadow is parallel to itself, and consequently has the same Vanishing Point, L. Wherefore, if from c, the Shadow of C, a Line be drawn to L, it will fall on this Side c d, the Shadow of CD; consequently it has no Shadow, here.

Therefore, draw d L, cuting DE; d e is the Shadow of a Line from D, parallel to CF (the Eave of the Roof) on the other Side of the Building.

SCHOL. This Example is according to the Shadow, on the direct Picture, in the Apparatus.

SECONDLY. Let the Sun be supposed beyond the Picture, and let S be its Image.

Draw ST, perpendicular; and through A, draw T b indefinite; through B,

Fig. 23. draw S b, giving A b, for the Shadow of AB.

Then BC being parallel to the Horizon, and L its Vanishing Point, through b, draw L c, and through C draw S c, giving b c, the Shadow of BC.

V being the Vanishing Point of CD, draw SV. If SV, be parallel to the Horizon, draw c d, also parallel; if SV, be not parallel, produce it, till it cuts the Horizontal Line, on one Side or the other, and draw c d, tending to that Point; also, through D, draw S d, cuting it at d, the Shadow of D.

Again. Y being the Vanishing Point of DE, draw SY, cuting the Horizontal Line at I, the Vanishing Point of the Shadow DE.

Draw d I, and SE till it cuts d I, at e; and lastly, join e F, which will tend to its Vanishing Point T, and compleats the Shadow, A b c d e F.

In this Case, it is obvious that the whole Building, save the Plane DB, is in Shade.

CASE 3d. When the Luminary is on this Side of the Picture.

Let be its transprojected Image. $\odot$K being drawn, perpendicular, K is the Vanishing Point of the Shadows of perpendicular Lines. (Prob. 2.)

Draw AK and B$\odot$, intersecting at *b*, the Shadow of B.

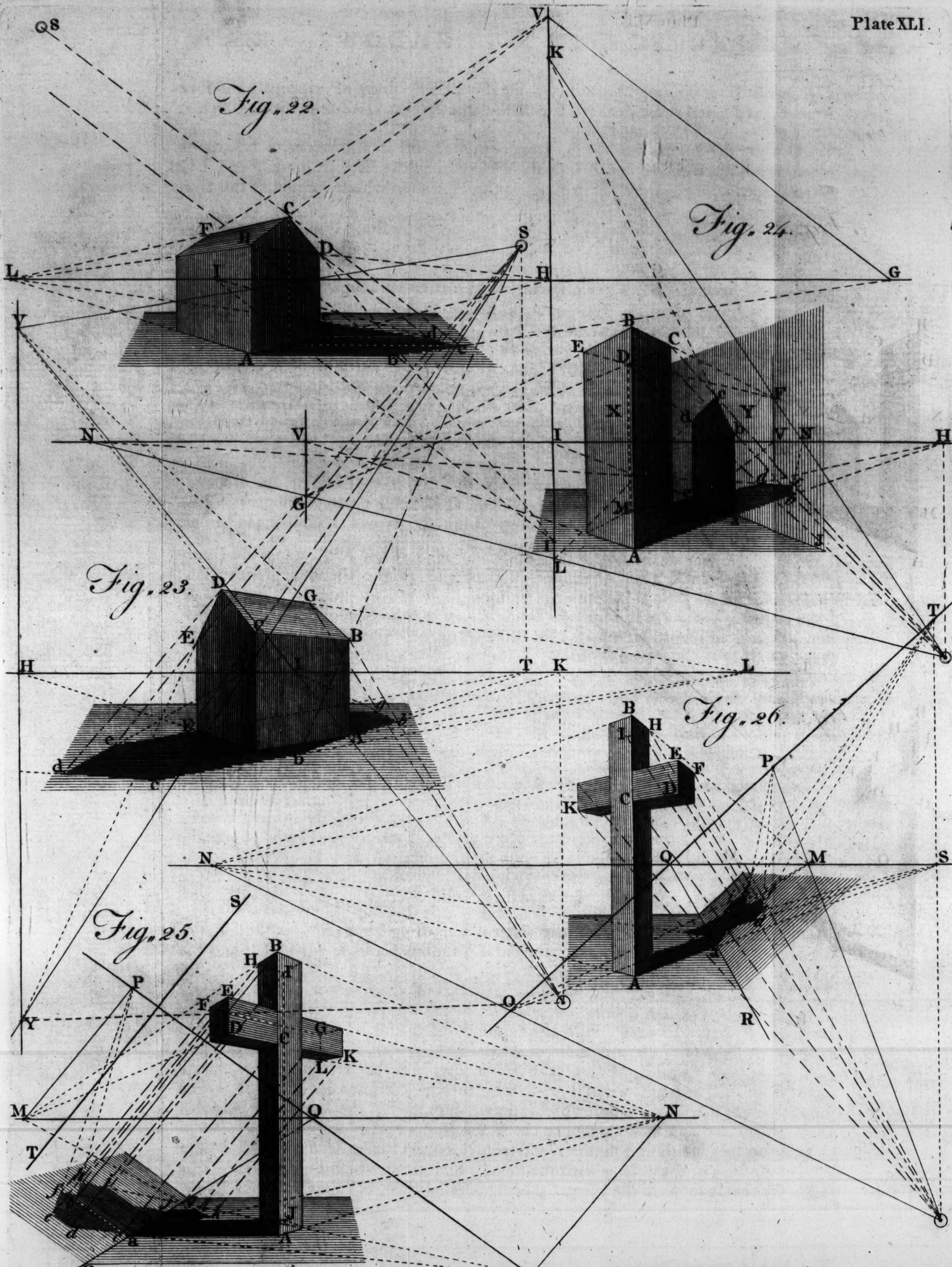
Then draw $\odot$V, cuting the Van. Line at I, the Van. Point, of the Shadow of BG.

Draw *b* I, and G$\odot$, cuting it at *g*, the Shadow of G.

Join $\odot$Y; and if it be not parallel, produce it to the Vanishing Line.

Through *g*, draw parallel, or tending to the Point in which $\odot$Y, cuts the Horizontal Line; which compleats the Shadow, as much as can be seen.

I presume, it is obvious, from these Examples, that the most judicious choice, is to suppose the Luminary on this Side of the Picture, as in this last Case. First, because the object is more advantageously situated to the Luminary, and consequently, has a more agreeable effect of Light and Shade. Also, because the Shadow is projected from the Eye, so that, it does not appear distorted and preposterous, as in the second Case; and it is far from being pleasing to have the Shadows of perpendicular Lines all parallel amongst themselves; as when the Luminary is in the Plane of the Picture. Therefore, as I have given Rules and Examples in each, I shall, hereafter, consider the Luminary only on this Side of the Picture, unless it be otherwise determined, for particular reasons.



Publish'd by T. Malton May 21st 1775.

EXAMPLE IV.

To project the Shadow of a long upright piece of Timber, or Block of Stone; part on the Ground, and part on a vertical Plane, inclined to the Picture, whose Intersection with the Ground is given.

Let X be the Object, whose Shadow is to be projected; let ⊙ be the transprojected Image of the Sun, JI the Intersection of a vertical Plane with the Ground, and KL its Vanishing Line. Fig. 24.

Draw ⊙H perpendicular to the Horizontal Line. Draw AH, and B⊙ cuting it at *b*, the Shadow of the Angle B on the Ground.

But, A*b* cuts the Intersection JI, at a; wherefore, because the Plane Y is vertical, AB is parallel to the Plane; and, being also parallel to the Picture, its Shadow is consequently parallel; draw a b, parallel to AB, till it cuts the Ray B⊙, at b, the Shadow of B on the vertical Plane; as *b* is, on the Ground.

The Vanishing Point of BC is F; which, being inclined to the Horizon is not in the Horizontal Line. Draw ⊙F, and produce it, till it cuts KL, the vertical Vanishing Line, at K, the Vanishing Point of its Shadow; then, draw bK, and C⊙ cuting it at c, the Shadow of C on the vertical Plane.

Then, G being the Vanishing Point of CD, a Line on the other Side, parallel to BE, which projects a Shadow, draw ⊙G, cuting the vertical Vanishing Line at L. Draw cL and D⊙ intersecting at d.

If DM be a perpendicular Line, draw d e parallel to it, cuting the Intersection at e, and join eM; which tends to the Vanishing Point, H, of Perpendiculars, and compleats the Shadow; Aaef on the Ground, and abcde on the vertical Plane, as it was proposed.

If the vertical Plane was out of the way, its Shadow on the Ground is A*bcd*f.

EXAMPLE V.

To project the Shadow of a Cross, perpendicular to the Horizon, part on the Ground; also, on a Plane inclined to the Horizon, and to the Picture, whose Vanishing Line, and Intersection with the Ground Plane are given.

Let AFBK be the Object, inclined casually to the Picture; whose Vanishing Points are M and N, in the Horizontal Line, MN. Let QR be the Intersection of the inclined Plane with the Ground, whose Vanishing Line is OP. ST is the given Ray of Light, the Luminary being in the Picture. Fig. 25.

Because the Object is inclined to the Picture, and the Luminary is in the Picture, the Side AB and its opposite, IJ, will project Shadows.

Draw A a, and J b parallel to the Horizon, cuting the Intersection, QR, at a and b. Draw a*b*, and b*i*, parallel to the Vanishing Line OP, indefinite; and draw B*b*, parallel to ST, cuting a*b*, at *b*, the Shadow of B.

For the Line AB being parallel to the Picture, the Plane of Shade occasioned by that Line is, in this Case, parallel to the Picture; consequently, its section with every Surface, on which the Shadow of AB falls, is parallel to the Picture; therefore, Aa is parallel to the Vanishing Line of the Ground Plane, and a*b* to OP, of the inclined Plane.

Draw NO and MP, parallel to the Ray of Light, ST; O is the Vanishing Point of the Shadows, of all Lines parallel to CD; and P, of all that are parallel to EF, on the inclined Plane.

Draw C c parallel to ST, cuting the Shadow of AB at c. Draw Oc indefinite, and D*d* parallel to the Rays, cuting it at *d*, the Shadow of D.

DE being parallel to the Picture, draw *de*, its Shadow, parallel to the Vanishing Line, OP, meeting the Ray of Light from E, at *e*.

Pl. XLI. Fig. 25. P being the Vaniſhing Point of the Shadow of EF, draw *e* P, meeting the Ray from F, at *f*; and draw *f* O, till it cuts the Interſection QR, at *j*; from which it tends to N, the Vaniſhing Point of FG; being parallel to the Ground.

d O cuts the Interſection at b, from which Point it alſo tends to N, till it meets the Ray, from K, at *k*. Draw *k* M, cuting the Ray from L; and LG being perpendicular, draw *l g*, parallel to the Horizon, which compleats the Shadow of that Arm, on the Ground.

The Shadow, *b h i*, of the Top, having the ſame Vaniſhing Points, reſpectively, it would be unneceſſary to deſcribe, being obvious, from the Figure.

Second. *When the Luminary is on this Side of the Picture.*

In the foregoing Caſe (as in other preceding) the Vaniſhing Point of the Shadow of every Right Line, on the inclined Plane, was produced, by drawing a Right Line, parallel to the given Ray, from the Vaniſhing Point of the Line whoſe Shadow was required, to the Vaniſhing Line of the inclined Plane; becauſe the Luminary being ſuppoſed in the Plane of the Picture, all its Rays are conſequently parallel amongſt themſelves.

In this, the Rays having a Vaniſhing Point, a Right Line joining the Vaniſhing Point of the Line, and the Vaniſhing Point of the Luminary, cuts the Vaniſhing Line, or being produced, would (if it be not parallel) cut the Vaniſhing Line of the Plane of Projection, in the Vaniſhing Point of the Shadow (Prob. 4.) for, the Rays, inſtead of being parallel amongſt themſelves (as in the former Caſe) converge in a Point, in which conſiſts the difference.

Fig. 26. ⊙ is the Vaniſhing Point of the Rays, MN the Vaniſhing Line of the Ground, and OP of the inclined Plane, of which, QR, is its Interſection with the Ground. M and N are the Van. Points of hor. Lines in the Object, as before.

S being the Seat of the Luminary, on the Ground, draw AS, cuting the Interſection, QR, at a. Then, becauſe AB is parallel to the Picture, produce ⊙S, (being parallel to AB) till it cuts the Vaniſhing Line of the inclined Plane, at T.

Draw a T; B⊙, a Ray of Light, cuts a T at *b*, the Shadow of B; and C⊙ cuts it at *c*; through which, a Line drawn from O gives the Shadow of DK. Rays from D and K, tending to ⊙, determine the extremes of the Shadow.

It would be uſeleſs to deſcribe the whole, the ſame Letters of reference, as in the foregoing Caſe, ſhew how the Shadow of every Line is produced, by adhering to the former Leſſons, in this Caſe.

E X A M P L E VI.

To project the Shadow of a Right Line, inclined to the Horizon and to the Picture, on ſeveral Planes, variouſly inclined to both.

Plate XLII. Fig. 27. AC repreſents a Ladder, leaning againſt a Building, on which its Shadow is to be projected. ⊙ is the tranſprojected place of the Luminary; H and I are the Vaniſhing Points of horizontal Lines in the Building, and E of the Ladder. FI is the Vaniſhing Line of the Roof of the Building; and GI of the adjoining Shed; on which, the Shadow is alſo projected.

Draw E⊙, and produce it to the vertical Vaniſhing Line (KI) of the Sides of the Building, cuting it at K; the horizontal Vaniſhing Line is cut at L, and the inclined Vaniſhing Lines at M and N.

K, L, M, N, and E, are all Vaniſhing Points of the Shadow. (Prob. 4.)

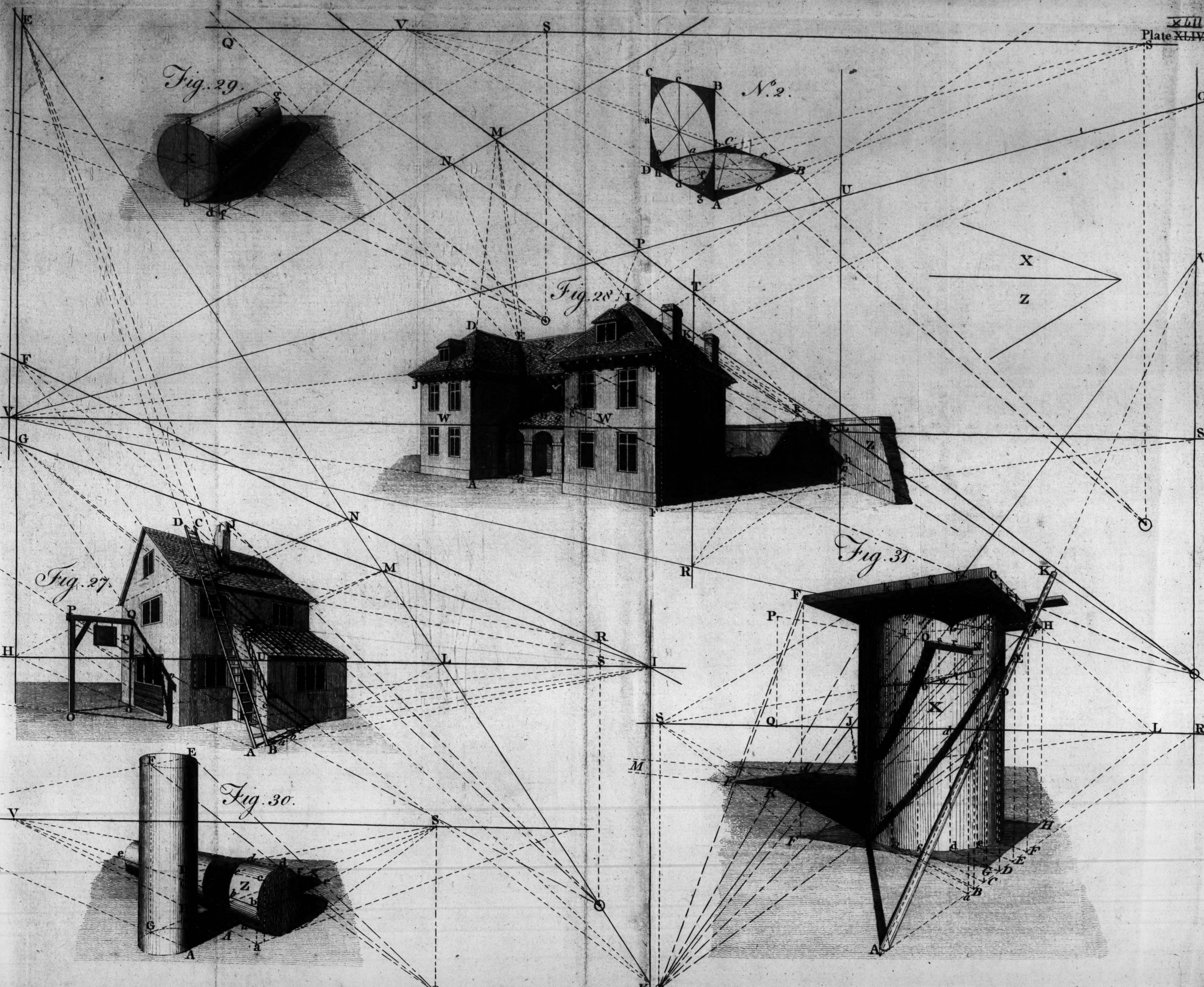
From the foot of the Ladder, draw Lines to the Vaniſhing Point I, cuting the Interſection of the Plane V, with the Ground, at a and b; A a b B, is its Shadow on the Ground.

K being the Vaniſhing Point of the Shadow, on the vertical Planes, draw Lines from K, through a and b, till they cut the common Interſection of the Planes V and U, at c; from which, draw Lines to E, the Vaniſhing Point of the Ladder; becauſe it is parallel to the Plane U.

M being the Vaniſhing Point of the Shadow, on the Plane X, draw d M, cuting the Interſection of that Plane with the next, at e; from which Points draw Lines to K; for the Shadow on the Plane Y, being parallel to V, has the ſame Van. Point.

From

XLII
Plate ~~XLIV~~



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From the Points ff (where the Ladder touches the Eaves of the Roof) draw to N; and from C and D the ends of the Ladder, draw to $\odot$, cuting fN at g, which compleats the Shadow of the Sides of the Ladder, save one End which falls on the Chimney; its Vanishing Point is E.

If from each Step, on either Side, Lines are drawn to $\odot$, cuting the Shadow of that Side, at a, b, c, &c. and from those Points, Lines are drawn to I, the Vanishing Point of the Steps, their Shadows will be compleated, on the Planes V, X, Y, and Z, as well as on the Ground; because the Steps are parallel to all those Planes; being parallel to the Horizon.

On the vertical Plane U, their Vanishing Point is where a Right Line drawn from I, through $\odot$, cuts the vertical Vanishing Line EH, of that End.

The Chimney being perpendicular, $\odot$S, drawn perpendicular, cuting the Vanishing Line (GI) of the Roof, at R, gives its Vanishing Point; J$\odot$ determines its length.

OPQ represents a Sign Post, whose Shadow, Oo, on the Ground, and opQ, on the Wall, is projected by the same Point ($\odot$) whose Seat is S; which, being so near the Vanishing Point I, consequently, the Luminary is nearly in the Plane OPQ; wherefore, the Shadow of both, on the Wall, are almost in one Line.

In this situation of this Object, the Shadow is more pleasing supposing the Sun on the other Side of the Picture, as O*o* on the Ground, *op* of the Post, on the Wall, and *p*Q of the Bearer. The Sun being on the left hand, G, may be the Vanishing Point, of the Shadow *p*Q; and, the Seat of the Luminary, is the Vanishing Point of O*o*, on the Ground (Prob. 2.) OP being perpendicular. The Rays of Light, P*p*, &c. indicate the rest.

EXAMPLE VII.

To project the Shadow of a Building, and the several parts of it on the other; the Luminary being on this side of the Picture, having its Altitude and Inclination given.

Let the Angle of the Sun's Elevation be equal X; and the Inclination to the Picture, of a vertical Plane passing through its Center, equal Z. Fig. 28.

Find $\odot$, the transprojected place of the Luminary; by Prob. I.

Draw $\odot$S perpendicular, giving S, the Seat of the Luminary, on the Ground.

Then, draw AS, cuting the Intersection of the Plane of the Arches, on the Ground, at a; from which, draw a a perpend. the Shadow of AB on that Plane.

Produce $\odot$S, till it cuts VO, the Vanishing Line of the inclined Plane X, at O; the Vanishing Point of the Shadow of AB, on that Plane.

Draw aO indefinite; and, because of the projecture of the Cornice at C, casting a Shadow (JB) on the Plane W, draw B$\odot$, cuting aO at b, the Shadow of B; from which, draw bc, to the Vanishing Point V; because, bC, projecting that Shadow is parallel to the Plane X.

Then, M being the Vanishing Point of CD, draw $\odot$M, and produce it, till it cuts the Vanishing Line of the Planes V and W, at Q; and, VO being also cut at P, draw cP, cuting the next plane at e, and draw eQ.

L being the Vanishing Point of DE, draw $\odot$L, and produce it to the Vanishing Line VM, of the Roof, at N. Draw NE, and D$\odot$ cuting it at d, and draw Md, to f, which compleats the Shadow of that part.

By means of the Vanishing Points, S and V, of the Shadows of FG, and the Ridge, KE, on the Ground; and, drawing Lines from all the Angles, G, H, I, and K, to $\odot$, the Shadow of the Building on the Ground, and Wall, Z (casually situated to the Building and Picture) is projected; by means of its Vanishing Line.

To determine the Shadow of the projecture of the Cornice. Let mn be the seat of the first Truss, on the Top of the Cornice, and m its utmost projecture, which projects the Shadow; or, take any other Point, in mY.

L being the Vanishing Point of the Truss, i. e. of mn, draw from $\odot$ thro' L, till it cuts the Vanishing Line of the Planes W; from which Point, draw no and m$\odot$, cuting it at at o, the Shadow of m, on the Plane W, produced.

Draw

Plate XLII. Fig. 28. Draw o V, cuting both Planes W, in G r, and B J, the Shadow of the extreme projecture of the Cornice, on those Planes.

Having, by the same means, obtained the Shadow of the lower edge of the fronts of the Trusses, their places are obtained by drawing Lines, from each to ⊙. The first falls on the Wall, at *p*, and the first on the other Side, at *q*, on the low Roof.

The Shadows of the Piers of the Arcade, against the wall, within, are projected by means of the Vanishing Point S, till they meet the wall, where they are upright. Lines drawn to ⊙, determine their height. (See Sect. IV. for the Arches.)

Thus, the Shadow of the whole Building and every part of it, is compleated; in which are all the variety of Examples, necessary for projecting the Shadows of right lined Objects.

SECTION IV.

Of the Shadows of curved Lines and curve-lined Objects, on Plane and curved Surfaces.

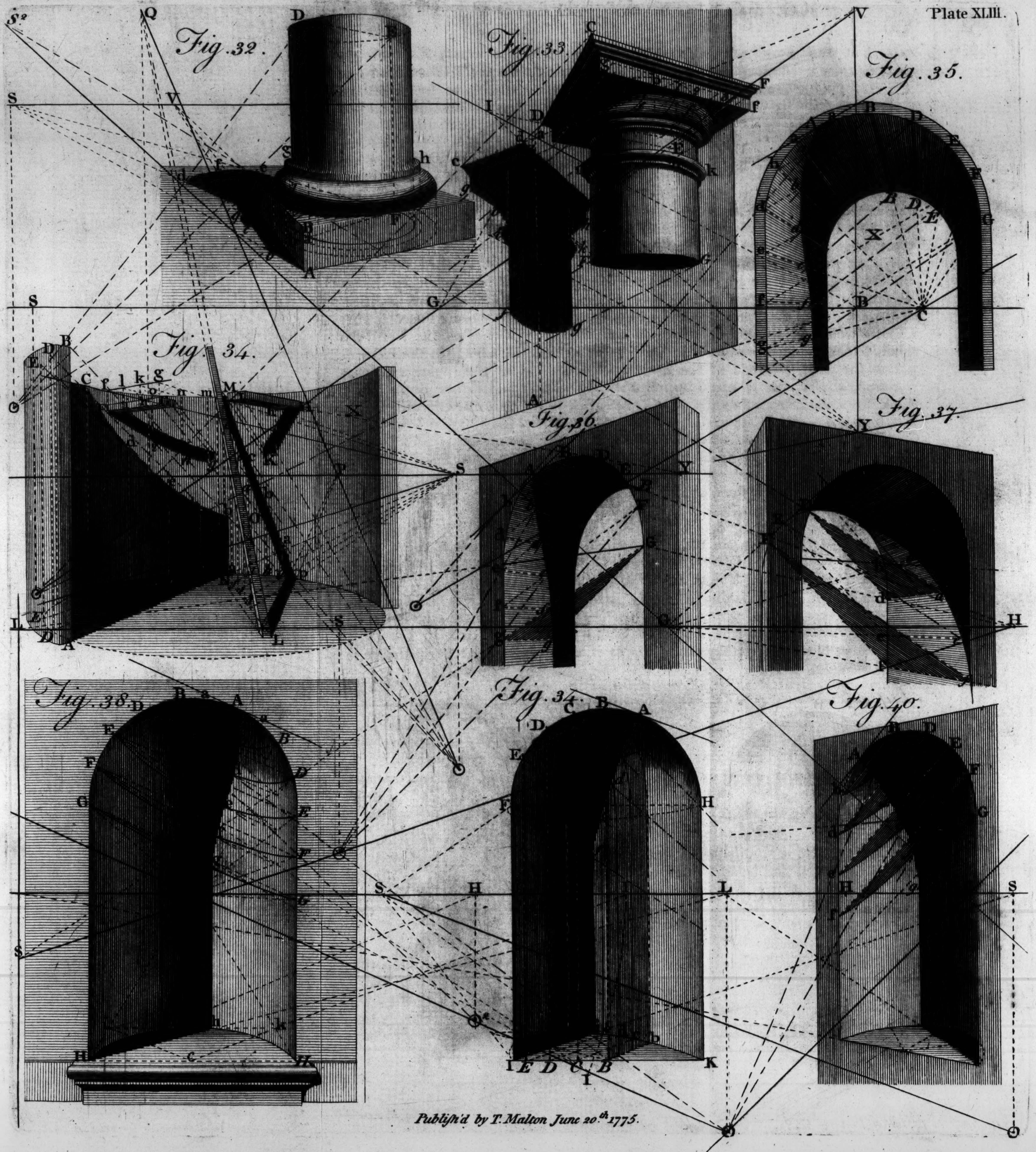
AS the Perspective of curve-lined Objects is more difficult and liable to error than right-lined, on account of the continual bending, and varying in the direction of curve Lines; so, their Shadows are with more difficulty projected. As there are no vanishing Points of the Original Lines, so neither can there be vanishing Points of the Shadows; also, when the Shadows, either of right or curved lines, are projected on a curved Surface, there being no Vanishing Line, the figure of the Shadow cannot, so certainly, be described thereon. Nevertheless, Rules may be prescribed, for projecting Shadows, in such Cases, which, if followed, will produce the true projection of as many Points, in the Curve, as are necessary, for describing the contour of the Shadow, with tolerable accuracy. The principal Shadow being determined, the minutias, of mouldings, &c. may, by a Person who has judgment, be done by Hand, and in many Cases they must; for, I fairly own, that it is more than human patience can bear, nor indeed is it possible, to project Shadows, in many Cases, with mathematical exactness

The Shadow of a Circle, projected upon any Plane, to which the Circle is parallel, is a Circle, of equal dimensions; in all situations of the Luminary, whatever.

For, whether its situation be on this, or on the other Side, or in the Plane of the Picture; whether it be more or less elevated, 'tis still the same; the projecting Rays, being parallel amongst themselves, generate a Cylinder, for the most part oblique; consequently, being cut by a Plane parallel to the Circle, its projection must also be a Circle, seeing it is the opposite Base of the Cylinder; but, being cut oblique it is an Ellipsis. So that, in Arches, &c. parallel to the Picture, the Luminary being on this Side, and projecting the Shadow on a vertical Plane, parallel to the Arches, the Center of each being determined, the representation of the Shadow of each Arch, may be described with Compasses; which will be in proportion to the Arches, as the distance of the Plane of the Arches, to the distance of the Plane of projection. But, being projected on the Floor, or any other Plane, not parallel to the Arches, their Representations are Ellipses; except when the Cylinder, projecting the Shadow, is cut subcontrary*; in which Case the Shadow will be a Circle.

* An oblique Cylinder or Cone is cut subcontrary, when the Plane of section is so situated, that the Axe of the Cylinder or Cone has the same inclination to it, contrarywise. As, if the Sun was elevated 45 Degrees, and be situated so, that a vertical Plane, passing through its Center, is perpendicular to the Plane of the Arches, *i. e.* to a vertical Circle; then, if the shadow be projected on the Floor, or any horizontal Plane, it will be a Circle; for the Rays cut the Floor, in the same Angle, *viz.* 45 Degrees.

Plate XLIII.



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In all other positions of the Arches, to the Picture, and in all situations of the Luminary, being projected on a parallel Plane, the Shadows being Circles, their Representations are Ellipses, described as the Arches, themselves; and also, being projected on any other Plane, the Shadows being Ellipses, their representations are also Ellipses, more excentric, generally, though not always so.

The Shadow of a Sphere or Globe (as well as its perspective Projection) on a Plane, is an Ellipsis, in all Cases, whatever; except when a Right Line passing through the Center of the Luminary, and of the Sphere, is perpendicular to the Plane of Projection.

EXAMPLE VIII.

To project the Shadows of Cylinders, on a horizontal Plane, lying along; another upright, so situated, that its Shadow crosses one of the former.

Let the Cylinder, X, be inclined to the Picture; its Vanishing Point, is V. Fig. 29.

The Luminary is on this Side of the Picture, and ⊙ its transprojected Image.

The Ends or Bases of the Cylinder, X and Y, being inclined to the Picture, their representations are Ellipses; their Shadows, may be thus projected.

Let A B C D be the representation of a Square, circumscribing a Circle, whose Sides are parallel and perpendicular to the Horizon. Draw the Diagonals AC and B D, also the two Diameters a b and c d, parallel and perpendicular to the Horizon; and, from the Points e and f, draw perpendiculars to the Base, AD.

Find the Shadow of the Square ABCD (by Prob. 2. and 3.) and draw the Diagonals, A*c* and D*d*; draw d S, also g S and h S, cuting the Diagonals; and, through their Intersection, draw *a b*, to the Vanishing Point of a b, &c.

Then, if a Curve be described through the eight Points, *a*, *e*, d, *f*, *b*, *c*, and *d*, it will be the true representation of the Shadow of the Circle, a c b d.

By the same means, the Shadow of the Base of the Cylinder, X, may be projected; or, because there is but half its Shadow required, proceed thus.

Draw a b perpendicular, through its Center; also, c d and e f parallel to a b.

Find the Shadows of all the Points, a, c, e, &c. (by Prob. 2. Case 3.) S being the Vanishing Point of perpendiculars to the Horizon; and describe the Curve *aeb*.

Then, the Sides of the Cylinder being parallel to the Plane of projection, draw *a*V, a Tangent to the Curve at *a*; and, the Shadow of the other End being described by the same means, compleats the Shadow of that Cylinder.

SECOND. The Shadow of the Cylinder Z, on account of its situation, is but little seen; which is projected after the same manner. Fig. 30.

THIRD. The Sides of the upright Cylinder being perpendicular to the Horizon, draw AS, a Tangent to its Base, at A, cuting a V, the Seat of the Side b e, on the Ground, at *A*; from which Point, draw *Ab* perpendicular, cuting b e at *b*.

But, on account of the inclination of the Rays of Light, the Shadow of the Side AE will be continued on the Ground, till it falls into that of the other, at *a*; and it is continued down the Side, from *b* towards *a*, till it is lost, insensibly, in the Shade of the other Cylinder.

Then, take as many Points, b, c, d, in the Curve of the Base, as are necessary; and others, answering to them, that is, of equal height, perspectively, at B, C, D, on the Side, AE, of the upright Cylinder; from all which, draw Lines to S, cuting others, drawn from the corresponding Points b, c, d, to the Vanishing Point V, at *b*, *c*, *d*, &c. through which, a Curve being described, will be the Shadow of the Side AE, on the Cylinder Z.

For, the Side of the Cylinder (AE) projecting the Shadow, is perpendicular to the Plane of projection; wherefore, the Plane of Shade, occasioned by that Line, vanishes in ⊙S; consequently, all horizontal Lines (in that Plane) from the Points A, B, C, D, &c. vanish at S; and, consequently, they must cut others, b *b*, &c. drawn in the Side of the Cylinder Z, being of equal height. Therefore, *a b c d* is the section of the Plane of Shade, with the surface of the Cylinder Z, made by the Side, AE, of the upright Cylinder; and consequently, *a b c d* is its Shadow on that Surface.

Plate XLII. Fig. 30. The upper Base (EF) of the upright Cylinder, is above the Eye, and, being parallel to the Plane of projection, its Shadow is a Circle; wherefore, having obtained the Shadow of any Diameter, as e f, of EF, by drawing E⊙ and F⊙; the former cuting AS, at e, and drawing e f to the Vanishing Point of EF; then, the Shadow being a Circle, and projected on a Plane, not parallel to the Picture, its representation is an Ellipsis (Theo. 2. Sect. 5.)

Describe the representation of a Circle, e g f, whose Diameter found is e f (Prob. 2. Sect. 8.) and through f draw a Tangent, tending to S, the Shadow of the other Side of the upright Cylinder, which compleats it.

EXAMPLE IX.

To project the Shadows of Right Lines, on a convex cylindrical Surface.

Fig. 31. Let AK be a Right Line, any how situated, in respect of the Cylinder X; its Vanishing Point is V. Its Shadow on the Cylinder is required; ⊙ being the transprojected Image of the Luminary.

Draw V⊙, cuting the Horizontal Line at J, the Vanishing Point of the Shadow on the Ground. Draw AJ, cuting the Base of the Cylinder at a; and, through a, draw ⊙*A*, cuting AK at *A*.

A a, on the Ground, is the Shadow of A*A*, part of the Line AK; beyond which, its Shadow is projected on the Cylinder.

Having obtained S, the Seat of the Luminary, (Prob. 2.) also, R, the Vanishing Point of the Seat of AK, on the Ground, draw AR; and, from various Points, B, C, &c. at discretion, beyond *A*, draw B*B*, C*C*, &c. perpendicular; from all which Points, *B*, *C*, &c. so obtained, draw Lines to S, cuting the Base of the Cylinder, at b, c, &c. from which, draw perpendicular up the Cylinder; and from the several Points, B, C, &c. draw lines to ⊙, cuting the corresponding Perpendiculars, at *b*, *c*, *d*, &c. through all which, a Curve, a *b c d e*, being described is the Shadow of *A* E, part of AK, on the cylindrical Surface.

The remaining part of its Shadow, falls on the Ground, behind the Cylinder.

Second. FH, on the top of the Cylinder, is a Square, which may represent the Abacus of the Tuscan Capital; to project its Shadow on the Cylinder and Floor.

Take as many Points f, g, h, i, and k, as are necessary, in the lower Lines, which cast their Shadows on the Cylinder; from all which, draw to S; also, from the Angle G, cuting the upper Base, or Curve, at 1, 2, 3, &c.

Draw perpendiculars from each Point, down the Cylinder, which are cut, by Rays from each Point, f, g, &c. to ⊙, at *f*, *g*, &c. the Angle G is projected to *G*. The Ray k⊙, touches the Cylinder, at *k*, whence, it is projected to *K*, on the Floor.

Curve Lines, drawn through *G f g h*, and *G i k*, are the Shadows of the Lines G h and G k on the Cylinder; k F is projected on the Floor.

Draw S a, a Tangent at the Point a, cuting the Ray, from k at *K*; a *K* is the Shadow of the Side of the Cylinder on the Floor.

Draw *KF* to the Vanishing Point of GF, and F⊙, cuting it at *F*, the Shadow of the Angle F; draw *F* L, to the Vanishing Point of G H, and F I, the upper Line, on the other side, which casts the Shadow; I⊙ cuts it at *I*; through which, draw *IL*, tending to the vanishing Point of H I, and compleats the Shadow on the Floor.

Third. N O is a Line projecting from the Cylinder, at pleasure; let P be its Vanishing Point; to project its Shadow, on the Cylinder.

Draw Oo and N n perpendicular, also P Q, cuting the horizontal vanishing Line, in the vanishing Point of its Seat n o, on the Ground; or any other horizontal Plane, imagined at discretion, cuting the Cylinder.

The rest is performed as above, for the Shadow of AK; as shewn in the Figure.

EXAMPLE X.

To project the Shadow of a Tuscan Base on the Floor, casually situated to the Picture.

⊙ is the transprojected Image, and S the Seat of the Luminary. Plate XLIII. Fig. 32.

Draw AS and B⊙, cuting at *b*, and draw *b*V; C⊙ cuting *b*V determines the Shadow of the edge of the Plinth, BC, provided nothing else interfered. But, the Torus, &c. and Shaft of the Column unite their Shadows with it.

For the Column, draw S a, a tangent to its Base, on the Ground; from the point of contact a, draw a D perpendicular; which Line determines where the total Shade, on the Shaft, begins.

Draw S F, a Tangent to the other side, and FE perpendicular; BD, and EF, are the Sides of the Column which project the Shadow, on the Ground.

Draw D⊙, and E⊙, cuting a S, and F S, at d and e, the Shadows of D and E.

Draw DE, which is a Diameter of the Column; d e being drawn, will tend to the same Point, and may be considered as its Shadow; on which, as a Diameter found, describe the representation of a Circle (Prob. 3. Sect. 8.) or, of a Semicircle, d f e, only, which terminates the Shadow. The Column being supposed cut, by a horizontal Plane, through DE, its Shadow is, consequently, a Circle; being projected on a Plane to which it is parallel.

The Shadow of the Fillet g h, may be projected after the same manner; but, as so little of it is seen, it would be unnecessary trouble.

Draw a Tangent to its Base, at b, from S, and draw b g perpendicular, and g⊙, cuting b S, at *g*, the Shadow of the Point g. By the same means the Shadows of other Points, as *i*, of i, may be obtained.

The Shadow of the Torus is not easy to describe; seeing its Surface is continually varying, it is scarce possible to determine what part of it projects the Shadow; except the Point *d*, of the greatest swell, determined, as the Fillet, by a perpendicular from c. No other Point in that Circle projects a Shadow, on this side; to determine, absolutely, what other Points do, is what I shall not attempt, being persuaded, my Readers will excuse me that trouble.

Its Shadow, on the Ground is mostly beyond *e*, the Shadow of *d*, and falls into the Shadow of the Fillet.

N. B. The Shadow of the Shaft falls, first, on the Cavetto, before it reaches the Ground; the Fillet on the Torus, and the Torus on the upper Plane of the Plinth; which the Figure describes.

The Shade on the Torus, and on the Column, &c. is more hard and edgy when the Sun shines on them, than otherwise; yet there is no edge defined; also, the Light is broader, and graduates more suddenly into the Shade; and the Reflections are much stronger; but that, I shall reserve to another Section.

EXAMPLE XI.

To project the Shadow of a Doric Capital, on a vertical Plane, whose Vanishing Line (BV) and Intersection (AB) with the Ground Plane are given.

CD cuts the Plane at a; find its Shadow c d (Prob. 4.) ⊙ being the Vanishing Point of the Rays, and G, of the Line; which, being joined and produced, cuts the Vanishing Line, BV, at V, the Vanishing Point of the Shadow. Fig. 33.

After the same manner, the Shadow d e, of DE, &c. may be obtained; and the whole Shadow, c d e f, of the square of the top of the Abacus, in which draw the Diagonals; and let the lesser Trapezium *a b c d*, be the supposed Shadow of the square of the Column, parallel to the former, and found by the same means; in which the Ellipsis, inscribed, may be the Shadow of the top of the Column in the Plane of the Abacus, from which draw Perpendiculars; or, rather (because of the swelling of the Column) somewhat diverging, and gently curved; as *a f*, and *c g*.

The Shadows of the Angles, g and h, are obtained as the Square of the top; and, if great accuracy was requisite, the square of the Astragal, i k may be also projected, in its true place. But, as nothing more of its Shadow is defined than what projects beyond the Column, it may be dispensed with, the Points, *i* and *k*, being determined, is sufficient.

Plate XLIII. Fig. 33. Rays of Light, passing by the Ovolo to ⊙, may be some guide for describing its Shadow; but, on account of the continual variation of its Surface, every way, 'tis not easy to determine at what Points they touch it.

Neither can the curve of the Shadow of f g, be described on the Ovolo, with certainty, as on a Cylinder, yet, somewhat may be done towards it; for example; the Shadow of the point b is required.

In this process, it is necessary to have the perspective curve (*aef*) of the seat of the small Fillet, from which the Ovolo projects, on the under Plane (g h) of the Abacus.

Then, draw b S, cuting it at *a*; draw *ab*, perpendicular, and describe the curve *bcd*, a section of the Ovolo, by the Plane of Shade; b⊙ being drawn, cuting that Curve, determines the Shadow of b, on the Ovolo, at *c*; which, because it touches the Curve only, is the limit of the Shadow on the Ovolo.

The rest of the Shadow of that Line, from b to g, falls on the vertical Plane.

Thus may as many Points be determined (as *e*, the Shadow of e) as the Student has patience for; and when done, 'tis a great chance if it be more exact than a judicious Person would describe by guess; nevertheless it is useful, to give an Idea how such Shadows are projected, and what kind of curves they describe.

EXAMPLE XII.

A vertical, concave, cylindrical Surface being so situated, that the Shadow of one part is projected on the other; how to determine that Shadow; also, of Right Lines any how situated to the Surface, and to the Luminary.

Fig. 34. Let ABX be a Wall, inclosing a circular Area, &c. ⊙ is the Vanishing Point, of the Rays of Light, and S, of the Shadows of Perpendiculars.

Draw SC, a tangent to the Curve, which determines the point where the Shadow begins, whatever be the Altitude of the Luminary.

Draw AS, cuting the Curve on the Ground, at a; draw ab perpendicular, and B⊙, cuting it at b, the Shadow of B.

By the same means, the Shadows of as many Points, D, E, &c. as are requisite, may be found; through which, the Curve C d e b, may be described; which is the true Shadow of the Curve CDEB, on the concave Surface.

Second. Let FG be a piece of Timber, projecting perpendicularly from the Wall, whose Shadow is required; its Vanishing Point is V.

Draw E f perpendicular, either to the top or bottom of the Wall; and, through f, draw Vg, cuting a perpendicular from G. f g is the Seat of FG, on the Plane of the top, being horizontal.

From several Points, h, i, &c. draw perpendicular, cuting f g, at k, l, &c. from g, k, and l, draw to S, cuting the Curve of the Top, at m, n, o, from which, draw Perpendiculars, indefinite; and from the Points g, h, i, &c. draw to ⊙, cuting them, respectively, in the Points *g*, *h*, *i*, which determine the Shadows of those Points on the Wall; through which, a Curve, being described, is the Shadow of the lower edge of the Timber. The rest is evident.

HI is another piece, also perpendicular to the Wall, its Vanishing Point is Y. Its Shadow is described after the same manner; or by means of a section of the Wall (H h) parallel to the Horizon. Draw I S, cuting it at h; from which, draw perpendicular, and I⊙ cuting it at K, which determines its length; being so nearly perpendicular it deviates but little from a Right Line.

If its Vanishing Point was at S, the Shadow would be right lined.

Third. L M is a piece of Timber, inclined to the Wall, its Van. Point is Q.

Draw Q⊙, cuting the Horizontal Line at P, the Vanishing Point of the Shadow on the Ground. Draw LP, cuting the Curve at the bottom, at p; through p, draw ⊙O, cuting LM, at O. L p is the Shadow of LO, on the Ground.

Draw

Draw MN, perpendicular; and LN, being drawn, is the Seat of LM, on the Ground. From several Points *a*, *b*, *c*, (between O and M) draw perpendicular to the Seat LN, at *d*, *e*, *f*; from which, draw to S, cuting the Curve at the bottom, at *g*, *h*, *i*, and from them draw perpendiculars up the Wall; also, from *a*, *b*, *c*, draw to $\odot$, cuting them, respectively, at a, b, c; through which, a Curve being described is the Shadow of OM, on the concave Wall.

For the Shadow of each Perpendicular *a d*, &c. is projected first, from *d* to *g*; and *g* a, &c. is parallel to *a d*, *a d* being parallel to the Wall.

EXAMPLE XIII.

To project the Shadows of Arches; or, the Shadow of the circular outline, on the inside, cylindrical, and plane Surface.

This Example is the same, in every respect, as the first part of the last, except in the position of the Cylinder to the Picture; their position to the Horizon is of no signification.

FIRST; the Arch (X) is parallel to the Picture; C is the Center. Fig. 35.

The Luminary being on this side, and $\odot$ its Image; draw C$\odot$, indefinite.

Then, because the plane of the Circle is parallel to the Picture, draw a Tangent to the Curve, at A, also, from various Points B, D, &c. at discretion, draw Ordinates, parallel to C$\odot$, cuting the opposite side of the Arch, at b, d, &c. from all which, draw to C; and lastly, draw B$\odot$, D$\odot$, &c. cuting bC, dC, &c. respectively, at *b*, *d*, &c. through which, a Curve being described, is the Shadow of all that part of the Circle, from A (where the Shadow begins) to G, where it falls into the perpendicular.

If from A, to *b*, there be too much space, for, describing the curve accurately, take another point (a) and project its Shadow to *a*, and more if necessary.

SCHOL. In this, the affinity to the former Example is obvious. The Right Lines, B*B*, D*D*, &c. tending to C, (the Center of the Picture) represent perpendiculars to the Bases of the Cylinder; as, in the former they are perpendicular, being parallel to the Picture; also, the Lines B b, D d, &c. which are, in this Case, parallel to C$\odot$, in the former they vanish in S; because they represent parallel Lines, in a Plane not parallel to the Picture; as, in this Case, they are parallel, being in a Plane parallel to the Picture. The Rays of Light, in both Cases, tend to their Vanishing Point.

2. The Shadow, from A to *f*, notwithstanding its apparent convexity towards the Front, is really concave, the same as the other; the convexity arises only from its position to the Eye, owing to the exterior Curve A d e; which, in the former, is concave, the Eye being between the two Bases, nearly opposite to that part of the Surface on which the Shadows falls; in which position of the Eye, the true curve of the Shadow is seen.

Fig. 36. represents the same thing, inclined to the Picture; in which, the perpendiculars B*B*, &c. vanish in H, in the Horizontal Line HL, as before in the Center; consequently, b *b*, d *d*, &c. vanish in the same Point. Fig. 36.

Also B b, D d, &c. vanish at S, in the Vanishing Line of the Arch, where H$\odot$ cuts it; H being the Vanishing Point of the Shadows of Right Lines perpendicular to the Plane of the Arch (Prob. 2.) The Rays of Light vanish at $\odot$; consequently, HS is the Vanishing Line of the Plane of Shade, for Lines perpendicular to the Plane of the Arch, and S is its Seat on that Plane.

Fig. 37. represents another Arch, with the Light flowing inwardly (as in Example 42. Book 3. of the Piazza.) In which Case, it is the exterior Curve, from A to F, which projects the Shadow, the Sun being on the other side of the Picture (in this Case) at S^2; and, G is the Vanishing Point of the Shadows of Perpendiculars, as before. Fig. 37.

The Figure, having the same Letters of reference, explains the rest.

Here, the curve of the Shadow is seen properly, that is, concave; and it is the same in both the other, in the Object, though represented convex.

In Example 42, of the Piazza, the Luminary is on this Side of the Picture, although it may to some, appear absurd; but, the direction of the Shadows indicate it. It is indeed very much inclined, nearly approaching to parallel; that is, the Luminary is nearly in the Plane of the Picture.

Plate XLIII.

EXAMPLE XIV.

To project the Shadow of the circular outline of a Nich, on the interior, spherical, and cylindrical Surface.

Fig. 38. FIRST; let the Front of the Nich be parallel to the Picture.

C being the Center, draw C⊙, indefinite, the Intersection of a Plane passing through the Luminary, perpendicular to the Picture.

Draw a Tangent to the Curve at A, parallel to C⊙, and several Ordinates, B*B*, D*D*, &c. at discretion. On the Diameters B*B*, D*D*, and E*E*, describe the representations of Semicircles, in Planes whose Vanishing Line is C⊙, (CI is its Distance) which are sections of the Head, parallel amongst themselves, and to their Plane of Shade.

As the Section, through F*F*, cuts the cylindrical part of the Nich, also, and that through G*G* entirely, those Sections are not Semicircles; the former, where it cuts the Head, is circular, but where it cuts the Cylinder (which only is wanted) it is elliptical; and that through G*G* is a semi-Ellipses, whose transverse Diameter is G*G* and its Conjugate, the Diameter of the Nich.

Describe the representations of such Ellipses; which being done, from B, D, &c. draw Rays to ⊙, cuting the Curves, respectively, at b, d, &c. through which, describe the Curve A b d e f g, the Shadow of the exterior Curve (from A to G.)

Draw g h, perpendicular, the Shadow of the edge GH, to which it is parallel, and join H h; which compleats the whole Shadow, in that situation of the Sun.

If the Luminary be less inclined to the Plane of the Nich, but in the same Plane, of which CI is the Vanishing Line, its place being ⊙²; A*abdefgh* is the Shadow, which is not so convex as the former, being farther removed from the Curve of the Head A*BE*.

Fig. 39. SCHOL. This process, though apparently troublesome, is the shortest of any that has yet been published, and much more simple than by vertical Sections, as exhibited in Fig. 39;* in which, the several Points, B, D, &c. in the Arch, by means of perpendiculars to the right Line IK, at the bottom, have corresponding Points, *B*, *D*, &c.

Then (S, being the Seat of the Luminary) lines are drawn from *B*, *D*, &c. to S, cuting the curve of the bottom of the Nich, at b, d, &c. from which, perpendiculars are drawn to the Curve FGH, where the spherical Surface begins; through which, the representations of parallel Sections are made, (as in the Figure) a most difficult and laborious operation. Perpendiculars from each Point, B, C, &c. tending to ⊙, (as in the former Figure) give the Shadow of each Point in its respective Section; through which, the contour of the Shadow is described.

Sir William Chambers (if Fame reports true) was the first who took notice, to the World of this seeming paradox, *viz.* a convex Shadow of a concave Line; which, since, has been most servilely copied, by almost every Architect, or other Artist, who had occasion to represent a Nich, either in external or internal Designs. Nay, to such an extravagant excess it is carried, that, in geometrical Sections, and where it is impossible that the Sun should ever shine, almost every Nich is so shaded; which, I do maintain, is, in such Case, most unnatural; nor is the Shadow really convex, but only apparently so; yet it is frequently exaggerated, preposterously.

But, I presume that, in Sun-shine, it has not always the appearance of convexity; for, I have shewn (Fig. 38.) that, the less the Inclination of the Sun, to the Plane of the Front, the less convex is the Curve. There is, also, another circumstance which occasions a different appearance. Those Authors, who have favoured the World with the method of describing it, have always given it direct, with the Center of the Picture in the middle of the Nich, as in Fig. 38. But, certainly, in the Front of a Building, perspectively delineated, the Eye cannot be

* In this Example, Fournier is most egregiously mistaken; for, he makes all these vertical Sections pass through B, the Vertex of the Nich, when it is evident, that they are parallel amongst themselves, and to the Plane of Shade of perpendicular Lines. By which means, the contour of his Shadow is more convex than it can ever possibly appear, in any situation of the Luminary, and of the Spectator.

opposite to Niches at each extreme. Suppose, then, the Eye obliquely situated to a Nich, in a Plane parallel to the Picture, on the same side with the Sun; I presume, the convexity will not be so great, as in direct opposition; yet, it is customary to shade all alike, which is most palpably absurd.

Fig. 39. represents a Nich so situated and shaded; the Luminary being on the left hand, its transprojected Image is, consequently, on the Right, at ⊙; the Center of the Picture is at S, and the Distance is SH.

Now, although the altitude of the Sun, in this, is less, and its Inclination more, than in Fig. 38. yet, the contour of the Shadow is concave, where the other is convex; and this is solely owing to the situation of the Eye, in respect of the Nich; for, being seen directly opposite, it would appear more convex than the other.

I think, none have ventured to describe this Shadow, when the Nich is in a Plane inclined to the Picture; which, in delineating, more frequently happens so than otherwise, if the Picture be properly situated. Yet, in such Case, many, not thinking properly about it, would give the contour of the Shadow the same, which can never possibly happen, except when the Rays of Light are very much inclined to the Plane of the Nich.

Fig. 40. is the representation of a Nich, in a Plane inclined to the Picture; C is the Center, the Distance is CH, and ⊙ the Image of the Luminary. Fig. 40.

The same Letters of reference indicate the process the same, as already described, with only this difference; that, as, in the former, the Ordinates were parallel (being parallel to the Picture) so, in this, they vanish at V, where C⊙ cuts the Vanishing Line of the Plane of the Nich.

The outline of the Nich being delineated, and several Points, in the hither side of the Arch, taken at discretion, draw the Ordinates, tending to V; from which Point, draw the Tangent, at A, where the Shadow begins.

Then, representations of Circles being described on those Diameters (Prob. 3. Sect. 8.) and Rays drawn from the several Points, to ⊙, cuting them respectively, give the contour of the Shadow, as before; which, in this Case (as in the last Example) the Eye being opposite to the Surface where the Shadow is defined, is really concave; and which, in other Points of view, would appear convex.

N B. As the Line which projects the Shadow is concave, the System of Rays form a cylindrical Surface; consequently, the Shadow is concave, towards the Eye, on whatever Surface it falls.

SCHOL. It must have been observed, that the Center and Distance of the Picture are not necessary in the projection of Shadows, except in determining the Vanishing Point of the Rays; but, in this Example, as the Vanishing Line C⊙ is of Planes perpendicular to the Picture, it necessarily passes through the Center (Theo. 4.) and, the representations of the Sections, through the head of the Nich, could not be described without the Distance.

I have now, I presume, furnished the Reader with Rules and Examples, for the projection of Shadows, by the Sun, sufficient for any purpose almost whatever. The Shadows of streight Mouldings, on Planes, are Right Lines, save the contour of the Profile; which, in Mouldings above the Horizon, in Cornices, &c. are almost wholly shaded by the projecting Fillets, &c. and for circular Mouldings, or the Profile of others below the Eye, the projection of the Shadows of Circles, on plane or other Surfaces (as in this last Section) contain all that can be done in such Subjects.

He who has well digested what is there advanced will never be much at a loss, in the most difficult Cases (where it is possible to project the Shadow, at all, by Rule) in complex, finished Subjects; of which, Examples may be seen, throughout the Work, particularly in the Frontispiece; that being intended as a general Lesson. Yet, in that, I have designedly deviated from the rigid observance of mathematical exactness; which, as I have observed in the Preface, does not always produce the best effect of Light and Shade. For, Example.

According to the general direction of the Rays of Light given, in that Picture, the Vanishing Point of the Shadows of perpendicular Lines being determined,

mined, it must be obvious, that, as the Columns, in the Front, stand off, detached, their Shadows would cover more than the whole space between them, on the Wall; which, I am persuaded, would make that part appear heavy, and too hollow, as receding more than it really does. A little Light is also introduced on the circular Colonade, on the Right, in order to relieve it from the Building; although it would be wholly in Shade from the Building, according to the situation and altitude of the Luminary, the height, magnitude, and distance of the Object, occasioning the Shade. Such liberties may freely be taken, where it manifestly produces a better general Effect; and, when it does not clash with, and evidently contradict the invariable Law of Nature.

In that Piece, as in most other in this Work, the Luminary is on this side of the Picture; and though, in general, it is productive of the best effects, yet there may sometimes be reasons for supposing it on the other side, or in the Plane of the Picture, as in Plate 26, of Chelsea College. If that Piece had been shaded on the supposition of the Sun being on this Side, either it must be much inclined, and consequently the Vanishing Point at a great Distance, or every Face, in that Object, would be illumined, being on the Right; or, being on the Left, the whole Front would be immersed in Shade; with very little Light, in the Picture; but on the Roof, and end of the Building. Besides, from the situation of that Building, the Sun never can shine on both, the Front and hither End; and I think that circumstance should always be attended to, when a real Object is represented, which is the sole reason why the Queen's Palace (Plate 27.) is shaded from the Left; for I am of opinion, that it would have a better Effect being shaded from the other Hand; but, as we are more accustomed to see it so shaded than otherwise, it has certainly the most natural Effect.

Nevertheless, though there be no Face of this Object, as there represented, but what receives Light, imediately from the Sun, save one Face of the Library; yet, as some Faces are more opposed to the Luminary than others, there are various Teints, which distinguish one Plane from another, though, not so strongly. For, according as the Rays of Light are less or more inclined to the Face of any Object, it may have all the variety of Teints, from the strongest Light to total Shade; and, till the inclination is such, that they are nearly parallel to the Plane, it is still illumined, though in a very small Degree; and, when they become parallel, that is, when the Sun is in any Plane, it is as much deprived of Light, as if it was, in reality, on the other Side; and perhaps more so, as it then receives the least advantage from Reflection.

SECTION V.

Of SHADOWS projected by a Torch or Candle.

IF the Theory of Shadows, in general, (Sect. 2nd.) and the Practice by Sunshine be well understood, little need be said respecting Candle-light Shadows, for in reality there is no difference, except in the Distance; which, on account of the short Distance, and being situated amidst various Objects, its Light is diffused in all directions; as it is from the Sun, respecting the whole System of Planets: also, in respect of any single Object, of the same magnitude, or nearly, as the Flame, the Rays may be considered as parallel.

In projecting Shadows by Candle-light, there is but one Case, or situation of the luminous Point, in respect of the Picture; for, it is always supposed on the other side. In reality it is an Object delineated in the Picture; and, when determined, it is the Center of the Rays of Light; from which, the Shadows of other Objects are projected all around, in every respect the same as by the Sun, in that Case. Therefore, the Theory being the same, and the Practice having but little variation, few Examples will suffice.

I ſhall not ſuppoſe it neceſſary to ſhew how to find the Vaniſhing Point of the Rays of Light, which is the ſame as finding any other Point, in any Object whatever. The Vaniſhing Point of the Shadows of Perpendiculars, to any Plane, is the Seat of the luminous Point on that Plane; which, in this Caſe, is finite. In reſpect of the Sun, being ſuppoſed at an infinite Diſtance, it is, conſequently, in the Vaniſhing Line of the Plane; but in this Caſe, it muſt be found on the Plane of Projection; which being found, the Shadows are readily determined.

Shadows projected by a luminous Point, at a ſhort Diſtance, it is manifeſt muſt neceſſarily be larger than the Object; ſeeing that, all the Rays diverge from that Point, the ſame as from the Sun, when it appears in the Picture; which, on account of its immenſe Diſtance, the Rays have not ſo much divergency. But, the Objects whoſe Shadows are projected by them, are generally of much greater Magnitude; the one being confined to ſmall Objects only, as Chairs, Tables, &c. within a Room, the other is of Buildings, &c. external.

P R O B L E M I.

The repreſentation of a luminous Point, being given, and its Seat on the Ground, or other horizontal Plane, together with the Interſections of other Planes, with that Plane, whether they be perpendicular, parallel, or inclined to the Picture, or to the Horizon, or to both; to determine its Seat on the other Planes, and the Shadows of Right Lines perpendicular to them.

Let O be the luminous Point, given; and S its given Seat. Plate XLIV. Fig. 41.

Let AB be the Interſection of a vertical Plane with the Ground Plane, parallel to the Picture, and BG of a vertical Plane perpendicular to the Picture. Alſo, let BD be the Interſection of the two vertical Planes; and DE, DH, EI, EF, &c. Interſections of various Planes with each other.

In ſhort, let LABG, repreſent a Floor, GBDH a vertical Plane, HDEI the Cieling, and IEFK an inclined Cieling, as in a Garret; AFEDB is the farther End of the Room, parallel to the Picture, all the other Planes are perpendicular to it. O is the repreſentation of the luminous Point (as the Flame of a Candle) and S is its Seat on the Floor.

First. To find the Seat of the luminous Point, on any of the other Planes.

OS, being perpendicular to the Floor, conſequently parallel to the Picture, imagine a Plane paſſing through OS, parallel to the Picture, cuting all the Planes (ſave AFEDB) perpendicularly; the Section of which, with them, is GHIKLG.

Now, becauſe the luminous Point is in that Plane, and it is perpendicular to the Planes BDHG, &c. a Perpendicular from O, to each Plane, muſt neceſſarily be in that imaginary Plane; and conſequently, muſt cut each Plane, in its Interſection with the Plane; wherefore, Perpendiculars (O ſ) to each Interſection, GH, HI, IK, &c. give the Seat (ſ) of the luminous Point, on each Plane.

The proceſs is ſo very obvious, that I ſhall avoid the deſcription.

To find its Seat, on the Plane AFEDB; draw SC, cuting the Interſection, AB, at r; draw r s, perpendicular, and OC, cuting it at s, the Seat of O on that Plane. For, OC repreſents a perpendicular to the Plane, cuting it at s.

Secondly. To find the Shadows of Right Lines perpendicular to thoſe Planes.

Through the Point *B*, or b, where the Perpendicular *AB*, or a b, cuts the Plane, draw S*B*, indefinite; and, through the extreme *A*, or a, draw Oa, cuting it at *C*, or c, the Shadow of the Point *A*, or a.

Conſequently, *BC*, or b c, is the Shadow of *AB*, or a b.

This proceſs, it is obvious, is the ſame in each Plane; and, changing S for ſ, the deſcription, given, ſerves for all alike.

Plate XLIV. The Perpendiculars to the horizontal Cieling are in the ſame poſition as on the Floor; to the vertical Planes they are horizontal; and to the inclined Cieling, IEFK, they are (as in all the other) perpendicular to the Interſections, IK and EF.

The Perpendicular *a b*, to that Plane, is ſo ſituated, that the Eye is in the Plane of Shade; wherefore, its Shadow cannot be determined as the reſt, being in a continuation of the Line, but may be thus found.

Through the Seat, ſ, of the Light on that Plane, draw ſ *B*, at pleaſure; and through *b*, where *a b* cuts the Plane, draw *b B*, perpendicular to *a b*, cuting ſ *B*, at *B*, and draw *B d*, parallel to *a b*, and *a d* to *b B*. Then, through *d*, draw O *e*, cuting ſ *B* produced, at *e*, and draw *c e* parallel to *b B*; which determines, *b c*, for the Shadow of *a b*, which could not be determined by the Ray O *c*.

Secondly. When the Plane is vertical, and inclined to the Picture.

Fig. 42. Let AB, be the Interſection of a vertical Plane, with the Floor, and D its Vaniſhing Point. The Center of the Picture is C; O is the luminous Point, and S, its Seat on the Floor.

To determine its Seat on the vertical Plane, draw CE, perpendicular, and equal to the Diſtance of the Picture. Join DE, and draw EF, perpendicular to DE, cuting the Horizontal Line at F; the Van. Point of perpendiculars to the Plane.

Then, draw SF, cuting AB, at r, and r ſ, perpendicular, cuting OF at ſ, the Seat of O, on that Plane; for, O ſ, repreſents a Perpendicular to the Plane.

Let ab, be a Line, perpendicular to the vertical Plane, its Vaniſhing Point is F.

To project its Shadow on the Plane; draw ſ b, through the Point b, where it cuts the Plane, indefinite; and through a, draw O c, cuting ſ b, produced, at c. b c, is the Shadow of a b, on the vertical Plane.

Thirdly. When the Plane is inclined to the Horizon, and to the Picture.

Fig. 43. Let AB, be the Interſection of a Plane, inclined to the Horizon and alſo to the Picture; C is the Center, and D the Vaniſhing Point of AB. O is the luminous Point, and S its Seat, on the Floor.

Find the Vaniſhing Point, F, of Lines perpendicular to the Interſection AB, (Prob. 4.) and draw SF, cuting it at r, as in the laſt.

Then, the Inclination of the Plane to the Horizon being known, find its Vaniſhing Line, DG, (by 5th of the ſame) and the Vaniſhing Point H, of Lines perpendicular to the Plane, (Prob. 2. Sect. 12.) which will be that Point, where a Perpendicular to the Vaniſhing Line, paſſing through C, the Center of the Picture, cuts GF, produced.

Draw r G, and OH, cuting it at ſ, the Seat of O, (the luminous Point) on that Plane. For, H is the Vaniſhing Point of Perpendiculars to the Plane, and r G repreſents a perpendicular to AB; wherefore OH cuts the Plane at ſ.

Becauſe, OS being perpendicular to the Horizon, and F, the Vaniſhing Point of S r, is perpendicular to AB; conſequently, GH, paſſing through F, alſo perpendicular (that is, parallel to OS) is the Vaniſhing Line of a Plane O ſ r S, paſſing through OS, perpendicular to the inclined Plane; and H, being the Vaniſhing Point of Perpendiculars to the Plane, OH muſt cut in r G, the common Section of the two Planes; therefore, ſ, where OH, cuts r G, is the Seat of the Point O, on the inclined Plane.

The Seat ſ, of the luminous Point being found, and a b, a given Line perpendicular to the Plane, its Shadow is determined, by drawing ſ c, through b, indefinite, and O c, through a, interſecting at c; giving b c, the Shadow of a b, as in all the foregoing.

In this Problem is contained the whole Theory of Projecting Shadows by Candle Light, in which there is not any difference to that of projecting Shadows by the Sun; in reſpect of Lines parallel or inclined to the Plane of Projection, there is very little difference in the Proceſs.

The chief difficulty in projecting Shadows, by Candle Light, is to find the Seat of the luminous Point; for which, the Rules given will anſwer in all Caſes, and in all poſitions of Planes, whatever, having the Interſection of the Plane, with ſome other, on which, the Seat is already determined, and the Inclination of the Planes, to each other, known.

Fig. 41.

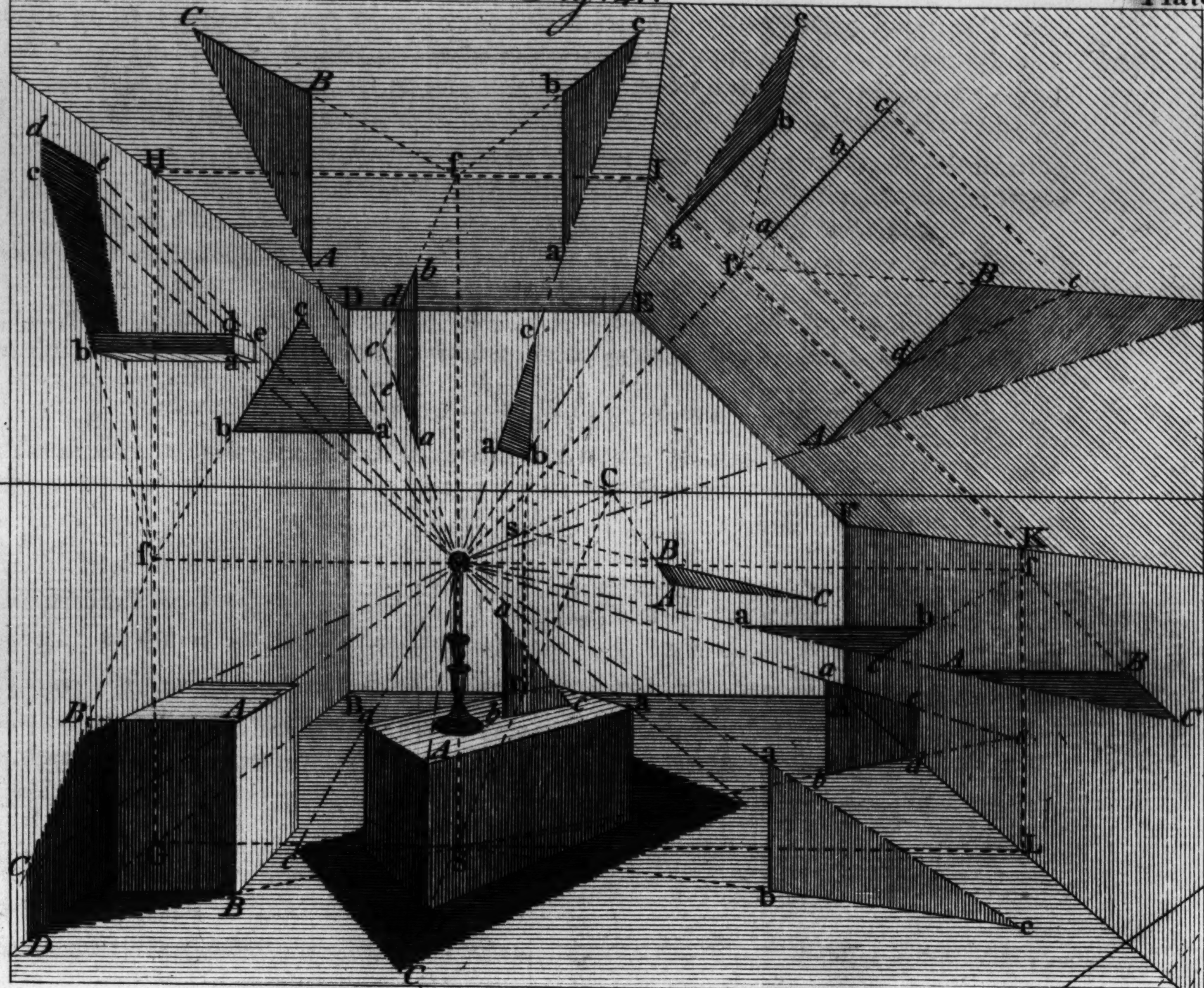


Fig. 43

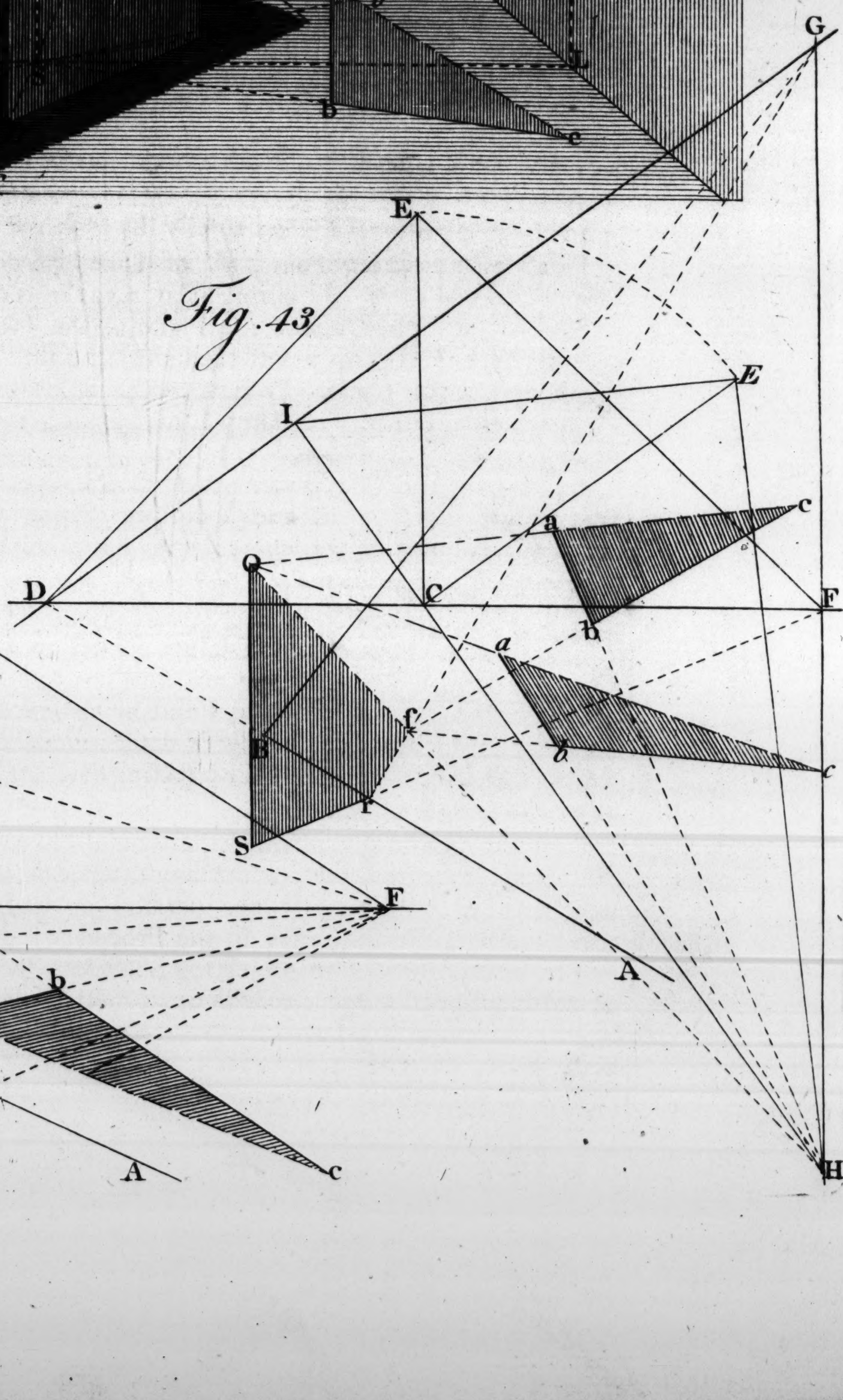
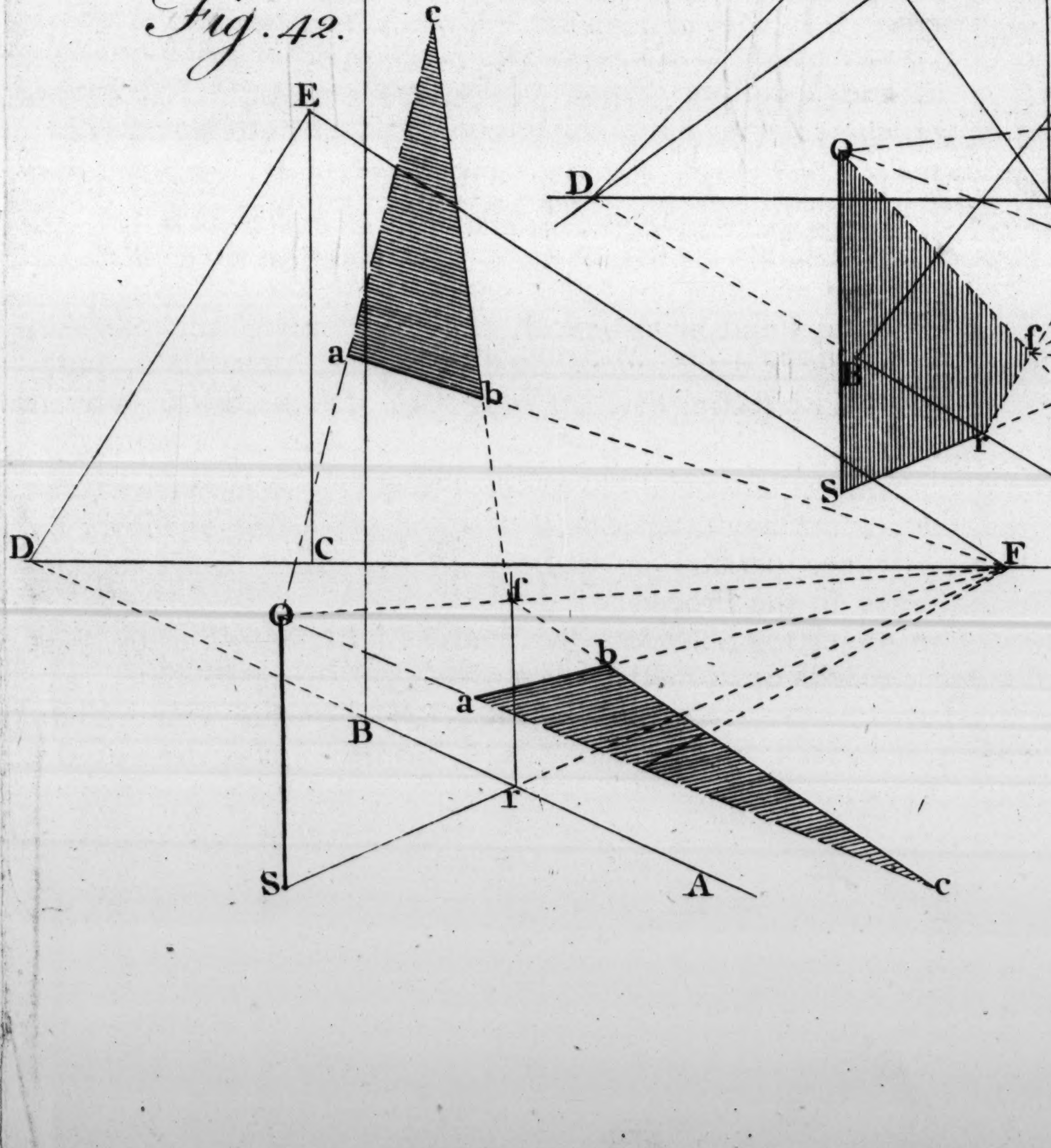


Fig. 42.



PROBLEM II.

Plate XLV.

To project the Shadows of Right Lines, parallel to the Plane of projection, however situated to the Picture.

The Shadows of Right Lines, on Planes to which they are parallel, whether they are projected by the Sun or by Candle Light, it is obvious, are always parallel to the Lines projecting the Shadows.

Fig. 44.

For, however the luminous Point be situated, or at whatever Distance, the Rays OA, OB, &c. generating the Plane of Shade, are all cut proportionally by the Line and the Plane of Projection,† consequently, seeing the extreme Rays, OA, OB, or any other, are so cut, as OA is to AC, so is OB to BD, it necessarily follows, that CD is parallel to AB (Case 2. 2. 6. El.)

† 2. 6. El.

Then, consequently, if the Line be parallel to the Picture, the Shadow is parallel on the Picture; that is, to the Vanishing Line of the Plane of projection; and consequently, however otherwise the Line be situated to the Picture, the Shadow, being parallel to the Line, has necessarily the same Vanishing Point. (Cor. 1. Theo. 5)

Let ACD be a right angled Parallelopiped, having the Face AC parallel to the Picture; the Sides CD, &c. are consequently perpendicular to the Picture, and parallel to the Floor, on which the Shadow is projected. Fig. 45. No. 1.

Let O, be the luminous Point, and S its Seat, on the Floor.

Through A, draw S b, indefinite, and Ob, cuting it at b; then, because BC is parallel to the Floor, and to the Picture, its Shadow is parallel to the Line.

Wherefore, draw b c, parallel to BC, and O c (through C) cuting it at c.

Then, because CD is perpendicular to the Picture, and parallel to the Plane of projection, C (the Center) is the Vanishing Point of its Shadow; wherefore, draw c C, which compleats the Shadow, as much as can be seen.

2. EH is another Parallelopiped obliquely situated to the Picture; having one Face so situated, that the luminous Point, is in its Plane, produced. No. 2.

Produce SE, and, through F, draw O f, cuting it at f, the Shadow of f.

Then, K being the Vanishing Point of FG, draw f K, and O g (through G) cuting it at g; f g is the Shadow of FG, to which it is parallel, having the same Vanishing Point. And, because GH vanishes at I, draw g I, cuting the Parallelopiped, which compleats the Shadow f g h.

To project the Shadows of Right Lines parallel to any other Plane, however situated to the Picture, has nothing more of variety in it; the seat of the Luminary being obtained on the Plane.

3 The Shadows of the Shelves, LM, &c. at the farther End of the Room, are thus obtained, being parallel to the Picture. No. 3.

Through the Feet, R, and T, draw right Lines, from S, cuting the Intersection, r t, at r and t; from which, draw Lines parallel to LR, and MT, indefinite. From O, draw through the extremes of the Shelves, L, N, &c. cuting them, at l, n, &c. from which Points, draw parallel to the Shelves, cuting the edge of the upright piece, at *l*, *n*, &c. and join, N *n*, O *o*, &c. as in the Figure.

4. The Shelves, against the side Wall, may have their Shadows projected after the same manner, with only this difference; that, the Shadows, instead of being drawn parallel to the edges of the Shelves, tend to the same Vanishing Point. No. 4.

As a b, the Shadow of AB, and c d of CD, tending to the Center.

5. For the Shadow of the single Shelf on the other Side, find the Seat, f, of the Light on the Wall (Prob. 1.) then, a b being perpendicular to the Plane, draw f a, indefinite, and O b, being produced, cuts it at *b*. Draw *b*V, cuting the Angle of the Room, at *c*, and join *c d*. No. 5.

Plate XLV. Fig. 45. No. 6. 6. For, the hanging Frame and Shelves, above, proceed thus.

Find *S*, the Seat of the Light on the Cieling; *AB* and *CD* being perpendicular, draw *SA*, *SC*, indefinite; and, through *B*, and *D*, draw O*B*, O*D*, cuting them at *b*, and *d*. Draw *bd*, the Shadow of *BD*; and, from the Center of the Picture (being the Vanishing Point of *BE* and *DF*) draw *be*, and *df*, the Shadows of *BE* and *DF*, on the Cieling.

Lastly; through *G* or *H*, draw O*g*, or O*h*, cuting *be*, or *df*, at *g* or *h*; through which, draw *gh*, parallel to *GH* (being parallel to the Horizon) and, through the extremes of the Shelves, draw Lines from O, cuting *gh* at *a* and *c*.

The rest, for that Shelf, is obvious.

The other, notwithstanding its inclination, is performed by the same means; only, instead of being parallel, the Shadow tends to the same vanishing Point, as the Shelf; or, both extremes being determined, the vanishing Point is useless. Their widths are determined after the same manner.

No. 7. 7. The Shadow of the Stool, on the Counter and Floor, is thus projected.

From S, the Seat of the luminous Point, draw Right Lines through the feet of the Frame, a, b, &c. till they cut the Intersection, AC, at *a*, *b*, *c*, *d*; from which Points, draw Perpendiculars; which, are the indefinite Shadows of the feet of the Frame, on the Side of the Counter, and if, through the Angles, e, f, &c. Lines are drawn from O, cuting the Perpendiculars, at *e*, *f*, &c. respectively, their lengths are determined.

The Shadow of the cross Frame is determined, by drawing Lines, from O, through their extremes, cuting the Shadows of the Legs, at *i*, *k*, &c. But, as the Shadow of the extreme i, falls on the Floor, if i k be parallel to the Picture, draw *ij*, parallel to the Horizon, or tending to the Vanishing Point of i k, till it cuts the Intersection AC at *j*; and join *jk*.

No. 8. 8. The Box on which the Candle stands has its upper Face, only, illumined, and consequently its Shadow, only, is projected, on the Floor, &c. by drawing Lines, from S, through the Seat of each Angle, on the Floor; and O m, O n, being produced, till they cut them, at *m* and *n*. O p, cuts the side of the Counter, in the perpendicular *rp*, and if *no* be drawn, to the Vanishing Point of n p, till it cuts AC, at *o*, and the point *q* being obtained, by the same means, *op* and *pq*, being joined, compleats its Shadow.

PROBLEM III.

To project the Shadows of Right Lines, any how inclined to the Plane of projection, and to the Picture.

This Problem I shall resolve also, by Examples, as in the foregoing.

The Vanishing Point of the Rays of Light, that is, the place of the luminous Point being determined, the process is the same as by Sun shine, having found the Vanishing Line of the Plane of Shade; which passes through the Vanishing Point of the Line, but not through the luminous Point.

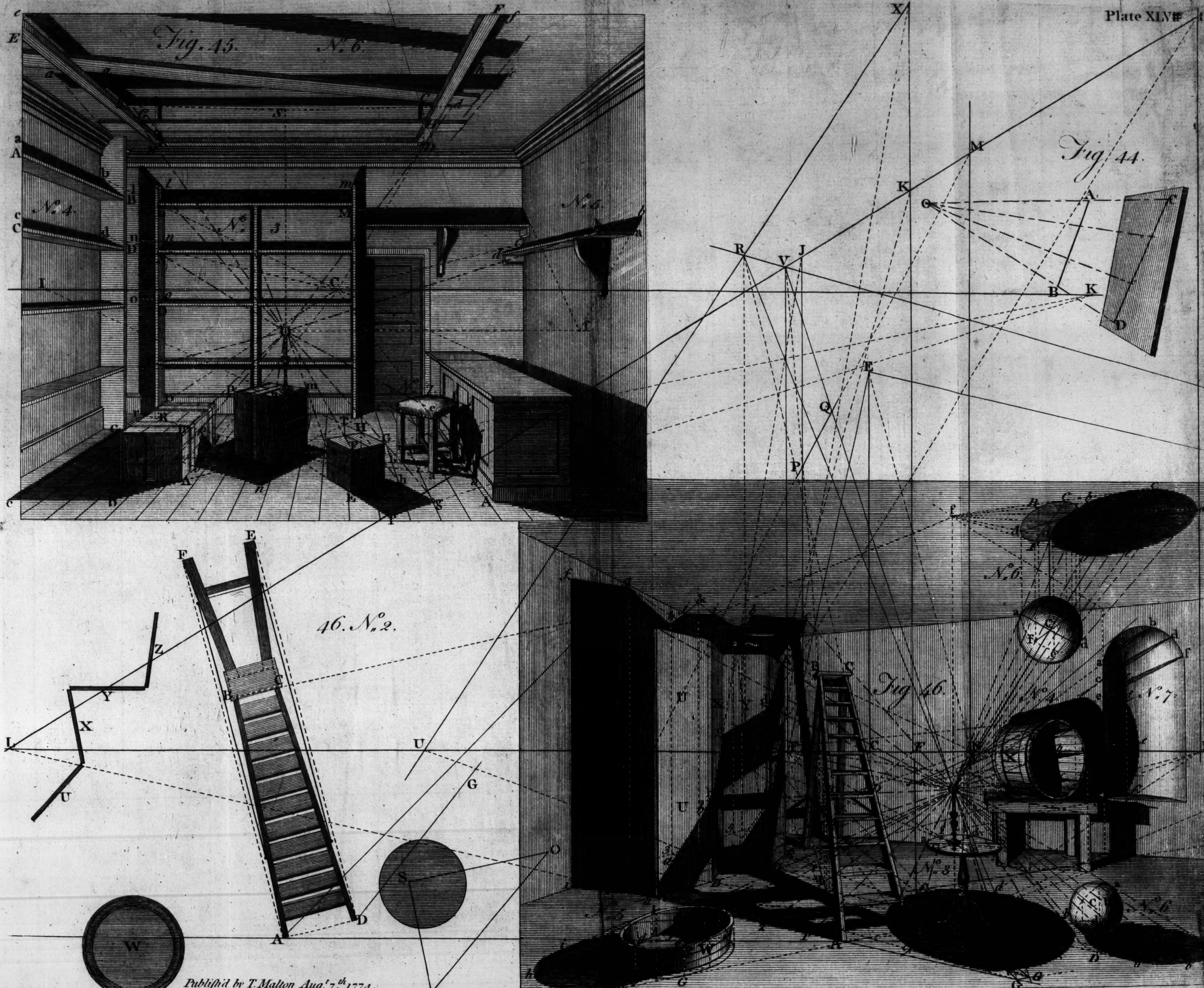
Fig. 46. ABCD represents a high pair of Steps, and UXYZ a large folding Screen, so situated, in respect of the Light, at O, that the Shadow of the inclined Sides, AB and CD, of the Steps, fall on the Screen.

Their position, in respect of the Steps, and also their inclination to each other, together with the Seat of the Light, on the Floor, are determined as in No. 2.

Let U, X, &c. be the Seats of the Leaves of the Screen; A, D, E, F, of the feet of the Steps, and, S the Seat of the Light; BC, is the Seat of the top of the steps, which being contracted, AB and CD, are the Seats of the inclination of the Sides, on the Floor, which cast their Shadows on the Screen; let their inclination to the Horizon be the Angle CDG.

Let V be the Vanishing Point of the Side, AB, of the Steps.

Plate XLVII



Publish'd by T. Malton Aug.t 7.th 1774.

Having found S, the Seat of the Light, also F, the Vanishing Point of the Seat of AB, on the Floor, draw FS, indefinite; VO, being produced, cuts it at G. Fig. 46.

Draw GA, and produce it to the Horizontal Line, cuting it at L, and draw VL, the Vanishing Line of the Plane of Shade, of AB.

AL cuts the Intersection, of the Leaf U with the Floor, at *a*; through H, the Vanishing Point of that Intersection, HI being drawn, perpendicular, is its Vanishing Line, which cuts VL at I; draw *a*I, cuting the Intersection of the Leaves, U and X, at b.

Then, where the Vanishing Line of the Leaf, X, cuts VL, at J, draw b J, cuting the Intersection of the Leaves X and Y, at c.

The Leaf Y, is parallel to the Picture; wherefore, draw c d parallel to the Vanishing Line, VL, cuting the next Intersection at d; and, lastly, draw d K, to the Point, K, where the Vanishing Line of the Leaf Z, cuts VL, which compleats the Shadow of AB, on the Screen.

The remainder of its Shadow falls behind the Screen, out of Sight.

The Shadow of the other Side, CD, whose Vanishing Point is R; falls only on the two Leaves, Y, and Z; and on the Wall, to g, mostly out of sight.

The Shadow of the Supporter, CE, is projected first to f, where it cuts the Wall; then, f g, being joined, compleats its Shadow.

For the Steps; from O, Lines are drawn through the extremes *A, B, C,* &c. on either Side, till they cut the Floor, or leaves of the Screen, at *a, b, c,* &c. and from, *a, b,* &c. to the Vanishing Point of the Steps, on the Floor, and on the Leaves, being parallel to them. Otherwise, they are drawn to that Point in the Vanishing Line, of each Leaf, in which, the Vanishing Line of the Plane of Shade of the Step cuts it. Or, drawing through both extremes, of each Step, as *C* and *D*, &c. cuting the Shadows of both Sides, the Vanishing Point is unnecessary; but when it falls within bounds, it is the most correct. Nevertheless, as every Line (although parallel to some other) has a different Vanishing Line of its Plane of Shade (the distance of the Light being finite) it may be dispensed with.

In this Case (viz. of Lines inclined to the Plane of Projection) the Vanishing Line of the Plane of Shade is necessary, when the Shadow is projected on various Planes, but is not so easily determined, as for Shadows projected by the Sun; nothing more being required than to draw a Right Line through the Vanishing Points, of the Line and of the Rays, because its distance is supposed infinite; and consequently, the same Vanishing Line of any Plane of Shade, serves for all Lines which are parallel. But, here, the Light being at a short distance, it cannot possibly be in the Vanishing Line, which is at an infinite distance; except apparently so, when it happens to be so situated, in respect of the Eye.

That VL is the Vanishing Line of the Plane of Shade of AB, is manifest; and is determined either by means of the horizontal Vanishing Line (HL) or the vertical Vanishing Line (MN) of the Wall, W, on which the Shadow would be projected, the Screen being out of the way. For, AB, cuts the Wall, at P, as the Floor at A; wherefore, VO represents a Line parallel to AB; which, is a Ray of Light in the same Plane with AB, cuting the Floor, at G, and the Wall, at Q.

Consequently, GA, produced, is the Shadow of AB, on the Floor; and, PQ produced, is its Shadow on the Wall; the former cuts the Vanishing Line of the Floor, at L, the latter cuts the vertical Vanishing Line at M; both which, are in the same Right Line passing through V; and, since L and M are Vanishing Points of the Shadow, VL is, consequently, the Vanishing Line of the Plane of Shade, occasioned by the Line AB (Theorem 11.)

2. The Shadow of the Screen, on the Wall and Cieling, may be thus projected.

Through, *A, B,* &c. at the bottom of the Screen, draw S*A*, S*B*, &c. and produce them to the Intersection of the Wall (W) with the Floor, cuting it at 1, 2, &c. from which Points, draw Perpendiculars up the Wall; and, through the Angles *F, G,* &c. at the top, draw O*F*, O*G*, &c. cuting the Perpendiculars, corresponding with *AF*, &c. at *f, g,* &c. and draw *f g, g h,* &c.

But the Ray O*I*, cuts the Cieling; wherefore, having found s, the Seat of the Light on the Cieling, draw s i, to that Point where the Perpendicular, from 4, cuts the Intersection of the Wall with the Cieling; and O*I*, produced, limits the Shadow, at *i*; let it be produced, also, till it cuts the Perpendicular at k, and join *h* k, cuting the Intersection at *j*, and join *j i*.

Plate XLV. Fig. 46. This process would be indispensibly necessary, provided the Lines, HI &c. were not parallel to the Ceiling; but being so, they are drawn either parallel to the Line (as for *HI*) or to the same Vanishing Point (as for *IK*). *A i f g h j i k l*, are the extremes of the Shadow of the Screen, on the Floor, Wall, and Cieling.

By the same means, the Shadow of AB, may be projected on the Screen, thus. Draw S*A*, S*B*, &c. cuting AF, the Seat of AB on the Floor, at n, o, p, and q; from which, draw Perpendiculars, cuting AB, at *A*, *B*, &c. through which Points, draw O*A* a, &c. cuting the corresponding angles of the Screen, at a, b, c, &c. which, being joined by Right Lines, give the same Shadow as before.

This process, though shorter, is by no means so correct and masterly; but, as it is performed in less room, it may be applied when the inclination is such, that the Vanishing Points are very remote.

3. The Shadow of the Table, on which the Candle stands may be thus determined.

No. 3. Having obtained the Seat (s) of the Light on the Table, and (S) on the Floor, through s, draw a b, parallel to the Horizon, also c d, and e f at pleasure, cuting the Horizontal Line, at H, and, F; and through S, on the Floor, draw H c, and F e, indefinite; also *a b*, parallel to a b.

Then, Rays drawn, from O, through the extremes of those Lines, viz. O a, O c, &c. (being produced) will cut the corresponding Lines in the Shadow of each, respectively, by which means as many points may be obtained as are necessary, for obtaining the true curve of the Shadow, which is a Circle, the Table being circular.

The Candle standing towards one edge of the Table, the Rays proceeding from it, around its Circumference, form a scalene, or oblique Cone; which being projected to the Floor (to which the Table is supposed parallel) the Section, thereon, (which is its Shadow) is consequently a Circle; and, being seen oblique, its Representation is an Ellipsis (Theo. 2. Sect. 5.)

No. 4. 4. The Shadow of the concave edge of the hollow Cylinder (X) on the interior Surface, may be projected, in the following manner. Find *s*, the seat of the Light on the Plane of its Base, and draw *s* a, *s* b, &c. at pleasure, cuting the circumference on both sides; from the Points, d, e, &c. draw Right Lines to the Vanishing Point of the sides of the Cylinder, and draw O a, O b, &c, cuting them, respectively, at *a*, *b*, &c. through which, the contour of the Shadow may be described.

No. 5. 5. For the Shadow of the edge, of the conical Vessel (W) on the interior Surface.

If Right Lines be drawn from *S*, the Seat of the Light, on the Plane of its upper Base, cuting it on both sides, and making vertical Sections through them; then, draw O a, &c. and produce them, cuting the opposite, corresponding lines, at *a*, *b*, &c. through which, the curve of the Shadow may be described.

Where they fall on the Bottom, join *c d*, &c. O e, O f, &c. cuts those Lines, at *e*, *f*, &c. through which, the curve of the Shadow is described.

By taking several Points (g, h, &c.) in the exterior Curve, and finding their Seats on the Floor, its Shadow (*g h i*) may be described thereon.

6. The Shadows of Spheres, it is not easy to describe, with certainty.

No. 6. If a Tangent be drawn, from the Light (O) to any part of its Surface (as a) and, through that Point, a Section, by a Plane, be described, perpendicular to the Axis, (O C), (which, I freely own, is not easy to do, being obliquely situated) then, take as many Points in its Circumference as are necessary (a, b, d, &c.) and find their Seats, (*A*, *B*, *D*, &c.) on the Floor, or Cieling; and, through S, or s, the Seat of the Light, draw S *A*, S *B*, &c. and O a, O b, &c. cuting them, respectively, at *a*, *b*, *d*, &c. a Curve described through *a*, *b*, &c. will be an Ellipsis, the true Shadow of the Sphere; being an oblique section of the Cone of Rays.

7. The Shadow in the Nich is described as by Sunshine, with little variation.

No. 7. Find the Seat (*s*) of the Light on its Plane, (Prob. 1.) from which, draw the Ordinates, a b, c d &c; on which Ordinates describe Sections through the Head, &c. perpendicular to the Plane, and draw O a, O c, &c. giving the Points *a*, *c*, &c. A Curve described through those Points is the true contour of the Shadow, in the Head of the Nich. The rest is evident.

2

SECTION VI.

Of reflected Light; and of the reflected Images of Objects, on Water, and polished Surfaces; Keeping, &c.

Plate XLVI.

IN treating on this Subject, it may be necessary to consider, in the first place, what is meant by Reflection, simply, or considered abstractedly.

REFLECTION, in a physical sense, signifies a rebounding of Matter, by Percussion. i. e. When an elastic Body,* in motion, strikes another Body, also elastic; it rebounds, from that other Body, in a different direction, from that in which it was at first impelled; making an Angle, with its first direction, greater or less, according to the obliquity in which it strikes the Surface of the other Body.

If a Globe strikes a Plane, or the Surface of another Globe (or any Surface whatever) perpendicularly, it will rebound from the other Surface, perpendicularly, in direct opposition to its first, or incident motion; as if A falls, perpendicular, to the Plane EF, striking it at B, it will rebound again, from B, towards A. Fig. 47. But, it is found, from experiment, that if the incident motion be from C to B, oblique to the Plane, or other Surface, it will rebound, or be reflected from B towards D, also oblique; making the Angle ABC, with a perpendicular, at that Point (called the Angle of Incidence) equal to ABD (called the Angle of Reflection) or, which is the same thing, the Angle CBE, in which it inclines to the Plane, in its incident motion, is equal to DBF, in which it reflects from the Plane. And this is the same, in all positions of the Plane, whether horizontal, vertical, or inclined. Hence, Light is said to be reflected, from one Surface to another; and, probably, from that consideration it is imagined to be material.

Without taking Matter into consideration, it is certain, that any Surface (not wholly opake) being opposed to a luminous Body, becomes illumined itself; and, in an inferior degree, illumines other Objects, in vicinity with it.

The first and grand instance, of which, is the Moon and other Planets; as they are more or less illumined, by the Sun, or rather, the more their illumined Surfaces are towards the Earth, the more the Earth is illumined; even to such a degree (though but reflected) as to project Shadows, strongly defined. The Case is perfectly similar in respect of other Bodies, on the Earth. For, however any Surface be situated to the Sun, being illumined by its Light, that Surface illumines others, which are near it, more or less, according as that Surface is situated to the Sun, and as they are situated in respect of each other. Without which, Bodies, or the Surfaces of Bodies, which are not illumined, directly from the Sun or other luminous Body, would be so totally immersed in Shade as not to be visible, excepting their exterior Figure or Outline.

Now, admiting Rays of Light to be emited from any luminous Body, at S, and falling, directly, on another Body, or plane Surface, at A, they are said to be reflected, directly, again, towards S; but, falling on it oblique, as at B, they are reflected towards D, making the Angle DBE equal SBA. Fig. 48.

Hence arises an objection to the Newtonian System, respecting the reflection of Light, from one Body to another, and from that other Body to the Eye; by which means, only, it is conjectured, Objects, not illumined, become visible.

Let AB be a Plane Surface, directly opposed to the Sun; and, suppose AB the utmost limits of the Plane. Let X be an Object, having one plane Face (CD) parallel to AB; and suppose no other Object, or Body, near.

* By Elastic, in this Place, is meant, simply, hard Bodies only, as Stones, Metals, &c. of which, some are more elastic than others. Lead, or pure Gold, yields to the stroke, and therefore, does not rebound like Tin, or Tin like Copper, nor that like Iron, or hard Steel. So Clay will not rebound like Free Stone, nor that like Marble; because, after Percussion, the more remote parts of Matter are still in motion, till the Body is compressed by the Stroke; which deprives it, either wholly or in part, of Motion; and consequently, it rebounds with less force, or not at all.

Now,

Plate XLVI. Now, the diſtance of the Sun being ſuppoſed infinite, its Rays, conſequently, fall perpendicularly on AB, in every part.

Fig. 48. Let SA, SB, &c. repreſent Rays of Light from the Sun. Then, according to the Maxim laid down, they are reflected, directly back again, towards S (which, being material, is ſomewhat repugnant to our Ideas of Matter, ſeeing that, it is continually flowing, with equal and unremited velocity, from the Sun to AB) conſequently, the Object X being ſituated ſo, that none of the reflected Rays can poſſibly fall on the Surface CD, that Surface is wholly inviſible, to any Eye at E, or *E*. Quere, whether it would be viſible or not.

I am of opinion that it would be clearly viſible, to any Eye, on this Side CD; not only in reſpect of its Figure, but that, the Surface, CD, would be illumined, by means of Reflection, from AB; when, according to the general Maxim, it could not, ſeeing no Rays, from AB, are reflected to CD. Yet, I am fully perſuaded that it may be ſeen; not only, directly, at E, but alſo oblique, in any direction, as at *E*; although it is manifeſt, that the Rays, from AB, muſt be reflected oblique, to CD, and again to E, or *E*, in various Angles and Directions; and yet, the original Rays all fall perpendicularly on AB.

Now, if CD be ſeen, at all, it is manifeſt it muſt be illumined; and it is evident, that it cannot be ſo from the Sun, directly, conſequently, it muſt be from Reflection; and ſince no other Body is near, it muſt be from the Surface AB, Hence, then, it is manifeſt, that, Light is reflected in other directions, from one Object to another. How or by what means it is reflected, I will not attempt to enquire; but ſhall only make a few Obſervations, reſpecting the effects it produces, on Objects, ſo eſſential to the perfecting a Picture, or true Portrait of Nature.

It had, formerly, been cuſtomary, with many, to repreſent Objects, immerſed in Shade, ſo very obſcurely, as ſcarce to be diſtinguiſhable; whereas, it is not ſo in Nature. In clear Day-light, when the Sun does not ſhine out, we ſee Objects in their true Colours, and every part is diſtinct; but, when the Sun breaks out, and darts its Rays on thoſe parts which are oppoſite, the Colours are more intenſely vivid, occaſioned by the refulgent luſtre of the Sun-Beams. Nevertheleſs, thoſe parts which are prevented from receiving that additional Light, and appear to be in Shade, cannot poſſibly be deprived of what Light they had before, but muſt rather receive additional Light on them; the difference, then, can only ariſe, from the ſplendor of the ſurrounding Light, which dazzles the Eye, and renders thoſe parts obſcure which, before, were diſtinctly ſeen. But, when out of the full glow of Sun-ſhine, we perceive every Object or parts of Objects, which are in Shade, as diſtinctly as before; and in the ſame Colours, though greatly different from thoſe on which the Sun ſhines.

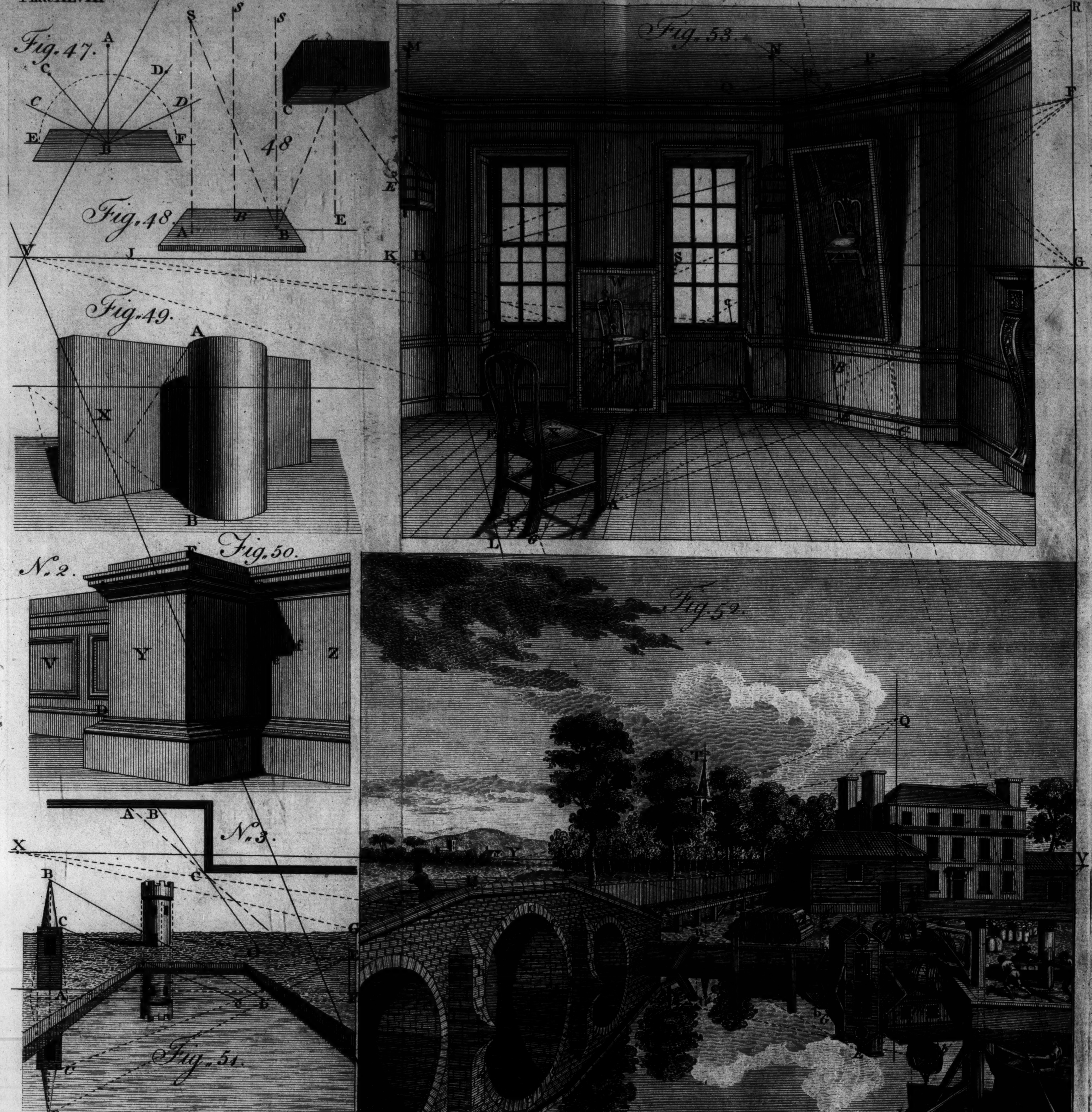
Some, again, of late Years, run into the oppoſite extreme, and make too little diſtinction, between the fulleſt Light and the ſtrongeſt Shade, by which means, their Pieces look flat, and do not produce, on the Mind, a juſt Idea of what is intended. For, although it is certain, that, in Sunſhine, every part receives additional Light, yet, the contraſt is ſo ſtrong, from the brilliancy of the Colours, where the Rays fall, that, the other Parts appear as if deprived of Light, in compariſon with theſe; and, ſince that vivid luſtre cannot be given by Art, and Colours, the difference muſt be made by keeping the other parts ſomewhat under.

Reſpecting the degree of reflected Light on Objects, it is not eaſy to determine; that being more or leſs, according to the ſituation of the Object, in reſpect of others; alſo, according as that other Object is ſituated to the Light.

If one Object be ſo ſituated to another, that the Surface of one, being directly oppoſed to the Light, is alſo much oppoſed to the ſhaded ſide of the other Object; the reflected Light, on the other, will be ſtronger than when they are more obliquely ſituated; either the one in reſpect of the Light, or the other, in reſpect of the illumined Surface.

Fig. 49. For example. The Cylinder, AB, being ſituated near the Wall X, on which the Light is direct; the ſhaided Edge (AB) has a much ſtronger reflection, than if the

Plate XLVIII



Miller sculp.

Publish'd by Thos. Malton Sept. 1st 1775.

the Plane (X) was more inclined to the Light; or, being removed, and no other reflecting Surface near, there would be no Reflection on the Column, in this position of the Light, save what is received obliquely, from the Ground.

Respecting plane Surfaces. We frequently see (in Prints and Drawings) the shaded face of a Pedestal, &c. very dark, at the hither edge, and gradated towards the other, so, as to give it the appearance of concavity; which cannot be, when no other Object is near. But, when one part of an Object projects from the other, as the Plane X from Z, the Plane Z, being much opposed to a strong Light, and consequently the Shadow of the Plane X, on Z, is very narrow; then, the Light, reflected from Z to X, will be strongest where it joins to Z, and gradually lessened, towards the other edge; but not in so extravagant a degree, as may be seen in various Prints; particularly in Mr. Kirby's Perspective of Architecture. Fig. 50

Now, the reason for this is extremely obvious; because, the vicinity of the part which joins to Z, is greater than the more remote; and consequently, the reflected Light is more languid on the remote parts. But, if the Light was very oblique on Z, so as to cast a great breadth of Shadow, on the Plane X; in that Case, the Light reflected, to the Plane X, will be more faint; insomuch that, there is barely a distinction between the Shadow, and the Object occasioning that Shadow; which there always should be, in some degree, the Shadow being darkest. Because it receives little or no advantage from Reflection, which the Object does.

In respect of Mouldings; the edge of the Facia, AB, being in full Light, casts its Shadow on the Cavetto below; consequently the Light, from below, is reflected again on the Mouldings; and the Ovolo, which otherwise would be darkest at the bottom, is reversed, the Shade being strongest on the upper part. On the returning Moulding; the under Facia or Planceer (BC) by means of Reflection, from below, is brighter than the vertical Facia, over it; also the vertical Face of each Fillet &c. is darker than the horizontal; which in other Cases, having the Light directly on them, is consequently brighter.

In a Nich (see Plate 43.) the Shadow is strongest at the edge or outline, and gradually softened towards that edge of the Nich, which occasions the Shadow; owing to the strong Reflection from the other Side, on which the Light falls, direct.

The Shade (without Sun-shine) on a convex or concave half Cylinder is the same; each may be the other, by supposing the Light on either Hand.

More may be said in respect of reflected Light; but, if the Observations I have made be well attended to, it will be found sufficient, for any purpose whatever.

Of the reflected Images of Objects on Water, and polished Surfaces.

The Reflections of Objects on the Surface of still Water gives (where Water is represented) transparency to the Water, and perfection to the Picture; which, if well managed, has a pleasing effect.

It is obvious, whether by means of Rays of Light, reflected to and again, from the Object to the Water, and from thence to the Eye, or otherwise, that, if AD be the Surface of Water (or a polished Mirrour) and AB, an Object, standing erect, we shall see the Image of the Point B, on the Water, at b (the Eye being at E) where the Right Lines, drawn from B and E to the Water, at b, make equal Angles with its Surface. Consequently, if Eb be produced, meeting BA, produced, at, *B*, *B*A, equal AB, is the apparent place of the Object, which appears on the Surface of the Water. Hence it is manifest, that the Image of the Object AB, represented on the Surface of the Water (and which, in real length, is Ab) is represented equal to the Object itself. For, it is evident, the Point A is common to both, and because B, appears at *B*, it appears just as much within the Water, as its real place is from the Water viz. A*B* equal AB, and A*C* equal AC, &c. Fig. 51

Hence we have an unerring Rule, for representing the reflected Images of Objects on polished Surfaces. But it must be observed, that, if the Object be at some distance, beyond the Water, the measure of the Object must be applied from its Seat, on the Plane of the Surface of the Water, not from the Water edge, as when it is in the Water or close to it. Also, that the Image represented on the Water, is not

Plate XLVI. ſimilar to the Repreſentation of the Object, on the Picture, which I ſhall ſhew; except when a plane Figure only is repreſented, parallel to the Picture.

E X A M P L E S.

Fig. 52. Let X be an Object in the Water, having one Face, X, parallel to the Picture.

No. 1. Produce the Sides, BA and CD, making *AB* and *CD* reſpectively equal to them; alſo, make the perpendicular *EF* equal to EF, and join *BE* and *CE*; i. e. make the Figure A*E*D ſimilar to AED, inverted, from the Line AD where the Surface of the Water cuts the Object. The Image of the Door, &c, is alſo ſimilar.

For the Face AG, being perpendicular to the Picture, the horizontal Line, BG, vaniſhes in the Center, C; wherefore, BG being parallel to the Water, its Image will be parallel to the Line, and conſequently, it has the ſame Vaniſhing Point.

Draw *B*C, meeting GH, produced at *G*, which compleats the Image of that Face, and which, it is obvious, is not ſimilar to the Repreſentation ABGH. Likewiſe, a ſide of the Roof is repreſented in the Object, which it is manifeſt cannot be ſeen in the Water, by Reflection.

No. 2. The Piles, which are perpendicular, have their Images alſo equal, each to the Repreſentation; likewiſe, the Pile W is reflected equal, and equally inclined, being parallel to the Picture. But, being otherwiſe inclined, as Y and Z, one hanging towards, the other from the Picture, the Seats of their tops, on the Surface of the Water, being determined, at a, produce the Perpendicular ba, making a*b* equal to ab, and draw from *b*, to the Timbers, where they are cut by the Surface.

The Vaniſhing Point of the reflected Image is at the ſame diſtance on the other ſide of the horizontal Line, as that of the Object, whether it be above or below it.

No. 3. The Image of the Bridge, on the Water, may be obtained after the ſame manner, in reſpect of the Piers. The images of all horizontal Lines tend to the ſame Point, in the horizontal Vaniſhing Line, as the Repreſentations; for, Right Lines, being parallel to the Surface of the Water, their Images have the ſame Vaniſhing Points, reſpectively.

The Images of the inclined Lines (IK &c. at the Top) are got by means of Perpendiculars at each extreme of the Line; or by their Vaniſhing Point (Q) which is always at the ſame diſtance from the Horizontal Line, on the contrary Side, as that of the Repreſentation (at P).

For the Arches; draw as many perpendicular Lines, a b, c d &c. as are neceſſary, and produce them in the Water; making each equal to its correſponding one, *a*b equal ab, &c. by which means, as many Points, in the Curve, as you pleaſe, may be obtained, and the image of the Arch deſcribed through them.

No. 4. The Crane &c, on the Wharf, hanging over the Water, has its Image reflected, by means of perpendicular Lines from each part, and finding their Seats on the Surface of the Water, repeating the ſame meaſure downwards.

For the Hogſhead; draw as many Lines (repreſenting lines parallel to the Horizon) as are neceſſary, tending to the Vaniſhing Point V of the horizontal Diameter a b; then, by means of perpendiculars from each extreme of thoſe Lines, their Images are acquired, and the curve of the Head deſcribed through them. If other Lines are drawn, from one head to the other, to the Vaniſhing Point of its Axe (X) the whole Image may be deſcribed, as in the Figure.

No. 5. The Warehouſes, &c. ſtanding from the front of the Wharf, have their reflected Images deſcribed as the other Objects; by ſuppoſing the Surface of the Water produced, and the Seat of each perpendicular Line determined thereon; as A, B, C, of the corners of the Warehouſe W; which are made equal, downward, from their Seats, (*a*, *b*, *c*,) reſpectively, and the horizontal Lines have the ſame Vaniſhing Points.

No. 6. The Spire of the Church, at T, is reflected on the Water; becauſe, its Seat S, on the level of the Water, being obtained, the height, ST, being applied downward reaches the Water, otherwiſe it would not.

Thus much may ſuffice, for Reflection on a horizontal Surface; which is the ſame, whether it be a clear Fluid (as Water) or other plane Mirrour.

2

Secondly. Of Reflection in Mirrours, vertical or inclined.

Let W be a Mirrour, parallel to the Picture, whose center is S; and X, an Object, whose reflected Image is required. Fig. 53.

In this Case, it must be observed, (as in the former) that the Image, of any Point in an Object, is reflected to, the same distance from its Seat on the Mirrour, as the Point from its Seat; so, that distance is, here, represented, and its Seat obtained, pespectively; and, because the reflecting Surface is parallel to the Picture, the Center is, consequently, the Vanishing Point of Lines perpendicular thereto.

Draw Lines, from the Feet, A, D, &c. of the Object to S; and, where they cut the Intersection of the Mirrour with the Floor, at a, d, &c. make a *a* represent a length equal Aa, &c. and draw *a b* &c. perpendicular; then, BS and CS, cuting those Perpendiculars, respectively, give their reflected Images. For, as every Object appears to be as far on the other side, as it really is on this side, from the Mirrour; consequently, in this Case, it must be so perspectively.

The reflected Images of horizontal Lines, as the rails of the Chair, &c. (in this Case) tend to a Vanishing Point in the Horizontal Line, at the same distance from the Center of the Picture, on the contrary side, as the Vanishing Point of the Representation. If V be the Vanishing Point of BC, the front Rail; then, SY being made equal to SV, Y in the Vanishing Point of b c, the reflected Image of B C.

In a vertical Mirrour, inclined to the Picture there is no difference in the process, but only in the Vanishing Point; and, if the Vanishing Point of Perpendiculars to the Mirrour be determined, 'tis the same in all inclinations.

Let Z be a Mirrour inclined to the Horizon, and casually inclined to the Picture. Fig. 54.
It is required to find the reflected Image of the Chair X, on the Mirrour Z.

V being the Vanishing Point of the bottom of the Mirrour, which is horizontal, find the Vanishing Line, VL, of the Mirrour, its inclination to the Horizon being known (Prob. 5. Sect. 3.) and also, the Vanishing Point F, of Lines perpendicular to the Mirrour (Prob. 2. Sect. 12.) Then, having found the Intersection (*CD*) of the Mirrour with the Floor, produced, and G, the Vanishing Point of Perpendiculars thereto, draw AG, DG, &c. cuting *CD*, at *A*, *D*, &c. and through them, draw from L, the Vanishing Point of the sides of the Mirrour, indefinite.

Draw AF, DF, &c. cuting them, respectively, in the corresponding Points, a, d, &c. (their Seats on the Mirrour) beyond which, their Images are represented equal, in Perspective; by making *a* a, represent an equal measure as a A, &c.

Find the Vanishing Point (O) of the Image of AB, &c. i. e. of Lines making a known Angle with the Mirrour. Draw O*a*, O*d*, &c. indefinite and BG, CG, &c. cuting them in their respective Images on the Mirrour.

And thus, as many Points (as e, of E) may be determined as occasion requires, from their Seats on the Floor; by which means, the reflected Image of any Object, whatever, may be represented on the Mirrour; as the Cage, hanging from the Cieling, &c. or the Door, &c. on the opposite side of the Room.

Of Keeping; or the effect of Distance.

The Term, Keeping, in the Art of Painting, in general, is used to signify a just and proper subordination of all the parts of a Picture to the principal Object; in respect of Magnitude, Colour and distinctness of Parts.

The magnitudes of Objects, in respect of each other, perspectively, are various, according to the Station from which they are viewed; and consequently, the subordination of Colour, &c. is not in proportion to the Objects, but to their Distance from the Eye. It is absolutely impossible to lie down Rules, by which the Artist may, with certainty, produce the desired effect; seeing that, in hazy or foggy weather, or in a misty morning, &c. Objects are less distinct, at a short Distance, than, in a clear Day, at a much greater; wherefore, no proportion or degree can be determined. If Objects, of known magnitude, appear not to be far distant, in the Picture, and yet, their parts not distinctly defined, in comparison of others, in the Fore-ground; it implies, that the Air is more gross and hazy, than if the parts were more perfect.

Keeping

Keeping is, in a great measure, synonimous with Aireal Perspective; which signifies a diminution of Light, Colour, and distinctness of the parts of Objects, in a regular gradation, as linear Perspective of Magnitude; owing to the effect of Air, between the Eye and the Object; which, being a Medium, obstructs the sight, in some degree, at any Distance; consequently, as there must be a greater quantity of Air between the Eye and distant Objects than near ones, so their parts are less discernable; the Lights and Shades are insensibly mixed, and, at a great Distance, it is all a confused jumble, of Light, Shade, and Colour, without distinction.

It is customary with many, in delineating, and may frequently be seen in Architectural Designs, &c. to make a considerable difference, in Teint, between one plane Surface and another; when the one, in the Original Object, recedes but a few Inches, which is absurd; for, if that sudden gradation of effect was continued to the distance of a few Yards, it would become totally black, before it was possible for the Air to have any apparent effect on it. Now it is far from being so in Nature, which is obvious to any Eye; for, if the materials are clean, and of an even and uniform Colour (without which no judgment can be made) I will ven-
Fig. 50 ture to affirm, that two Plane Surfaces (V and Y) parallel between themselves, at several Feet distance from each other, and having a full Light on them, cannot be
No. 2. distinguished one from the other, nor so much as the Line or Edge (AD) seen, at a proper distance for delineating; provided the Light be not (on this Side) so very oblique (as at S, No. 3.) as to shade the other, beyond where the Line BC cuts it. I say, the Eye being so situated (at E) that the Angle C, appears to cut the receding Plane, at B, they will appear as one continued Surface. Nevertheless, it may be necessary, in delineating, to make a distinction, in proportion to the Distance; but, if the Plane V has Mouldings, &c. on it, being cut, apparently, by the edge of the other, the Line is sufficiently defined without it.

As it must be obvious, that no positive Rule can be prescribed, I shall just give an Example or two, and conclude the Subject; and with it, the Book.

In respect of Magnitude; it is evident, that Objects may appear, in Perspective, to have any proportion to each other, although not greatly distant. For Example; in Plate 27, of Chelsea College, the farthest Building, with a Pediment, is not above two eleventh parts of the hither one (to which it is equal) in height; and yet it does not offend the Eye, nor appear at too great a Distance. But, if the gradation of Light was in proportion, it would be too great; because, not merely the height is to be considered, but the square of the height, which would reduce it to the proportion of thirty to one; but where we shall fix the standard for unity, I am at a loss to devise.

Now, if we were at twice the Distance from the hither Building (the distance between the two Objects being about twice that) the proportion of one to the other, would not be much greater than one to three; the gradation of Light, in that case, would be nearly as one to ten, but one third part of the former. Yet, I presume, that in two Pictures so delineated, it would not be advisable to make that difference in the effect of Aireal Perspective.

In short, as it is absolutely impossible to fix any criterion to determine the Ratio of the gradation of Light and the effect of Distance, but must ever be at the discretion of the Artist, I shall only observe, that the Objects in the Fore-ground, as the hither end of the Bridge (Plate 46) being supposed near the Picture, must have its parts distinct and perfect, with strong Lights and Shades, which gradate to the other End. The Buildings on the Wharf, &c. are less perfect, each being less so, as it recedes from the Eye. The Church, at a Distance, the Trees and Hills, one beyond another, must consequently be less and less perfect, according to their Distance; till at last they are scarce distinguishable from the Sky. For all which, the Artist can have no other Rule, than carefully and judiciously to copy Nature (the best Mistress) which, in that Case, is not always uniform, but infinitely variable.

F I N I S.